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MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-5

Sequence & Series

ANSWERKEY (CAPS-5)

1.	(A)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(C)	**6.**	(A)	**7.**	(A)
8.	(AB)	**9.**	(ABCD)	**10.**	(BD)	**11.**	(BC)	**12.**	(BCD)	**13.**	(ABC)	**14.**	(ABC)
15.	54	**16.**	51	**17.**	4	**18.**	50	**19.**	8	**20.**	0	**21.**	65
22.	7	**23.**	(C)	**24.**	$-\dfrac{a(p+q)q}{p-1}$			**25.**	$-(p+q)$				

SCQ (Single Correct Type) :

1. The sequence a_1, a_2, a_3 ,....... satisfies $a_1 = 1, a_2 = 2$ and $a_{n+2} = \dfrac{2}{a_{n+1}} + a_n$, n = 1, 2, 3, ... the value of $\dfrac{2^{2009}}{2011}\cdot a_{2012}$ is

(A) ${}^{2010}C_{1005}$ (B) ${}^{2011}C_{1006}$ (C) ${}^{2011}C_{1005}$ (D) ${}^{2012}C_{1006}$

Ans. (A)

Sol. $a_{n+2} = \dfrac{2}{a_{n+1}} + a_n$

$\Rightarrow a_{n+2}\cdot a_{n+1} = 2 + a_n \cdot a_{n+1}$

Let $T_n = a_n \, . \, a_{n+1}$

$\Rightarrow \quad T_1 = 2$

And $T_{n+1} = 2 + T_n$

(This is A.P. with first term 2 and common difference 2)

$\Rightarrow \quad T_n = 2 + (n-1)2 = 2n$

$\Rightarrow \quad a_n \, . \, a_{n+1} = 2n$

$\Rightarrow \quad a_n = \dfrac{n-1}{n-2} a_{n-2}$

Now, $a_{2012} = \dfrac{2011}{2010}\cdot\dfrac{2009}{2008}\cdots\dfrac{3}{2}\cdot a_2$

$= \dfrac{2(2011!)}{(2^{1005}\cdot 1005!)^2} = \dfrac{2011}{2^{2009}}\cdot\dfrac{2010!}{(1005!)^2} = \dfrac{2011}{2^{2009}}\cdot{}^{2010}C_{1005}$

2. Let $a_1, a_2, a_3, \ldots a_n$ be in G.P. If the area bounded by the curves $y^2 = 4a_n x$ and $y^2 = 4a_n(a_n - x)$ be A_n, then the sequence $A_1, A_2, A_3, \ldots, A_n$ are in

(A) A. P. (B) G. P. (C) H. P. (D) None of these

Ans. (B)

Sol. $A_n = 2\int_0^{a_n/2} 2\sqrt{a_n}\sqrt{x}dx + 2\int_{a_n/2}^{a_n} 2\sqrt{a_n}\sqrt{a_n - x}dx$

$A_n = 4\sqrt{a_n}\times\dfrac{2}{3}\left[x^{2/3}\right]_0^{a_n/2} - 4\sqrt{a_n}\times\dfrac{2}{3}\left[(a_n - x)^{2/3}\right]_{-a_n/2}^{a_n}$

$$A_n = \frac{8}{3}\sqrt{a_n}\frac{a_n\sqrt{a_n}}{2\sqrt{2}} - \frac{8}{3}\sqrt{a_n}\left(0 - \frac{a_n\sqrt{a_n}}{2\sqrt{2}}\right)$$

$$A_n = \frac{4\sqrt{2}}{3}a_n^2 \text{ sq.units}$$

3. The sum of first 'n ' terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.......$ is

(A) 2^{n-1} (B) $1 - 2^{-n}$ (C) $2^{-n} - n + 1$ (D) $2^{-n} + n - 1$

Ans. (D)

Sol. $S = \frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.......$ to 'n' terms

$S = \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+....$ to 'n' terms

$= (1 + 1 + 1 + ...) - \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....\right)$

$$= n - \frac{1}{2}\cdot\left(\frac{1-\left(\frac{1}{2}\right)^n}{\left(1-\frac{1}{2}\right)}\right)$$

$$= n - \left(1-\frac{1}{2^n}\right) = 2^{-n} + n - 1$$

4. Find the value of $\sum_{r=1}^{n}\sum_{s=1}^{n}\delta_{rs}2^r3^s$ where $\begin{cases}\delta_{rs} = 0, \text{ if } r \neq S \\ \delta_{rs} = 1, \text{ if } r = S\end{cases}$

(A) $\frac{6}{5}(6^n - 1)$ (B) $6^n - 1$ (C) $\frac{1}{5}(6^n - 1)$ (D) None of these

Ans. (A)

Sol. $\sum_{r=1}^{n}2^r\left\{\sum_{s=1}^{n}\delta_{rs}3^s\right\}$

$= \sum_{r=1}^{n}2^r\left\{\delta_{r1}3^1 + \delta_{2r}3^2 + \delta_{r3}3^3 + ... + \delta_{rn}3^n\right\}$

$= 2^13^1 + 2^23^2 + 2^33^3 + ...+2^n3^n$

$= 6 + 6^2 + ...+ 6^n$

$= \frac{6}{5}(6^n - 1)$

5. Value of $\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)(r+3)} = \_\_\_$

(A) 1 (B) 1/2 (C) 1/18 (D) 0

Ans. (C)

Sol. $= \frac{1}{3}\sum_{r=1}^{\infty}\frac{(r+3)-r}{r(r+1)(r+2)(r+3)}$

$= \frac{1}{3}\sum_{r=1}^{\infty}\left[\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right]$

$$= \frac{1}{3}\left[\frac{1}{1.2.3} - \frac{1}{2.3.4} + \frac{1}{2.3.4} - \frac{1}{3.4.5} + \frac{1}{3.4.5} - \frac{1}{4.5.6} + \ldots.\infty\right]$$

$$= \frac{1}{3}\left(\frac{1}{1.2.3}\right) = \frac{1}{18}$$

6. If α, γ are the roots of $t_1 x^2 - 4x + 1 = 0$ and β, δ are the roots of $t_2x^2 - 6x + 1 = 0$ and $\alpha, \beta, \gamma, \delta$ are in H.P. then

(A) $-t_1 + t_2 = 5$ (B) $t_1 + t_2 = 12$ (C) $t_1 = 8$ (D) $t_2 = 5$

Ans. **(A)**

Sol. $\alpha + \gamma = \frac{4}{t_1}$; $\alpha\gamma = \frac{1}{t_1}$ .....(1)

And $\beta + \delta = \frac{6}{t_2}$; $\beta\delta = \frac{1}{t_2}$ ......(2)

$\therefore$ $\alpha, \beta, \gamma, \delta$ are in HP

$\Rightarrow$ $\beta = \frac{2\alpha\gamma}{a+\gamma} = \frac{1}{2}$ = $\frac{1}{2}$ (from (1)) $\gamma = \frac{2\beta\delta}{\beta+\delta} = \frac{1}{3}$ (from (2))

But β satisfies $t_2x^2 - 6x + 1 = 0$ $\Rightarrow t_2 = 8$

And γ satisfies $t_1x^2 - 4x + 1 = 0$ $\Rightarrow t_1 = 3$

7. If $1^2 + 2^2 + 3^2 + \ldots\ldots. + 2003^2 = (2003)\ (4007)\ (334)$ and

(1) (2003) + (2) (2002) + (3) (2001) + ... + (2003) (1) = (2003) (334) (x)., then x equals

(A) 2005 (B) 2004 (C) 2003 (D) 2001

Ans. **(A)**

Sol. If $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 2003^2 = (2003)\ (4007)\ (334)$

(1) (2003) + (2) (2002) + (3) (2001) + ... + (2003) (1) = (2003) (334) (x)

$\Rightarrow$ $\sum_{r=1}^{2003} r(2003 - r + 1)$ = (2003) (334) (x)

$\Rightarrow$ $2004 . \sum_{r=1}^{2003} r - \sum_{r=1}^{2003} r^2$ = (2003) (334) (x)

$\Rightarrow$ $2004 . \left(\frac{2003.2004}{2}\right) - 2003 . (4007) . 334 = (2003)\ (334)\ (x)$

$\Rightarrow$ x = 2005 Ans.

MCQ (One or more than one correct) :

8. Consider the sequence a_n given by $a_1 = \frac{1}{2}, a_{n+1} = a_n^2 + a_n$,

Let $S_n = \frac{1}{a_1+1} + \frac{1}{a_2+1} + \ldots\ldots + \frac{1}{a_n+1}$ then find the value of $[S_{2012}]$, where [.] denotes greatest integer function.

(A) 1 (B) [e / 2] (C) [e] (D) $[\pi - 1]$

Ans. **(AB)**

Sol. $\frac{1}{a_{n+1}} = \frac{1}{a_n(a_n+1)}$

$\Rightarrow$ $\frac{1}{a_{n+1}} = \frac{1}{a_n} - \frac{1}{a_n+1}$

$\Rightarrow \quad \dfrac{1}{a_n+1} = \dfrac{1}{a_n} - \dfrac{1}{a_{n+1}}$

$\Rightarrow \quad \dfrac{1}{a_1+1} + \dfrac{1}{a_2+1} + \ldots\ldots + \dfrac{1}{a_{2012}+1} = \dfrac{1}{a_1} - \dfrac{1}{a_{2013}}$

$= 2 - \dfrac{1}{a_{2013}}$ & $0 < \dfrac{1}{a_{2013}} < 1$

9. Given that x + y + z = 15 when a, x, y, z, b are in A. P. and $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{5}{3}$ when a, x, y, z, b are in H. P. Then

(A) G. M. of a and b is 3 (B) one possible value of (a + 2b) is 11

(C) A. M. of a and b is 5 (D) H. M. of a and b is $\dfrac{9}{5}$

Ans. **(ABCD)**

Sol. $x + y + z = 3\left(\dfrac{a+b}{2}\right)$

$\Rightarrow \quad 15 = 3\left(\dfrac{a+b}{2}\right)$

$\Rightarrow \quad a + b = 10 \qquad ....(1)$

$\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{3\left(\dfrac{1}{a} + \dfrac{1}{b}\right)}{2}$

$\Rightarrow \quad \dfrac{5}{3} = \dfrac{3(a+b)}{2ab} = \dfrac{3 \times 10}{2ab}$

$\Rightarrow \quad ab = 9 \qquad ....(2)$

From (1) and (2), a = 9, b = 1 or a = 1 and b = 9

Hence GM = $\sqrt{ab}$ = 3, a + 2b = 11 or 19

10. The product of two positive real numbers a and b is 192. The quotient of A.M. by H.M. of their G.C.D and L.C.M is $\dfrac{169}{48}$. The smaller of a and b can be

(A) 2 (B) 4 (C) 6 (D) 12

Ans. **(BD)**

Sol. Let g, ℓ be the G.C.D and L.C.M then

$g.\ell = 192$ and $\dfrac{g+\ell}{2} \times \dfrac{g+\ell}{2g\ell} = \dfrac{169}{48} \quad \Rightarrow (g + \ell)^2 = 13^2 . 4^2$

$\therefore \quad g = 4, \ell = 48 \quad \Rightarrow \quad a = 4, b = 48$ or $a = 12, b = 16$

11. Let a, b, c are distinct real numbers such that expression $ax^2 + bx + c$, $bx^2 + cx + a$ and $cx^2 + ax + b$ are always positive then possible value(s) of $\dfrac{a^2+b^2+c^2}{ab+bc+ca}$ may be:

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(BC)**

Sol. $D_1 : b^2 - 4ac < 0$

$D_2 : c^2 - 4ab < 0$

$D_3 : a^2 - 4bc < 0$

$D_1 + D_2 + D_3 : a^2 + b^2 + c^2 < 4(ab + bc + ac)$

$$1 < \frac{a^2 + b^2 + c^2}{ab + bc + ac} < 4$$

12. For ΔABC, if $81+144a^4 +16b^4 + 9c^4 =144abc$, (where notations have their usual meaning), then

(A) $a > b > c$ (B) $A < B < C$

(C) Area of $\Delta ABC = \frac{3\sqrt{3}}{8}$ (D) Traiangle ABC is right handled

Ans. **(BCD)**

Sol. $81+144a^4 +16b^4 + 9c^4 =144abc$

A.M $\geq$ G.M.

$$\frac{81 + 144a^4 + 16b^4 + 9c^4}{4} \geq (81 \times 144a^4 \times 16b^4 \times 9c^4)^{1/4}$$

$81+144a^4 +16b^4 + 9c^4 \geq 4 \times 3 \times 2\sqrt{3}a \times 2b \times \sqrt{3}c$

$\Rightarrow$ $81+144a^4 +16b^4 + 9c^4 \geq 144abc$

Here equality holds

$\Rightarrow$ $81=144a^4 =16b^4 = 9c^4$

$\Rightarrow$ $3 = 2\sqrt{3}a = 2b = \sqrt{3}c$

$\Rightarrow$ $a = \frac{\sqrt{3}}{2}, b = \frac{3}{2}, c = \sqrt{3}$

Here, $c > b > a$

$\Rightarrow$ $C > B > A$

Now, $a^2 + b^2 = \frac{3}{4} + \frac{9}{4} = 3 = c^2$

$\Rightarrow$ Triangle ABC is right angled

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}ab = \frac{3\sqrt{3}}{8}$

13. Let $x, y, z \in \left(0, \frac{\pi}{2}\right)$ are first three consecutive terms of an arithmetic progression such that $\cos x + \cos y + \cos z = 1$ and $\sin x + \sin y + \sin z = \frac{1}{\sqrt{2}}$, then which of the following is/are correct?

(A) $\cot y = \sqrt{2}$ (B) $\cos(x - y) = \frac{\sqrt{3} - \sqrt{2}}{2\sqrt{2}}$

(C) $\tan 2y = \frac{2\sqrt{2}}{3}$ (D) $\sin(x - y) + \sin(y - z) = 0$

Ans. **(ABC)**

Sol. x, y, z are in A.P.

Let $x = y - \theta$ and $z = y + \theta$

$$\cos(y - \theta) + \cos y + \cos(y + \theta) = 1 = \frac{\sin\frac{3\theta}{2}}{\sin\frac{\theta}{2}} \cdot \cos(y)$$

$$\sin(y-\theta) + \sin y + \sin(y+\theta) = \frac{1}{\sqrt{2}} = \frac{\sin\frac{3\theta}{2}}{\sin\frac{\theta}{2}} \cdot \sin(y)$$

$\Rightarrow \quad \cot y = \sqrt{2}$

$$\frac{\sin\frac{3\theta}{2}}{\sin\frac{\theta}{2}} = \sqrt{\frac{3}{2}} = 3 - 4\sin^2\frac{\theta}{2} \qquad \Rightarrow \cos\theta = \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$$

14. a_1, a_2 .... are distinct terms of an A.P. We call (p, q, r) an increasing triad if a_p, a_q, a_r are in G.P. where $p,q,r \in \mathbb{N}$ such that $p < q < r$. If (5, 9, 16) is an increasing triad, then which of the following option is/are correct

(A) If a_1 is a multiple of 4 then every term of the A.P. is an integer

(B) (85, 149, 261) is an increasing triad

(C) If the common difference of the A.P. is $\frac{1}{4}$, then its first term is $\frac{1}{3}$

(D) Ratio of the $(4k + 1)^{th}$ and $4k^{th}$ term can be 4

Ans. **(ABC)**

Sol. $a_9^2 = a_5 \cdot a_{16}$

$\Rightarrow (a+8d)^2 = (a+4d)(a+15d)$

$\Rightarrow a^2 + 64d + 16ad = a^2 + 60d^2 + 19ad$

$\Rightarrow 4d^2 = 3ad$

$\Rightarrow 4d = 3a$

(A) If $a_1 = 4k$; $k \in I$

then $4d = 3(4k) \Rightarrow d = 3k$

$\Rightarrow$ every term of A.P. is an integer

(B) (85, 149, 261) is an increasing triad

if $a_{149}^2 = a_{85} \cdot a_{261}$

if $(a+148d)^2 = (a+84d)(a+260d)$

if $(a^2 + 296ad + 21904d^2 = a^2 + 344ad + 21840d^2$

if $64d^2 = 48ad$

if $4d = 3a$

which is true

Hence, (85, 149, 261) is an increasing triad.

(C) If $d = \frac{1}{4}$, then $a = \frac{1}{3}$

(D) $\frac{a_{4k+1}}{a_{4k}} = \frac{a+4kd}{a+(4k-1)d} = \frac{3a+12kd}{3a+3(4k-1)d} = \frac{4d+12kd}{4d+3(4k-1)d} = \frac{4(1+3k)}{(1+12k)}$

Let, $\frac{a_{4k+1}}{a_{4k}} = 4 \;\Rightarrow\; \frac{4(1+3k)}{1+12k} = 4$

$\Rightarrow 1+3k = 1+12k \Rightarrow 9k = 0 \Rightarrow k = 0$

which is not possible

Hence, ratio of (4k +1) and $(4k)^{th}$ term can't be 4

Numerical based Questions :

15. If $\frac{25}{k} = 1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \ldots\ldots \infty$, then find the value of k

Ans. **54**

Sol. $S = 1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \ldots\ldots\ldots\ldots\infty$ ...(i)

$-\frac{1}{5} S = -\frac{1}{5} + \frac{2^2}{5^2} - \frac{3^2}{5^3} + \frac{4^2}{5^4} - \frac{5^2}{5^5} + \ldots\ldots\ldots\ldots\infty$ ...(ii)

(i) – (ii) we get

$\frac{6}{5} S = 1 - \frac{3}{5} + \frac{5}{5^2} - \frac{7}{5^3} + \frac{9}{5^4} - \frac{11}{5^5} + \ldots\ldots\ldots\ldots\ldots\infty$ ...(iii)

$-\frac{6}{25} S = -\frac{1}{5} + \frac{3}{5^2} - \frac{5}{5^3} + \frac{7}{5^4} - \frac{9}{5^5} + \ldots\ldots\ldots\ldots\ldots\infty$ ...(iv)

(iii) – (iv) we get

$\frac{6s}{5}\left(\frac{6}{5}\right) = 1 - \frac{2}{5} + \frac{2}{5^2} - \frac{2}{5^3} + \frac{2}{5^4} - \ldots\ldots\infty$

$\frac{36}{25} S = 1 - \frac{2}{5}\left[\frac{1}{1+\frac{1}{5}}\right]$; $\frac{36}{25} S = 1 - \frac{2}{5}\left(\frac{5}{6}\right) = \frac{2}{3}$; $S = \frac{25}{36} \times \frac{2}{3} = \frac{25}{54}$ **Ans**

16. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid he dies leaving a third of the debt unpaid. Find the value of the first installment.

Ans. **51**

Sol. Let first installment be = 'a' and the common difference of the A.P. be 'd'

So $a + (a + d) + (a + 2d) + \ldots\ldots + (a + 39d) = 3600 \Rightarrow \frac{40}{2}[2a + 39d] = 3600$

$\Rightarrow 2a + 39d = 180$ ..... (1)

and $\frac{30}{2}[2a + 29d] = 2400$

$\Rightarrow 2a + 29d = 160$ ..... (2)

By equations (1) & (2), we get

$d = 2$ and $a = 51$ **Ans.**

17. If $x > 0$, and $\log_2 x + \log_2\left(\sqrt{x}\right) + \log_2\left(\sqrt[4]{x}\right) + \log_2\left(\sqrt[8]{x}\right) + \log_2\left(\sqrt[16]{x}\right) + \ldots\ldots = 4$, then find x.

Ans. **4**

Sol. $\log_2 x + \log_2(\sqrt{x}) + \log_2 (x)^{1/4} + \log_2 (x)^{1/8} + \ldots\ldots = 4$

$\Rightarrow \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x + \ldots\ldots = 4$

$\Rightarrow \frac{\log_2 x}{1 - \frac{1}{2}} = 4 \Rightarrow \log_2 x = 2 \Rightarrow x = 4$

18. If $x_i > 0$, $i = 1, 2, ..., 50$ and $x_1 + x_2 + .. + x_{50} = 50$, then find the minimum value of $\frac{1}{x_1} + \frac{1}{x_2} + ...+ \frac{1}{x_{50}}$.

Ans. **50**

Sol. $x_1 + x_2 + x_3 + ...+ x_{50} = 50$

AM $\geq$ HM

$$\frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{1}{\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + ... + \frac{1}{x_{50}}}{50}}$$

$$\Rightarrow \frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{50}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}}}$$

$$\Rightarrow \frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}} \geq 50$$

so min value of $\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}} = 50$

19. The number of terms in an A.P. is even ; the sum of the odd terms is 24, sum of the even terms is 30, and the last term exceeds the first by 10½; find the number of terms.

Ans. **8**

Sol. Let AP has 2n terms

Sum of odd term = 24 $\Rightarrow \frac{n}{2}[a_1 + a_{2n-1}] = 24$ ..... (1)

and sum of even terms = 30 $\Rightarrow \frac{n}{2}[a_2 + a_{2n}] = 30$ ..... (2)

and $a_{2n} = a_1 + \frac{21}{2}$

$a_1 + (2n - 1)d = a_1 + \frac{21}{2} \Rightarrow (2n - 1)\, d = \frac{21}{2}$ ..... (3)

By equation (1) & (2)

$a_1 + a_{2n-1} = \frac{48}{n}$ and $a_2 + a_{2n} = \frac{60}{n}$

So $a_1 + (n - 1)d = \frac{24}{n}$ and $a_1 + nd = \frac{30}{n}$ So $d = \frac{6}{n}$

Now $(2n - 1)\frac{6}{n} = \frac{21}{2} \Rightarrow n = 4,\ d = \frac{6}{4} = \frac{3}{2}$

So no. of terms = 2n = 8 and $a_1 = 3/2$. Numbers are $\frac{3}{2}$, 3, $\frac{9}{2}$ ....

20. If a, b, c are in GP, a – b, c – a, b – c are in HP, then the value of a + 4b + c is

Ans. **0**

Sol. a, b, c are in G.P. $\Rightarrow b^2 = ac \Rightarrow (a - b), (c - a), (b - c)$ are in H.P.

So $\frac{1}{a-b}, \frac{1}{c-a}, \frac{1}{b-c}$ are in AP. Let a, b, c are $\frac{b}{r}$, b, br

So $\dfrac{1}{\frac{b}{r}-b}, \dfrac{1}{br-\frac{b}{r}}, \dfrac{1}{b-br}$ are in AP.

So $\dfrac{2}{br-\frac{b}{r}} = \dfrac{1}{\frac{b}{r}-b} + \dfrac{1}{b-br}$ $\Rightarrow$ $\dfrac{2r}{r^2-1} = \dfrac{r}{1-r} + \dfrac{1}{1-r}$

$\Rightarrow$ $-\dfrac{2r}{(r+1)} = (1 + r)$ $\Rightarrow$ $(1 + r)^2 = -2r$

$\Rightarrow$ $r^2 + 1 + 4r = 0$ $\Rightarrow$ $\dfrac{c}{a} + 1 + \dfrac{4b}{a} = 0$ $\Rightarrow$ $a + 4b + c = 0$

21. If $S = \dfrac{5}{13} + \dfrac{55}{(13)^2} + \dfrac{555}{(13)^3} + \dfrac{5555}{(13)^4} + \ldots$ up to ∞, then find the value of 36S.

Ans. **65**

Sol. $S = \dfrac{5}{13} + \dfrac{55}{(13)^2} + \dfrac{555}{(13)^3} + \ldots\ldots\infty$; $S = \dfrac{5}{9}\left[\dfrac{(10-1)}{13} + \dfrac{10^2-1}{(13)^2} + \dfrac{10^3-1}{(13)^3} + \ldots\infty\right]$

$$= \frac{5}{9}\left[\frac{\frac{10}{13}}{1-\frac{10}{13}} - \left(\frac{\frac{1}{13}}{1-\frac{1}{13}}\right)\right] = \frac{5}{9}\left[\frac{10}{3} - \frac{1}{12}\right] = \frac{5}{9}\left[\frac{39}{12}\right] = \frac{65}{36}$$ **Ans**

22. If $S = \dfrac{1}{1+1^2+1^4} + \dfrac{2}{1+2^2+2^4} + \dfrac{3}{1+3^2+3^4} + \ldots\ldots\infty$, then find the value of 14S.

Ans. **7**

Sol. $T_n = \dfrac{n}{1+n^2+n^4} = \dfrac{1}{2}\left[\dfrac{(2n)}{(1+n+n^2)(1-n+n^2)}\right]$;

$T_n = \dfrac{1}{2}\left[\dfrac{1}{1-n+n^2} - \dfrac{1}{1+n+n^2}\right]$

$T_1 = \dfrac{1}{2}\left[\dfrac{1}{1} - \dfrac{1}{3}\right]$, ; $T_2 = \dfrac{1}{2}\left[\dfrac{1}{3} - \dfrac{1}{7}\right]$,

$T_3 = \dfrac{1}{2}\left[\dfrac{1}{7} - \dfrac{1}{13}\right]$,

$T_n = \dfrac{1}{2}\left[\dfrac{1}{1-n+n^2} - \dfrac{1}{1+n+n^2}\right]$

$S_n = \sum T_n = \dfrac{1}{2}\left[1 - \dfrac{1}{1+n+n^2}\right] = \dfrac{n+n^2}{2(1+n+n^2)}$

Matrix Match Type :

23. **Match Column I with Column II:**

	Column-I		Column-II
A	$\lim\limits_{n\to\infty}\sum\limits_{k-1}^{n}\frac{1}{(k+1)\sqrt{k}+k\sqrt{k+1}}$ is equal to	p	1
B	$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$ is equal to	q	2
C	$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{k^2-\frac{1}{2}}{k^4+\frac{1}{4}}$ is equal to	r	3
D	$x_1=\frac{1}{2}$ and $x_{k+1}=x_k^2+x_k$. $T=\sum\limits_{k=1}^{n}\frac{1}{x_i+1}$ then [T] is equal to (where [.] denotes G.I.F.)	s	–1
		t	0

Code :
(A) A-p; B-r; C-q; D-s
(B) A-r; B-p; C-s; D-r
(C) A-p; B-q; C-p; D-p
(D) A-r; B-p; C-q; D-r

Ans. **(C)**

Sol. **(A)** On rationalizing, $\sum\limits_{k=1}^{n}\frac{(k+1)\sqrt{k}-k\sqrt{k+1}}{k(k+1)}=1-\frac{1}{\sqrt{n+1}}$

(B) $\sum\limits_{k=1}^{n}\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}=\frac{3}{3-2}-\frac{3^n}{3^n-2^n}$

(C) $\sum\limits_{k=1}^{n}\frac{k-\frac{1}{2}}{k^2-k+\frac{1}{2}}-\frac{k+\frac{1}{2}}{k^2+k+\frac{1}{2}}=\sum\limits_{k=1}^{n}\frac{k-\frac{1}{2}}{k^2-k+\frac{1}{2}}-\frac{(k+1)-\frac{1}{2}}{(k+1)^2-(k+1)+\frac{1}{2}}=1-\frac{2n+1}{2n^2+2n+1}$

(D) $\frac{1}{x_{k+1}}=\frac{1}{x_k(x_k+1)}=\frac{1}{x_k}-\frac{1}{x_k+1}$

$\therefore\ \frac{1}{x_k+1}=\frac{1}{x_k}-\frac{1}{x_{k+1}}$

Subjective Type Questions :

24. In an A.P. of which 'a' is the Ist term, if the sum of the Ist 'p' terms is equal to zero, show that the sum of the next 'q' terms is $-\frac{a\,(p+q)\,q}{p-1}$.

Sol. Given $\frac{p}{2}[2a + (p-1)d] = 0$

so $2a + (p-1)d = 0$

$\Rightarrow \quad d = -\frac{2a}{(p-1)}$

Now sum of next q terms are = sum of (p + q) terms of this A.P.

$= \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}[2a + (p-1)d + qd]$

$= \frac{p+q}{2}.\ q.\ d = -\frac{a(p+q)q}{p-1}$

25. The sum of first p-terms of an A.P. is q and the sum of first q terms is p, find the sum of first (p + q) terms.

Ans. $-(p + q)$

Sol. $q = \frac{p}{2}[2A + (p-1)d] \Rightarrow \quad \frac{2q}{p} = 2A + (p-1)d \quad$ .....(i)

$p = \frac{q}{2}[2A + (q-1)d] \Rightarrow \quad \frac{2p}{q} = 2A + (q-1)d \quad$ .....(ii)

on subtracting equation (i) from (ii), we get

$\frac{2}{pq}(q^2 - p^2) = (p-q)d$

$\Rightarrow \quad d = \frac{-2}{pq}(p+q)$

$\therefore$ Sum of (p + q) terms is

$= \frac{p+q}{2}[2A + (p+q-1)d]$

$= \frac{p+q}{2}[2A + (p-1)d + qd]$

$= \frac{p+q}{2}\left[\frac{2q}{p} + q\left\{\frac{-2}{pq}(p+q)\right\}\right]$

$= \frac{p+q}{2}\left[\frac{2q}{p} - 2 - \frac{2q}{p}\right] = -(p+q)$ Ans.