14. The values of x in [–2π, 2π], for which the graph of the function $y = \sqrt{\dfrac{1+\sin x}{1-\sin x}}$ – secx and $y = -\sqrt{\dfrac{1-\sin x}{1+\sin x}}$ + secx, coincide are

(A) $\left[-2\pi, -\dfrac{3\pi}{2}\right) \cup \left(\dfrac{3\pi}{2}, 2\pi\right]$ (B) $\left(-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$

(C) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ (D) $[-2\pi, 2\pi] - \left\{\pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}\right\}$

Ans. **(AC)**

Sol. $$y = \sqrt{\frac{1+\sin x}{1-\sin x}} - \frac{1}{\cos x} = \frac{1+\sin x}{|\cos x|} - \frac{1}{\cos x} = \begin{cases} \dfrac{\sin x}{\cos x} & \text{if } \cos x > 0 \\ -\dfrac{2+\sin x}{\cos x} & \text{if } \cos x < 0 \end{cases}$$

again $$Y = -\sqrt{\frac{1-\sin x}{1+\sin x}} + \frac{1}{\cos x}$$

$$\frac{1}{\cos x} - \frac{1-\sin x}{|\cos x|} = \begin{cases} \dfrac{\sin x}{\cos x} & \text{if } \cos x > 0 \\ \dfrac{2-\sin x}{\cos x} & \text{if } \cos x < 0 \end{cases}$$

Hence for identifical graph cos x > 0

15. Let f(x) = $[x]^2$ + [x + 1] – 3, where [x] denotes greatest integer less than or equal to x, then which of the following statement(s) is/are **CORRECT**?

(A) f(x) is many one function.

(B) f(x) vanishes for atleast three values of x.

(C) f(x) is neither even nor odd function.

(D) f(x) is aperiodic.

Ans. **(ABCD)**

Sol. We have f(x) = $[x]^2$ + [x] – 2 ⇒ f(x) = ([x] + 2) ([x] – 1) = $\begin{cases} -2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \end{cases}$ and so on.

⇒ f(x) is many one function.

Also f(x) = 0 ⇒ x ∈ [–2, –1) ∪ [1, 2) ⇒ f(x) = 0 has infinite solutions.

As f(–x) ≠ f(x) and f(–x) ≠ –f(x) ⇒ f(x) is neither even nor odd function.

Now f (x) = $[x]^2$ + [x] – 2, where $[x]^2$ + [x] is aperiodic.

⇒ f (x) is aperiodic.

16. Let f : I → R, defined as f (x) = 5 cos 4πx – 13 sin7πx + 2. Then which of the following alternative(s) is/are **TRUE**?

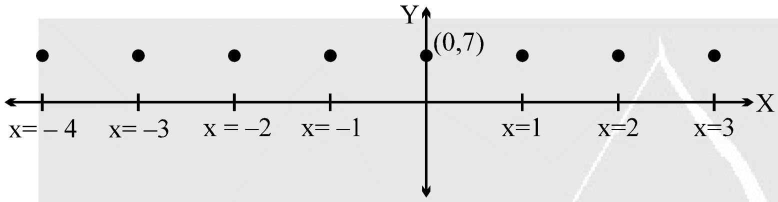
(A) Range of f is a singleton set.
(B) f is an even function.
(C) $f\left(f(x)\right) = f(x)\ \forall\ x \in I$
(D) Inverse function of f is non existent.

Ans. **(ABCD)**

Sol. Clearly f (x) = 7, $\forall\ x \in I$

$\therefore$ $R_f = \{7\}$

The graph of f (x) is shown below.



Note : f is periodic function with period 1.]

17. Which of the following pair(s) of function have same graphs?

(A) $f(x) = \dfrac{\sec x}{\cos x} - \dfrac{\tan x}{\cot x}$, $g(x) = \dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\operatorname{cosec} x}$

(B) f (x) = sgn $(x^2 - 4x + 5)$, g(x) = $\operatorname{sgn}\left(\cos^2 x + \sin^2\left(x + \dfrac{\pi}{3}\right)\right)$ where sgn denotes signum function.

(C) f (x) = $e^{\ln(x^2+3x+3)}$, g(x) = $x^2 + 3x + 3$

(D) $f(x) = \dfrac{\sin x}{\sec x} + \dfrac{\cos x}{\operatorname{cosec} x}$, $g(x) = \dfrac{2\cos^2 x}{\cot x}$

Ans. **(ABCD)**

Sol. **(A)** We have $f(x) = \dfrac{\sec x}{\cos x} - \dfrac{\tan x}{\cot x}$, $g(x) = \dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\operatorname{cosec} x}$

Clearly both f(x) and g(x) are identical functions as $x \neq \dfrac{k\pi}{2}\ \forall\ k \in I$.

(B) As $x^2 - 4x + 5 = (x - 2)^2 + 1 > 0$

Hence f(x) = 1 $\forall\ x \in R$.

Also $\cos^2 x + \sin^2\left(x + \dfrac{\pi}{2}\right) > 0$

Hence g(x) = 1 $\forall\ x \in R$.

$\Rightarrow$ f(x) and g(x) are identical.

(C) f(x) = $e^{\ln(x^2+3x+3)}$

As $x^2 + 3x + 3 = \left(x + \dfrac{3}{2}\right)^2 + \dfrac{3}{4} > 0\ \forall\ x \in R$.

Hence f(x) = $x^2 + 3x + 3\ \forall\ x \in R$.

$\Rightarrow$ f(x) is identical to g(x).

(D) We have $f(x) = \dfrac{\sin x}{\sec x} + \dfrac{\cos x}{\operatorname{cosec} x}$, $g(x) = \dfrac{2\cos^2 x}{\cot x}$

Clearly both f(x) and g(x) are identical functions as $x \neq \dfrac{k\pi}{2} \forall\ k \in I$.

Numerical based Questions :

18. N is the set of all natural number and R is the set of all real numbers. A function f : N → R is given by:

$f(n) = \dfrac{4n + \sqrt{4n^2 - 1}}{\sqrt{2n-1} + \sqrt{2n+1}}$, then the value of $\left[\dfrac{\sum_{r=1}^{40} f(r)}{40}\right]$ is equal to ______.

(where [·] represents greatest integer function)

Ans. **(9.00)**

Sol. $f(n) = \dfrac{(2n+1)^{3/2} - (2n-1)^{3/2}}{2}$

Hence f(1) + f(2) + ...+ f(40) = $\dfrac{(81)^{3/2} - (1)^{3/2}}{2} = 364$

19. If $f(x) = \begin{cases} -x+1, & x \le 0 \\ -(x-1)^2, & x \ge 1 \end{cases}$, then the number of solutions of the equation $f(x) - f^{-1}(x) = 0$ is /are_____.

Ans. **(4.00)**

Sol. $f^{-1}(x) = \begin{cases} -x+1, & x \ge 1 \\ 1+\sqrt{-x}, & x \le 0 \end{cases}$

Solutions are –1, 1, 0, 2

20. f (x) and g (x) are linear function such that for all x, $f(g(x))$ and $g(f(x))$ are Identity functions. If f (0) = 4 and g (5) = 17, compute f (136).

Ans. **(12)**

Sol. Let f (x) = ax + b and g (x) = cx + d

as $g(f(x))$ = x hold for all x, we have

$f(g(x)) = a(g(x)) + b = a(cx + d) + b = acx + ad + b$

∴ acx + (ad + b) = x for all x, on comparing coefficients

∴ ac = 1 and ad + b = 0

c = 1/a and d = – b/a

∴ $g(x) = \dfrac{x}{a} - \dfrac{b}{a} = \dfrac{x-b}{a}$ also f (x) = ax + b

now f (0) = 4 ⇒ b = 4 and

g (5) = 17 ⇒ $\dfrac{5-4}{a} = 17$ ⇒ $a = \dfrac{1}{17}$

∴ $f(x) = \dfrac{x}{17} + 4 \Rightarrow f(136) = \dfrac{136}{17} + 4 = 12$. **Ans.**

21. Let $f(x) = \left|x^2 - 9\right| - \left|x - a\right|$. Find the number of integers in the range of a so that f (x) = 0 has 4 distinct real root.

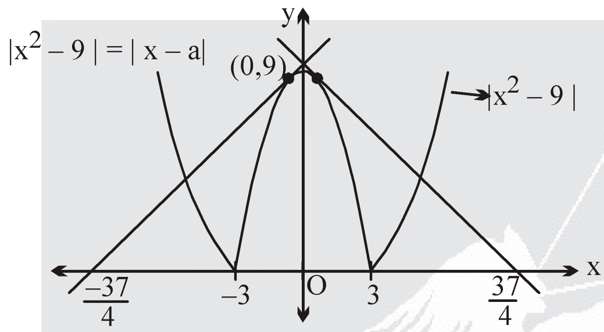
Ans. **(17)**

Sol. For tangency, $x^2 - 9 = x - a$

$\Rightarrow x^2 - x + a - 9 = 0$

Put $D = 0 \Rightarrow 1 - 4a + 36 = 0 \Rightarrow a = \frac{37}{4}$

|||ly $a = \frac{-37}{4}$



$\therefore$ For 4 distinct solution, $a \in \left(\frac{-37}{4}, -3\right) \cup (-3, 3) \cup \left(3, \frac{37}{4}\right)$

Hence, number of integers are 17 **Ans.**

Comprehension Type Question:

Paragraph for question nos. 22 to 23

Let $f(x) = x^2 - 2x - 1\ \forall\ x \in R$. Let $f : (-\infty, a] \to [b, \infty)$, where 'a' is the largest real number for which f (x) is bijective.

22. The value of (a + b) is equal to

(A) – 2 (B) – 1 (C) 0 (D) 1

Ans. **(B)**

Sol. $f(x) = (x - 1)^2 - 2$ a = 1, b = – 2

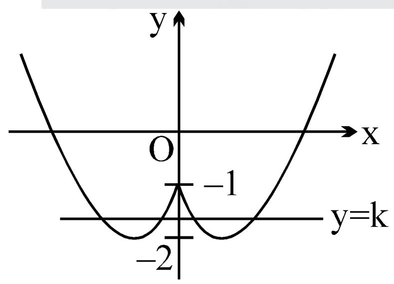
a + b = – 1 Ans.

23. Let $f : R \to R$, then range of values of k for which equation f (| x |) = k has 4 distinct real roots is

(A) (– 2, – 1) (B) (– 2, 0) (C) (– 1, 0) (D) (0, 1)

Ans. **(A)**

Sol.



; from graph $k \in (-2, -1)$ Ans.

Paragraph for question nos. 24 to 26

An even periodic function $f : R \to R$ with period 4 is such that

$$f(x) = \begin{cases} \max.(|x|, x^2) & ; 0 \le x < 1 \\ x & ; 1 \le x \le 2 \end{cases}$$

24. The value of $\{f(x)\}$ at $x = 5.12$ (where { } represents fractional part), is

(A) $\{f(7.88)\}$ (B) $\{f(3.26)\}$ (C) $\{f(2.12)\}$ (D) $\{f(5.88)\}$

Ans. (A)

25. The equation of circle with centre lies on the curve f(x) at x = 9 and touches x-axis, is

(A) $x^2 + y^2 - 14x - 2y + 49 = 0$ (B) $x^2 + y^2 - 18x - 4y + 84 = 0$

(C) $x^2 + y^2 - 18x - 2y + 81 = 0$ (D) $x^2 + y^2 - 18x + 2y + 81 = 0$

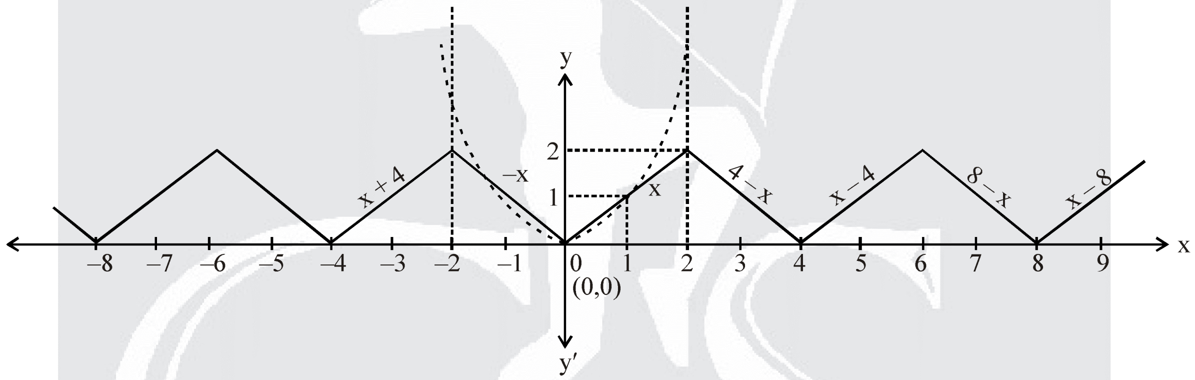
Ans. (C)

26. If $g(x) = |3\sin x|$, then the number of solutions of $f(x) = g(x)$ for $x \in (-6, 6)$, are

(A) 5 (B) 7 (C) 3 (D) 9

Ans. (B)

Sol.



(24) At $x = 5.12 \Rightarrow f(x) = x - 4$

Now $f(5.12) = 5.12 - 4 = 1.12$

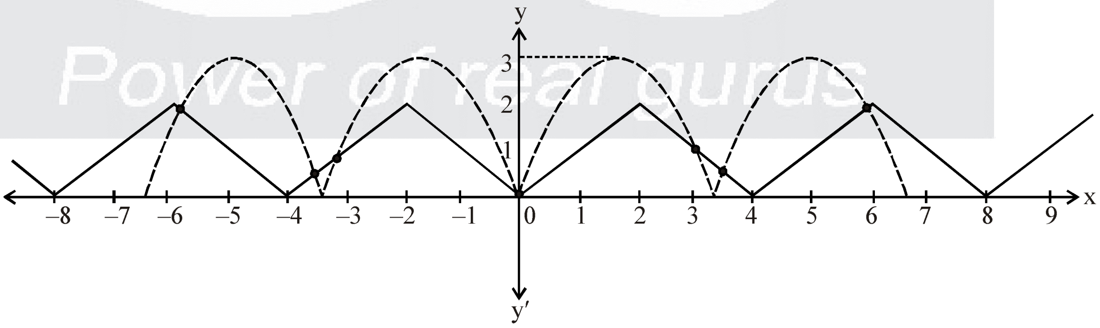
$\therefore \{f(5.12)\} = 0.12 = \{f(7.88)\}$

(25) At $x = 9$, centre of the circle lies on $f(x) = x - 8$, then $f(9) = 1$

So, centre is (9, 1) and radius = 1

Hence required equation of circle is $x^2 + y^2 - 18x - 2y + 81 = 0$

(26)



From above graph, clearly number of solutions are 7 **Ans.**

Matrix Match Type :

27. Match the following :

	Column-I		Column-II
A	Let f : [– 1, ∞) → (0, ∞) defined by f (x) = $e^{x^2+\|x\|}$ then f (x), is	P	one-one
B	Let f : (1, ∞) → [3, ∞) defined by f (x) = $\sqrt{10-2x+x^2}$, then f (x) is	Q	into
C	Let f : R → I defined by f (x) = $\tan^5\pi[x^2 + 2x + 3]$ where [ ] denotes greatest integer function, then f (x) is	R	many one
D	Let f : [3, 4] → [4, 6] defined by f (x) = $\|x-1\|+\|x-2\|+\|x-3\|+\|x-4\|$ then f (x)	S	onto
		T	periodic

Ans. (A) Q, R; (B) P, Q; (C) Q, R, T; (D) P, S

Sol. (A) f (x) = $e^{x^2+|x|}$

Clearly function is many one as f (1) = f (–1)

Into as minimum value of f (x) = $e^{x^2+|x|}$ is equal to 1 at x = 0

so range is [1, ∞) **Ans. Q, R**

(B) f (x) = $\sqrt{x^2-2x+10} = \sqrt{(x-1)^2+9}$

For x ∈ (1, ∞) range is (3, ∞). Hence into. One-one as line drawn 11 to x-axis cut's the graph at only one point.

Since x ∈ (1, ∞) **Ans. P, Q**

(C) f : R → I

f (x) = $\tan^5 \pi [x^2 + 2x + 3] = 0$ **Ans. Q, R, T**

(D) For x ∈ [3, 4]

f (x) ∈ [4, 6]

As f (x) = 2(x – 1) for x ∈ [3, 4]

28. Match the following :

	Column-I		Column-II
A	Let f : R^+ → {–1,0, 1} defined by f (x) = sgn $(x - x^4 + x^7 - x^8 - 1)$ where sgn denotes signum function, then f(x) is	P	Into
B	Let f : R → R and satisfies f (x) + x f (–x) = x + 1, then f (x) is	Q	One-one
C	Let f : [0, 4] → [0, 9] defined by f (x) = $\sqrt{6x-x^2}$, then f (x) is	R	Many-one
D	Let f : [0, 3] → [2, 8] defined by f (x) = $2^{\|x-1\|+\|x-2\|}$, then f (x) is	S	Even
		T	Onto

Ans. (A) P, R; (B) P, R, S; (C) P, R; (D) R, T

Sol. **(A)** Clearly $x - x^4 + x^7 - x^8 - 1 < 0 \ \forall \ x \in R^+$.

Hence sgn $(x - x^4 + x^7 - x^8 - 1) = -1 \ \forall \ x \in R^+$.

(B) We have $f(x) + x\,f(-x) = x + 1$ .... (1)

Replace x by –x in (1) , we get

$-x\,f(x) + f(-x) = 1 - x$ .... (2)

$\therefore$ From (1) and (2), we get

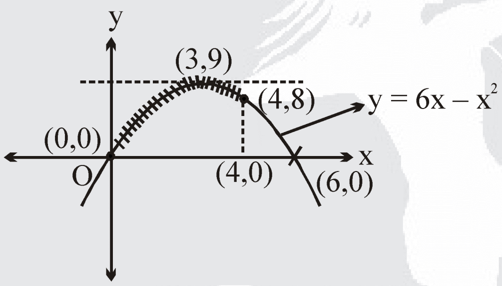
$x^2 f(x) - x\,f(-x) = x^2 - x$ .... (3)

$\Rightarrow (1 + x^2)\,f(x) = 1 + x^2$

Hence $f(x) = 1$

(C) We have $f(x) = \sqrt{6x - x^2}$

Clearly range of $f = [0, 3] \ \forall \ 0 \le x \le 4$.



(D) We have $f(x) = 2^{|x-1|+|x-2|}$

$$= \begin{cases} 2^{-2x+3} & ; \quad 0 \le x \le 1 \\ 2 & ; \quad 1 < x < 2 \\ 2^{2x-3} & ; \quad 2 \le x \le 3 \end{cases}$$

Clearly range of $f = [2, 8] \ \forall \ 0 \le x \le 3$.]