$\Rightarrow \quad \sin^2 B = \frac{(\sqrt{3}-1)^2}{8} \quad \Rightarrow \quad \sin B = \pm\frac{\sqrt{3}-1}{2\sqrt{2}}$

$\Rightarrow \quad \sin B = \frac{\sqrt{3}-1}{2\sqrt{2}}$ (negative value rejected)

$\Rightarrow \quad$ B = 15°, A = 45° and C = 120°

17. Let ABC be a triangle with $\angle BAC = 120°$ and AB.AC = 1. Also, let AD be the length of the angle bisector of angle A of the triangle. Then _____.

(A) Minimum value of AD is $\frac{1}{2}$

(B) Maximum value of AD is $\frac{1}{2}$

(C) AD is minimum when ΔABC is isosceles

(D) AD is maximum when ΔABC is isosceles

Ans. **(BD)**

Sol. Let AB = x

Then $BC^2 = x^2 + \frac{1}{x^2} + 1$ and $\cos B = \frac{2x^2+1}{2\sqrt{x^4+x^2+1}}$

that is, $B = \frac{\sqrt{3}}{2x^2+1}$

Also, $\frac{AD}{\sin B} = \frac{x}{\sin(B+60°)}$ given $AD = \frac{x}{\frac{1}{2}+\frac{\sqrt{3}}{2}\cot B} = \frac{x}{x^2+1}$

$\therefore \quad AD = \frac{1}{x+\frac{1}{x}} \le \frac{1}{2}$, with equality iff AB = AC = 1

Comprehension Type Question:

Let α, β and γ are the roots of the equation $x^3 + 6x + 3 = 0$

and $\quad A = \cos^{-1}\left(\sin\left((\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}\right)\right)$

$B = \cos\left(\tan^{-1}\left(\sin\left(\frac{\alpha+\beta+\gamma}{2}\right)\right)\right)$

$C = \sec^{-1}\left(\text{cosec}\left((1-\alpha)(1-\beta)(1-\gamma)\right)\right)$.

18. If the range of the quadratic trinomial g (x) = $x^2 - 2Bx + k$ is $[0, \infty)$, then range of k equals

(A) $[1, \infty)$ (B) $(1, \infty)$ (C) {1} (D) $(-\infty, 1]$

Ans. **(C)**

19. The value of (5A + B – C) is equal to

(A) 1 (B) 10 (C) 5 (D) 0

Ans. **(A)**

20. Range of the function $f(x)=\dfrac{(5A-C)x^5+6Bx^2}{x^4+(B-1)x^3+1}$, is

(A) $[3, \infty)$ (B) $[0, 3]$ (C) $[-3, 3]$ (D) $(-\infty, \infty)$

Ans. **(B)**

Sol. We have $x^3+6x+3=0 \begin{cases} \to \alpha \\ \to \beta \\ \to \gamma \end{cases}$

$\therefore \alpha + \beta + \gamma = 0$

Now, $A=\cos^{-1}\left(\sin\left\{(-\gamma)^{-1}+(-\alpha)^{-1}+(-\beta)^{-1}\right\}\right) = \cos^{-1}\left(\sin\left(-\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)\right)\right)$

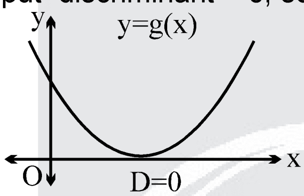
$= \frac{\pi}{2}+\sin^{-1}\left(\sin\left(\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}\right)\right) = \frac{\pi}{2}+\sin^{-1}\left(\sin\left(\frac{6}{-3}\right)\right) = \frac{\pi}{2}-(\pi-2)=\left(2-\frac{\pi}{2}\right)$

$B = \cos(\tan^{-1}(\sin 0)) = 1$

$C = \sec^{-1}(\operatorname{cosec}((1-\alpha)(1-\beta)(1-\gamma)))$ $\left(\begin{array}{l}\text{As } x^3+6x+3=(x-\alpha)(x-\beta)(x-\gamma) \\ \text{Putting } x=1, \text{ we get } (1-\alpha)(1-\beta)(1-\gamma)=10\end{array}\right)$

$= \frac{\pi}{2}-\operatorname{cosec}^{-1}(\operatorname{cosec}10) = \frac{\pi}{2}-(3\pi-10)=\left(10-\frac{5\pi}{2}\right)$.

(i) We have $g(x) = x^2 - 2Bx + k = x^2 - 2x + k$ (As $B = 1$)
For range of g (x) to be $[0, \infty)$,
put discriminant = 0, so



$4 - 4k = 0 \Rightarrow k = 1$. **Ans.**

(ii) On putting values of A, B and C, we get $(5A + B - C) = 5\left(2-\frac{\pi}{2}\right)+1-\left(10-\frac{5\pi}{2}\right)=1$.

(iii) We have $f(x)=\dfrac{(5A-C)x^5+6Bx^2}{x^4+(B-1)x^3+1}$

Now $(5A - C) = 0, B = 1$ $\left(\text{As } A=2-\frac{\pi}{2},\ B=1,\ C=10-\frac{5\pi}{2}\right)$

$\therefore \quad f(x) = \dfrac{6x^2}{x^4+1} = \begin{cases} 0 & , \ x=0 \\ \dfrac{6}{x^2+\frac{1}{x^2}} & , \ x\neq 0 \end{cases}$

Hence $f(x)|_{max.}=3 \Rightarrow$ Range of f(x) is [0, 3] $\left(\text{As } x^2+\frac{1}{x^2}\geq 2\right)$

Paragraph for question nos. 21 to 23

Let $f : A \to B$ be an onto function defined as $f(x) = \dfrac{\sin^{-1}x+\tan^{-1}x}{\cos^{-1}x+\cot^{-1}x}$.

21. If the minimum and maximum value of $f(x)$ be m and M respectively then the value of $\frac{M}{m}$ is

(A) – 1 (B) – 4 (C) – 7 (D) Infinite

Ans. (C)

22. Let $g : B \to A$ be a function such that $g(f(x)) = x\ \forall x \in A$ and $f(g(x)) = x\ \forall x \in B$, then the value of g(3) is

(A) – 1 (B) 0 (C) 0.5 (D) 1

Ans. (D)

23. The number of solutions of the equation $f(x^3 + 14x^2 + 13x - 5) = f(1 - x^2 + x^3)$ is

(A) 0 (B) 1 (C) 2 (D) 3

Ans. (B)

Sol. (i) We have $f(x) = \dfrac{\frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cot^{-1}x}{\cos^{-1}x + \cot^{-1}x} = \dfrac{\pi - \cos^{-1}x - \cot^{-1}x}{\cos^{-1}x + \cot^{-1}x} = \dfrac{\pi}{\cos^{-1}x + \cot^{-1}x} - 1$

Clearly A = [–1, 1]. So in [–1, 1], $\cos^{-1}x \in [0, \pi]$ and $\cot^{-1}x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$\therefore\ \cos^{-1}x + \cot^{-1}x = \left[\frac{\pi}{4}, \frac{7\pi}{4}\right] \forall x \in A$

Also $\cos^{-1}x + \cot^{-1}x$ is strictly decreasing on A

$\therefore\ m = f(x)_{min} = \dfrac{\pi}{(\cos^{-1}x + \cot^{-1}x)_{max}} - 1 = \dfrac{\pi}{\cos^{-1}(-1) + \cot^{-1}(-1)} - 1 = \dfrac{-3}{7}$

and $M = f(x)_{max} = \dfrac{\pi}{(\cos^{-1}x + \cot^{-1}x)_{min}} - 1 = \dfrac{\pi}{\cos^{-1}1 + \cot^{-1}1} - 1 = 3$

Hence $\dfrac{M}{m} = \dfrac{3}{\left(-\frac{3}{7}\right)} = -7$

(ii) Clearly g(x) will be inverse of f(x).

As $f(1) = 3 \Rightarrow f^{-1}(3) = 1$

Hence g(3) = 1

(iii) As f(x) is one-one function, so $f(x^3 + 14x^2 + 13x - 5) = f(1 - x^2 + x^3)$

$\Rightarrow x^3 + 14x^2 + 13x - 5 = 1 - x^2 + x^3 \quad \Rightarrow 15x^2 + 13x - 6 = 0$

$\Rightarrow 15x^2 + 18x - 5x - 6 = 0 \ \Rightarrow (3x - 1)(5x + 6) = 0$

$\Rightarrow x = \frac{1}{3}, \frac{-6}{5}$ but $x \in [-1, 1]$

Hence $x = \frac{1}{3}$ only.]

Numerical based Questions :

24. If $\sum_{r=1}^{10} \tan^{-1}\left(\dfrac{3}{9r^2 + 3r - 1}\right) = \cot^{-1}\left(\dfrac{m}{n}\right)$ (where m and n are coprime), then find (2m + n).

Ans. 32

Sol. We have $a_n = \tan^{-1}\left(\frac{3}{9n^2+3n-1}\right) = \tan^{-1}\left(\frac{3}{1+(3n+2)(3n-1)}\right) = \tan^{-1}\left(\frac{(3n+2)-(3n-1)}{1+(3n+2)(3n-1)}\right)$

$= \tan^{-1}(3n + 2) - \tan^{-1}(3n - 1)$

$\therefore$ Sum of first 10 terms $= \sum_{r=1}^{10} a_r = \sum_{r=1}^{10}\left(\tan^{-1}(3r+2) - \tan^{-1}(3r-1)\right)$

$= (\tan^{-1}5 - \tan^{-1}2) + (\tan^{-1}8 - \tan^{-1}5) + \ldots\ldots + (\tan^{-1}32 - \tan^{-1}29)$

$= \tan^{-1}32 - \tan^{-1}2 = \tan^{-1}\left(\frac{32-2}{1+32\cdot 2}\right) = \tan^{-1}\frac{30}{65} = \tan^{-1}\left(\frac{13}{6}\right) = \cot^{-1} = \cot^{-1}\left(\frac{m}{n}\right)$

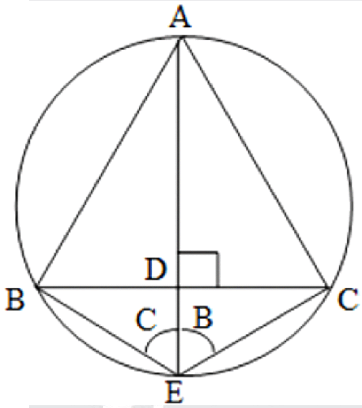
$\therefore$ m = 13 and n = 6.

Hence (2m + n) = 32 **Ans.**

25. Perpendiculars are drawn from the angles A, B, C of an acute-angled triangle on opposite sides and produced to meet the circumscribing circle. If these produced parts are α, β, γ respectively, then the value of $\frac{(a/\alpha)+(b/\beta)+(c/\gamma)}{\tan A+\tan B+\tan C}$ is _____.

Ans. **(2.00)**

Sol.



Let AD be the perpendicular from A onBC

When AD is produced, it meets circumscribibg circle at E

From equation, DE = α

Since angles in the same segment are equal,

$\angle$AEB = $\angle$ACB = $\angle$CB = $\angle$C and $\angle$AEC = $\angle$ABC = $\angle$B

$\tan C = \frac{BD}{DE}$ ......(1)

From the right –angled triangle CDE, we get

$\tan B = \frac{CD}{DE}$ ......(2)

Adding (1) and (2),

$\tan B + \tan C = \frac{BD+CD}{DE} = \frac{BC}{DE} = \frac{a}{\alpha}$ ......(3)

Similarly, tan C + tan A = $\frac{b}{\beta}$ ......(4)

And tan A + tan B = $\frac{c}{\gamma}$ ......(5)

$\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma} = 2$ (tan A + tan B + tan C)

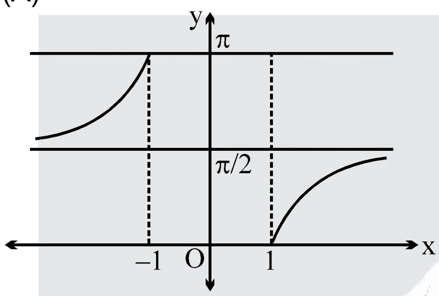
Matrix Match Type :

26. An function is defined on R or subset of R. Match the function given in column-I with their properties given in column-II. (Where [x] denotes the greatest integer less than or equal to x.)

Column-I		**Column-II**	
(A)	$\sec^{-1}x$	(P)	aperiodic
(B)	$\sqrt{\lvert x \rvert}$	(Q)	into
(C)	$\cos^{-1}(\cos x)$	(R)	odd or even
		(S)	neither odd nor even

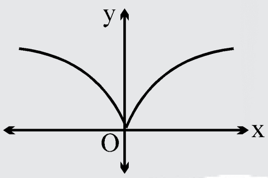
Ans. (A) P, Q, S; (B) P, Q, R; (C)]

Sol. (A)



Into; Also it is neither odd nor even and aperiodic.

(B) $\sqrt{|x|} = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$



Hence function is into as symmetric abou y-axis hence even.

27. Match the following :

	Column-I		**Column-II**
A	The number of solutions to the equation $\frac{\pi}{2}\left(\left|\sin^{-1}(\sin x)\right|\right) = \left|x^2 - \pi|x|\right|$ are	p	4
B	The least value of the expression $2\log_{10} x - \log_x 0.01$ for x > 1 is	q	2
C	The number of common terms in the series 1, 4, 7, 10, 13 .....91 and 4, 8, 16, 32 ..... 1024 are	r	5
D	Let A be a real number, and let $A = \frac{\sqrt{|x|-2} + \sqrt{2-|x|}}{|2-x|}$ then the unit digit of $[A]^{2013}$ is (where [.] denote greatest integer function)	s	3
		t	0

Code :

(A) A–p; B–r; C–q; D–s
(B) A–r; B–p; C–s; D–t
(C) A–s; B–q; C–p; D–r
(D) A–r; B–p; C–q; D–r

Ans. **(B)**

Sol. (A) Use graphs

(B) AM $\leq \Gamma$M

28. Match the folloiwng :

	Column-I		Column-II
A	If p2 – 2 p cos x = 673 and tan $\frac{x}{2}=7$, the integral value of p is	p	8
B	If sin θ + cos θ = m then the maximum value of m^2 is	q	9
C	r_1, r_2, r_3 are the radii of the circle drawn on the altitudes PD, PE and PF of ΔPBC, ΔPCA, ΔPAB respectively as diameter where P is the circumcentre of the acute angled ΔABC. The minimum value of $\frac{1}{18}\left[\frac{a^2}{r_1^2}+\frac{b^2}{r_2^2}+\frac{c^2}{r_3^2}\right]$ is (a, b, c are sides of ΔABC)	r	2
D	In ΔABC, a = 6, b = 3 and $\cos(A-B)=\frac{4}{5}$ then the area of the ΔABC is	s	25

Code :

(A) A→ (s); B→ (r); C → (p); D → (q)

(B) A→ (s); B→ (p); C → (p); D → (q)

(C) A→ (r); B→ (r); C → (p); D → (q)

(D) A→ (s); B→ (q); C → (p); D → (r)

Ans. (A)

Sol. (A) $\tan\frac{x}{2}=7 \Rightarrow \sin\frac{x}{2}=\pm\frac{7}{\sqrt{50}}$

$p^2 - 2p\cos x = 673$

$(p-1)^2 + 2p(1-\cos x) = 674$

$\Rightarrow$ $(p-1)^2 + 4p\sin^2\frac{x}{2} = 674$

$\Rightarrow$ $(p-1)^2 + \frac{98}{25}p = 674$

Put p = 25

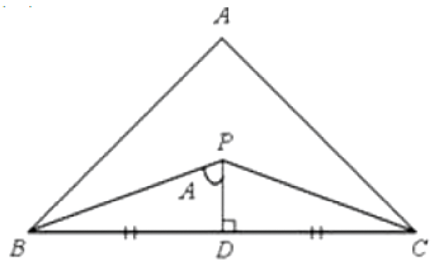
$(24)^2 + 98 = 674$ which is true

(B) sin θ + cos θ = m

$\sqrt{2} \leq \sin\theta+\cos\theta \leq \sqrt{2}$,

So the maximum value of m^2 is 2

(C)



$$\tan A = \frac{BD}{PD} = \frac{\frac{a}{2}}{2r_1} = \frac{a}{4r_1}$$

Similarly, $\tan B = \frac{b}{4r_2}$

$$\tan C = \frac{c}{4r_3}$$

So, $\frac{a^2}{r_1^2} + \frac{b^2}{r_2^2} + \frac{b}{r_3^2} = 16$ $(\tan^2 A + \tan^2 B + \tan^2 C)$

But $\tan^2 A + \tan^2 B + \tan^2 C \geq 3\ [\tan A \tan B \tan C]^{2/3}$ (using AM $\geq$ GM)

Also, in an acute angled ΔABC

$\tan A + \tan B + \tan C \geq 3\ [\tan A \tan B \tan C]^{1/3}$ (using AM $\geq$ GM)

But $\tan A + \tan B + \tan C = \tan A + \tan B + \tan C$

So, $(\tan A \tan B \tan C)^{2/3} \geq 3$

Therefore, $\tan^2 A + \tan^2 B + \tan^2 C \geq 9$

So, $\frac{a^2}{r_1^2} + \frac{b^2}{r_2^2} + \frac{c^2}{r_3^2} \geq 144$

The minimum value of $\frac{1}{18}\left[\frac{a^2}{r_1^2} + \frac{b^2}{r_2^2} + \frac{c^2}{r_3^2}\right]$

(D) $\cos(A - B) = \frac{4}{5}$

$$\Rightarrow \quad \frac{1 - \tan^2\left(\frac{A-B}{2}\right)}{1 + \tan^2\left(\frac{A-B}{2}\right)} = \frac{4}{5}$$

$$\tan\left(\frac{A-B}{2}\right) = \pm\frac{1}{3}$$

But $a > b \quad \Rightarrow A > B$

So, $\tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$

Again, $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \times \cot\frac{C}{2}$

$\Rightarrow \quad \frac{1}{3} = \frac{3}{9} \times \cot\frac{C}{2} \Rightarrow \cot\frac{C}{2} = 1 \quad \Rightarrow \frac{C}{2} = 45^\circ \Rightarrow C = 90^\circ$

So, the area of ΔABC = $\frac{1}{2}ab\sin C = \frac{1}{2} \times 6 \times 3 \times 1$

$\Rightarrow \quad \frac{1}{3} = \frac{3}{9} \times \cot\frac{C}{2} \Rightarrow \cot\frac{C}{2} = 1 \quad \Rightarrow \frac{C}{2} = 45^\circ \Rightarrow C = 90^\circ$

So, the area of ΔABC = $\frac{1}{2}a\sin C = \frac{1}{2} \times 6 \times 3 \times 1$