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MATHEMATICS	**CAPS-23**
TARGET : JEE- Advanced 2021	CIRCLE

ANSWER KEY OF CAPS-23

1.	(C)	**2.**	(C)	**3.**	(C)	**4.**	(B)	**5.**	(D)
6.	(D)	**7.**	(A)	**8.**	(B)	**9.**	(D)	**10.**	(B)
11.	(AC)	**12.**	(AB)	**13.**	(49)	**14.**	(2)	**15.**	(13)
16.	(1)	**17.**	(8)	**18.**	(75)	**19.**	(1)		

SCQ (Single Correct Type) :

1. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line y = x + 1 cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is :

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(0, \frac{1}{2\sqrt{2}}\right)$ (C) $\left(0, \frac{2-\sqrt{2}}{4}\right)$ (D) none of these

Ans. **(C)**

Sol. Let 'd' be the common difference

$\therefore$ the radii of the three circles be 1 – 2d, 1 – d, 1

$\therefore$ equation of smallest circle is $x^2 + y^2 = (1 - 2d)^2$ ....(i)

$\therefore$ y = x + 1 intersect (i) at real and distinct points

$\therefore$ $x^2 + x + 2d - 2d^2 = 0$ ....(ii)

$\because$ $D > 0 \Rightarrow 8d^2 - 8d + 1 > 0$

$\Rightarrow$ $d > \frac{2+\sqrt{2}}{4}$ or $d < \frac{2-\sqrt{2}}{2}$

but d can not be greater than $\frac{2+\sqrt{2}}{2}$

$\therefore$ $d \in \left(0, \frac{2-\sqrt{2}}{4}\right)$

2. A circle of constant radius 'r' passes through origin O and cuts the axes of coordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is :

(A) $(x^2 + y^2)(x^{-2} + y^{-2}) = 4r^2$ (B) $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = r^2$

(C) $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$ (D) $(x^2 + y^2)(x^{-2} + y^{-2}) = r^2$

Ans. **(C)**

Sol. Let the coordinates of P and Q are (a, 0) and (0, b) respectively

$\therefore$ equation of PQ is bx + ay – ab = 0 ....(i)

$\because$ $a^2 + b^2 = 4r^2$ .....(ii)

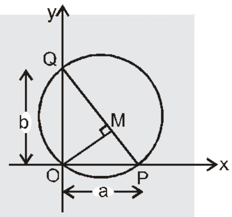
$\because$ OM $\perp$ PQ

$\therefore$ equation of OM is ax – by = 0 ....(iii)

Let M(h, k)

$\therefore$ bh + ak – ab = 0 ....(iv) and ah – bk = 0 ....(v)

On solving equations (iv) and (v), we get



$a = \dfrac{h^2+k^2}{h}$ and $b = \dfrac{h^2+k^2}{k}$

put a and b in (ii), we get

$(h^2 + k^2)^2 (h^{-2} + k^{-2}) = 4r^2$

$\therefore$ locus of M(h, k) is $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$

3. A pair of tangents are drawn from a point P to the circle $x^2 + y^2 = 1$. If the tangents make an intercept of 2 units on the line x = 1, then the locus of P is _____.

(A) a straight line (B) a pair of lines (C) a parabola (D) a hyperbola

Ans. **(C)**

Sol. Taking $P(x_1, y_1)$, we get the pair of tangents as

$$\left(x_1^2+y_1^2-1\right)\left(x^2+y^2-1\right)-\left(xx_1+yy_1-1\right)^2=0$$

Putting x = 1 and solving the quadratic equation in y, the diference of roots comes out to be 2.

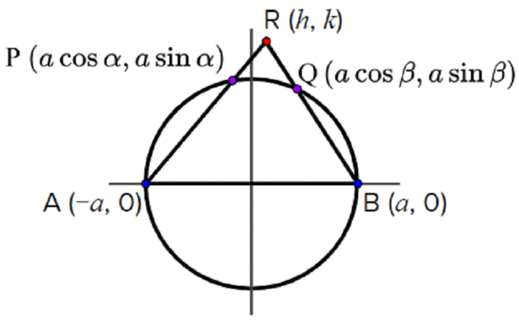
Hence the locus of P is $y^2 = 2(x+1)$, which is parabola

4. A circle with centre at the origin and radius equal to a meets the X axis at the points A(–a, 0) and B(a, 0) . P(α) and Q(β) are two points on this circle so that $\alpha - \beta = 2\gamma$, where γ is a constant. The locus of the point of intersection of AP and BQ is ______.

(A) $x^2 - y^2 - 2ay\tan\gamma = a^2$ (B) $x^2 + y^2 - 2ay\tan\gamma = a^2$

(C) $x^2 + y^2 + 2ay\tan\gamma = a^2$ (D) $x^2 - y^2 + 2ay\tan\gamma = a^2$

Ans. **(B)**



Sol.

Coordinates of $A(-a,0)$, $A(a\cos\alpha, a\sin\alpha)$

R (h,k) is the point of intersection A, P and R are collinear, So, $\dfrac{k-0}{h+a}=\dfrac{\sin\alpha}{\cos\alpha+1} \Rightarrow \dfrac{k}{a+h}=\tan\dfrac{\alpha}{2}$

B, Q and R are collinear,

So, $\dfrac{k-0}{h-a}=\dfrac{\sin\beta}{\cos\beta-1} \Rightarrow \dfrac{\beta}{2}=\dfrac{a-h}{k}$

Now, $\gamma=\dfrac{\alpha-\beta}{2}$

$$\therefore\ \tan\gamma=\frac{\tan\frac{\alpha}{2}-\tan\frac{\beta}{2}}{1+\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}=\frac{\left(\frac{k}{h+a}\right)-\left(\frac{a-h}{k}\right)}{1+\left(\frac{k}{h+a}\right)\left(\frac{a-h}{k}\right)}$$

Thus, $x^2+y^2-2ay\tan\gamma=a^2$

5. Let the lines $y-2=m_1(x-5)$ and $(y+4)=m_2(x-3)$ intersect at right angles at a point P, where m_1 and m_2 are parameters. If the locus of P is $x^2+y^2+gx+fy+7=0$, then the value of $(f-g)$ equals _____.

(A) 1 (B) 2 (C) 8 (D) 10

Ans. **(D)**

Sol. P lies on a circle with A(5, 2) and B(3, − 4) as diametric end points

$\Rightarrow$ Locus of point of intersection of lines is

$(x-5)(x-3)+(y-2)(y+4)=4$

$\Rightarrow x^2+y^2-8x+2y+27=0$

Hence, $(f-g)=2-(-8)=10$

6. The circle, which passes through the points of intersection of the circles $x^2+y^2-4x-6y+12=0$ and $x^2+y^2-8x+12y+50=0$, and also passes through the origin, is_____.

(A) $19x^2+19y^2-52x-222y=0$

(B) $19(x^2+y^2)-2(34x+111y)=0$

(C) $19(x^2+y^2)-117x+26y=0$

(D) such circle does not exist

Ans. **(D)**

Sol. $C_1=(2,3);\ r_1\sqrt{4+9-12}=1$

$C_2=(4,-6);\ r_2\sqrt{16+36-50}=\sqrt{2}$

$\therefore$ Both given circles does not intersect

So such required circle does not exist

7. Let P(α, β) be a point in the first quadrant. Circles are drawn through P touching the coordinate axes.

The relation between α and β, for which two circles are orthogonal, is _____.

(A) $\alpha^2+\beta^2=4\alpha\beta$

(B) $(\alpha+\beta)^2=4\alpha\beta$

(C) $\alpha^2+\beta^2=\alpha\beta$

(D) $\alpha^2+\beta^2=2\alpha\beta$

Ans. **(A)**

Sol. Let r_1 and r_2 be the radii of the two circles

Clearly, $(\alpha - r_1)^2 + (\beta - r_1)^2 = r_1^2$ and $(\alpha - r_2)^2 + (\beta - r_2)^2 = r_2^2$

$\Rightarrow$ r_1 and r_2 are the roots of the equation

$(\alpha - x)^2 + (\beta - x)^2 = x^2$, that is $x^2 + (-2\alpha - 2\beta)x + \alpha^2 + \beta^2 = 0$

For orthogonalityof two circles,

$2(-r_1)(-r_2) + 2(-r_1)(-r_2) = r_1^2 + r_2^2$

$\Rightarrow (r_1 + r_2)^2 = 6r_1r_2$

$\Rightarrow 4(\alpha + \beta)^2 = 6(\alpha^2 + \beta^2)$

$\Rightarrow \alpha^2 + \beta^2 = 4\alpha\beta$

8. The equation of circum-circle of a ΔABC is $x^2 + y^2 + 3x + y - 6 = 0$. If A = (1,−2), B = (−3,2) and the vertex C varies then the locus of ortho-centre of ΔABC is a

(A) Straight line (B) Circle (C) Parabola (D) Ellipse

Ans. **(B)**

Sol. Equation of circum-circle is $\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{17}{2}$

$C = \left(\frac{-3}{2} + \sqrt{\frac{17}{2}}\cos\theta, \frac{-1}{2} + \sqrt{\frac{17}{2}}\sin\theta\right)$

Centroid of ΔABC = $G = \left[\frac{-7}{6} + \sqrt{\frac{17}{18}}\cos\theta, \frac{-1}{6} + \sqrt{\frac{17}{18}}\sin\theta\right]$

Let orthocentre (O) be (h, k)

$\therefore\ h = \left(\frac{-1}{2} - \sqrt{\frac{17}{2}}\cos\theta\right)$

$k = \left(\frac{1}{2} - \sqrt{\frac{17}{2}}\sin\theta\right)$

$\therefore\ \left(h + \frac{1}{2}\right)^2 + \left(k - \frac{1}{2}\right)^2 = \frac{17}{2}$

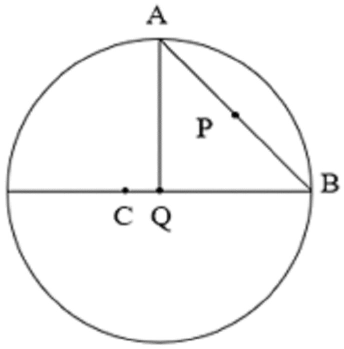
Which is a circle

9. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$which subtends an angle of 90º at the point (2,3) then the locus of the midpoint of AB is a circle whose centre is

(A) (1, 5) (B) $\left(1, \frac{3}{2}\right)$ (C) $\left(1, \frac{5}{2}\right)$ (D) $\left(2, \frac{5}{2}\right)$

Ans. **(D)**

Sol. Let midpoint of AB is P(h, k) & C be centre of given circle AB subtends 90° at Q(2, 3)



In ΔABC, AP = PB = PQ

$PC^2 + PB^2 = BC^2$

$(h-2)^2 + (k-2)^2 + (h-2)^2 + (k-3)^2 = 4 \quad \Rightarrow x^2 + y^2 - 4x - 5y + \frac{17}{2} = 0$

10. P and Q are two points on a line passing through (2, 4) and having slope m. If a line segment AB subtends a right angle at P and Q where A ≡ (0, 0) and B ≡ (6, 0), then range of m is

(A) $\left(\frac{2-3\sqrt{2}}{4}, \frac{2+3\sqrt{2}}{4}\right)$ (B) $\left(-\infty, \frac{2-3\sqrt{2}}{4}\right) \cup \left(\frac{2+3\sqrt{2}}{4}, \infty\right)$

(C) (–4, 4) (D) $(-\infty, -4) \cup (4, \infty)$

Ans. **(B)**

Sol. Sine AB subtends right angle at P and Q on variable line

So AB is a diameter of circle whose chord is a variable line

Equation of circle is $x^2 + y^2 - 6x = 0$ ..... (i)

Equation of line through (2, 4) is

$y - 4 = m(x - 2)$ ....(ii)

Line (ii) is chord if $\left|\frac{3m + 4 - 2m}{\sqrt{1+m^2}}\right| < 3$

$8m^2 - 8m - 7 > 0 \quad \Rightarrow m \in \left(-\infty, \frac{2-3\sqrt{2}}{4}\right) \cup \left(\frac{2+3\sqrt{2}}{4}, \infty\right)$

MCQ (One or more than one correct) :

11. If $a\ell^2 - bm^2 + 2d\ell + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$, then the line $\ell x + my + 1 = 0$ touches a fixed circle :

(A) which cuts the x–axis orthogonally

(B) with radius equal to b

(C) on which the length of the tangent from the origin is $\sqrt{d^2 - b}$

(D) none of these.

Ans. **(AC)**

Sol. $\because a\ell^2 - bm^2 + 2\ell d + 1 = 0$ ......(1)

and $a + b = d^2$ ....(2)

Put $a = d^2 - b$ in equation (1), we get

$(\ell d + 1)^2 = b(\ell^2 + m^2)$

$\Rightarrow \dfrac{|\ell d+1|}{\sqrt{\ell^2+m^2}} = \sqrt{b}$ ......(3)

From (3) we can say that the line $\ell x + my + 1 = 0$ touches a fixed circle having centre at (d,0) and radius $= \sqrt{b}$

12. Let A, B, C, D lie on a line such that AB = BC = CD =1. The points A and C are also joined by a semicircle with AC as diameter and P is a variable point on this semicircle such that $\angle PBD=\theta$, $0 \le \theta \le \pi$. Let R is the region bounded by are AP, the straight line PD and line AD.

(A) The maximum possible area of region R is $\dfrac{2\pi+3\sqrt{3}}{6}$

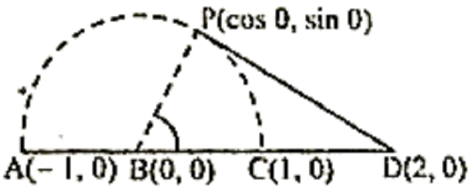
(B) If 'L' is the perimeter of region 'R', then L is equal to $3+\pi-\theta+\sqrt{5-4\cos\theta}$

(C) The maximum possible area of region R is $\dfrac{2\pi-3\sqrt{3}}{6}$

(D) If 'L' is the perimeter of region 'R', then L is equal to $3+\pi-\theta+\sqrt{5+4\cos\theta}$

Ans. **(AB)**

Sol.



$\text{Area} = \frac{1}{2}(1)^2(\pi-\theta) + \frac{1}{2}(2)\sin\theta = \frac{\pi}{2} + \sin\theta - \frac{\theta}{2}$

$A = \frac{\pi}{2} + \sin\theta - \frac{\theta}{2}$

$\frac{dA}{d\theta} = \cos\theta - \frac{1}{2},$

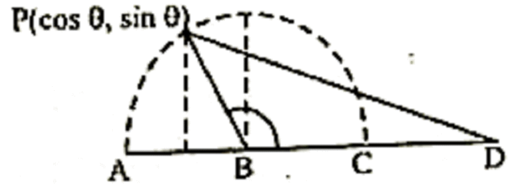
A is max, for $\theta = \frac{\pi}{3}$

$\Rightarrow A_{max} = \frac{\pi}{3} = \frac{\sqrt{3}}{2} = \frac{2\pi+3\sqrt{3}}{6}$

$A = \frac{1}{2}\sin\theta(2) + \frac{1}{2}(\pi-\theta)$

$L = 3 + (\pi-\theta) + \sqrt{(2-\cos\theta^2) + \sin^2\theta}$

$= 3 + (\pi-\theta) + \sqrt{5-4\cos\theta}$



Numerical based Questions :

13. The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms and the new equation $x^2 + y^2 = \frac{\lambda}{4}$, then find the value of λ.

Ans. **49**

Sol. $x^2 + y^2 - 5x + 2y - 5 = 0 \quad \Rightarrow \quad \left(x - \frac{5}{2}\right)^2 + (y + 1)^2 - 5 - \frac{25}{4} - 1 = 0$

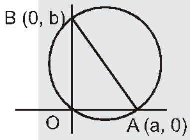
$\Rightarrow \quad \left(x - \frac{5}{2}\right)^2 + (y + 1)^2 = \frac{49}{4} \quad \Rightarrow$ So the axes are shifted to $\left(\frac{5}{2}, -1\right)$

New equation of circle must be $x^2 + y^2 = \frac{49}{4}$

14. A line meets the co-ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively and diameter of the circle is $\lambda_1 d_1 + \lambda_2 d_2$, then find the value of $\lambda_1 + \lambda_2$.

Ans. **2**

Sol.



Equation of circum circle of triangle OAB $x^2 + y^2 - ax - by = 0$.

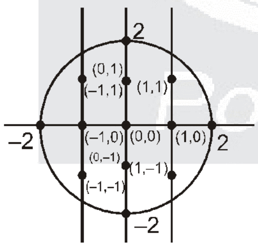
Equation of tangent at origin $ax + by = 0$.

$d_1 = \frac{|a^2|}{\sqrt{a^2 + b^2}}$ and $d_2 = \frac{|b^2|}{\sqrt{a^2 + b^2}} \quad \Rightarrow \quad d_1 + d_2 = \sqrt{a^2 + b^2}$ = diameter

15. Find the number of integral points which lie on or inside the circle $x^2 + y^2 = 4$.

Ans. **13**

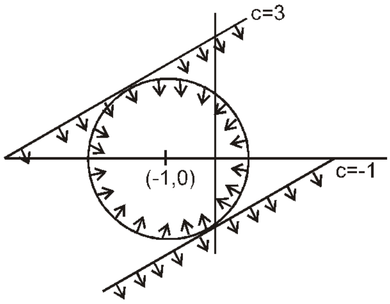
Sol.



16. Find number of values of 'c' for which the set,

$\{(x, y) \mid x^2 + y^2 + 2x \le 1\} \cap \{(x, y) \mid x - y + c \ge 0\}$ contains only one point is common.

Ans. **1**



Sol.

$\left|\frac{-1-0+c}{\sqrt{2}}\right| = \sqrt{2} \Rightarrow c - 1 = \pm 2 \Rightarrow c = -1, 3$

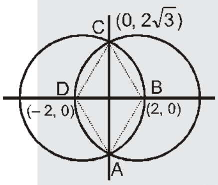
But c = –1 common point is one

c = 3 common point is infinite

Hence c = –1 is Answer.

17. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles and the area of the rhombus is $a\sqrt{3}$ sq. units, then find the value of a.

Ans. 8



Sol. Area of ABCD = $4\left(\frac{1}{2}.2.2\sqrt{3}\right)$.

18. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B (1, 7) & D (4, – 2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

Ans. 75

Sol. Given circle $x^2 + y^2 - 2x - 4y - 20 = 0$

Tangents at B(1, 7) is

x + 7y – (x + 1) – 2(y + 7) – 20 = 0

5y – 35 = 0 ⇒ y = 7

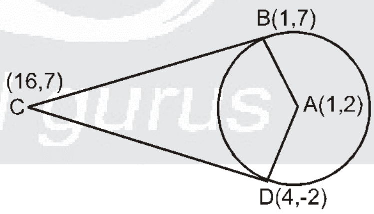
at D (4, –2)

4x – 2y – (x + 4) – 2(y – 2) – 20 = 0

3x – 4y = 20

Hence c(16, 7)

Area of quadrilateral ABCD = AB × BC = 5 × 15 = 75 square units



19. If a tangent of slope $\frac{1}{2}$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2 + y^2 + 4x + 2 = 0$, then the maximum value of ab is _____.

Ans. **(1)**

Sol. Equation of tangent of slope $\frac{1}{2}$ is $y=\frac{1}{2}x+\sqrt{\frac{a^2}{4}+b^2}$

This is normal to the $x^2+y^2+4x+2=0$

$\Rightarrow$ This tangent passes through (–2, 0)

So, $0=-1+\sqrt{\frac{a^2}{4}+b^2} \Rightarrow 2=\sqrt{a^2+4b^2} \Rightarrow a^2+4b^2=4$

Now using $AM \geq GM$, we get $\frac{a^2+4b^2}{2} \geq \sqrt{4a^2b^2} \Rightarrow ab \leq 1$

Subjective Type Questions:

20. Find the equation of the circle passing through the points A(4, 3), B(2, 5) and touching the axis of y. Also find the point P on the y–axis such that the angle APB has largest magnitude.

Ans. $x^2 + y^2 - 4x - 6y + 9 = 0$ OR $x^2 + y^2 - 20x - 22y + 121 = 0$, P(0, 3), $\theta = 45°$

Sol. Equation of circle touching y - axis is

$x^2 + y^2 + 2gx + 2fy + f^2 = 0$

$\because$ it passes through (4, 3) & (2, 5)

so $25 + 8g + 6f + f^2 = 0$

$29 + 4g + 10f + f^2 = 0$

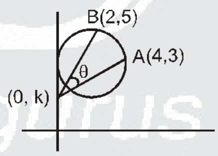
solving above two equations, we get

$(g, f) \equiv (-2, -3)$ & $(-10, -11)$.

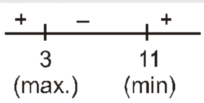
So equations of circles are $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 - 20x - 22y + 121 = 0$

for circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

$$\tan\theta = \frac{\frac{5-k}{2}-\frac{3-k}{4}}{1+\frac{5-k}{2}\left(\frac{3-k}{4}\right)} = \frac{14-2k}{23+k^2-8k}$$



$$\frac{d(\tan\theta)}{dk} = \frac{2(k-11)(k-3)}{(k^2-8k+23)^2}$$



So $\tan\theta$ is max at k = 3.

at k = 3 , $\tan\theta = 1 \Rightarrow \theta = 45°$

21. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$. Find the equations of the circles.

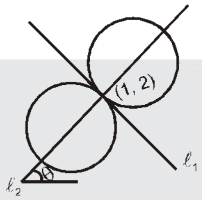
Ans. $x^2 + y^2 + 6x + 2y - 15 = 0$; $x^2 + y^2 - 10x - 10y + 25 = 0$

Sol. $\ell_1 \equiv 4x + 3y = 10$

$\ell_2 \equiv 3x - 4y = -5$

Let θ be the inclination of ℓ_2

$\therefore\ \tan\theta = \frac{3}{4}$



$\therefore$ equation of ℓ_2 in parametric form

$$\frac{x-1}{4/5} = \frac{y-2}{3/5} = \pm 5$$

co-ordinates of centres are (5, 5), (–3, –1)

22. The centre of the circle S = 0 lies on the line $2x - 2y + 9 = 0$ and S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points and also find their co–ordinates.

Ans. $(-4, 4)$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Sol. $\because$ centre lies over the line $2x - 2y + 9 = 0$

So let coordinate of centre be $\left(h, \frac{2h+9}{2}\right)$

Let the radius of circle be 'r'

So equation of circle is

$$(x - h)^2 + \left(y - \frac{2h+9}{2}\right)^2 = r^2$$

$x^2 + y^2 - 2hx - y(2h + 9) + 2h^2 + 9h - r^2 + \frac{81}{4} = 0$ $\because$ given circle cuts orthogonally to $x^2 + y^2 = 4$

so $2h^2 + 9h + \frac{65}{4} - r^2 = 0$ or $2h^2 + 9h - r^2 = -\frac{65}{4}$

so equation of required circle can be written as $x^2 + y^2 - 2hx - y(2h + 9) + 4 = 0$

$(x^2 + y^2 - 9y + 4) + h(-2y - 2x) = 0$

so this circle always passes through points of intersection of $x^2 + y^2 - 9y + 4 = 0$ and $x + y = 0$

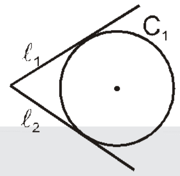
so fixed points are (–4, 4) and $\left(-\frac{1}{2}, \frac{1}{2}\right)$

23. The lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6 unit. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts of intercepts of length 8 on these lines.

Ans. $x^2 + y^2 - 10x - 4y + 4 = 0$

Sol. Centre of C_1 lies over angle bisector of ℓ_1 & ℓ_2

Equations of angle bisectors are



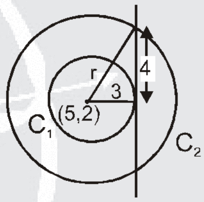
$$\frac{5x + 12y - 10}{13} = \pm \frac{5x - 12y - 40}{13}$$

$\Rightarrow$ $x = 5$ or $y = -\frac{5}{4}$

Since centre lies in first quadrant

so it should be on $x = 5$.

So let centre be $(5, \alpha)$



$\Rightarrow 3 = \frac{|25 + 12\alpha - 10|}{13}$ $\Rightarrow$ $\alpha = 2, -\frac{9}{2}$

From the figure $r = \sqrt{16 + 9} = 5$

But $\alpha \neq -\frac{9}{2}$ so $\alpha = 2$.

So equation of cirlce C_2 is

$(x - 5)^2 + (y - 2)^2 = 5^2$

$x^2 + y^2 - 10x - 4y + 4 = 0.$

24. Prove that the two circles which pass through the points (0, a) , (0, – a) and touch the straight line $y = mx + c$ will cut orthogonaly if $c^2 = a^2 (2 + m^2)$.

Sol. Let the equation of the circles be $x^2 + y^2 + 2gx + 2fy + d = 0$ ....(i)

$\because$ these circles pass through (0, a) and (0, –a)

$\therefore$ $a^2 + 2fa + d = 0$ ......(ii)

and $a^2 - 2fa + d = 0$ ......(iii)

solving (ii) and (iii), we get $f = 0$, $d = -a^2$

put these value of f and d in (i), we get

$x^2 + y^2 + 2gx - a^2 = 0$ ......(iv)

$\because$ $y = mx + c$ touch these circles $\Rightarrow \left| \frac{-mg + c}{\sqrt{m^2 + 1}} \right| = \sqrt{g^2 + a^2}$

$\Rightarrow$ $g^2 + (2cm)g + a^2(1 + m^2) - c^2 = 0$ ......(v)

equation (v) is quadratic in 'g'

$\therefore$ Let g_1 and g_2 are its two roots

$\therefore$ $g_1 g_2 = a^2(1 + m^2) - c^2$

$\because$ the two circles represented by (iv) are orthogonal

$\therefore \quad 2g_1g_2 + 0 = -a^2 - a^2$

$\Rightarrow \quad g_1g_2 = -a^2$

$\Rightarrow \quad a^2(1 + m^2) - c^2 = -a^2$

$c^2 = a^2(2 + m^2)$ Hence proved

25. Show that if one of the circle $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2g_1x + c = 0$ lies within the other, then gg_1 and c are both positive.

Sol. $\because$ One circle lies within the other circle $\Rightarrow C_1C_2 < |r_1 - r_2|$

$\Rightarrow \quad \sqrt{(g-g_1)^2} < \left|\sqrt{(g^2-c)} - \sqrt{g_1^2-c}\right|$

squaring both sides, we get

$-2gg_1 < -2\sqrt{g^2-c}\sqrt{g_1^2-c} - 2c$

$\Rightarrow \quad gg_1 > c + \sqrt{g^2-c}\,.\sqrt{g_1^2-c}$

$\Rightarrow \quad gg_1 - c > \sqrt{g^2-c}\,.\,\sqrt{g_1^2-c}$ ......(i)

$\Rightarrow \quad gg_1 - c > 0 \quad \Rightarrow \quad gg_1 > c$

again squaring both sides of (i), we get

$-2cgg_1 > -c(g^2 + g_1^2) \quad \Rightarrow c(g - g_1)^2 > 0$

$\Rightarrow \quad c > 0$ and from (i), we can say that

$\therefore \quad gg_1$ will also be > 0