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MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-22

Straight Line

ANSWER KEY OF CAPS-22

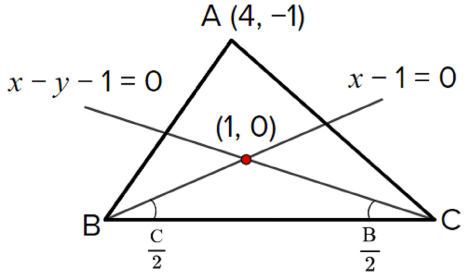
1.	(B)	**2.**	(A)	**3.**	(C)	**4.**	(D)	**5.**	(C)
6.	(C)	**7.**	(A)	**8.**	(ABC)	**9.**	(ABCD)	**10.**	(AD)
11.	(AD)	**12.**	(BC)	**13.**	(CD)	**14.**	(C)	**15.**	(D)
16.	(B)	**17.**	(30)	**18.**	(0)	**19.**	(1)	**20.**	(2)
21.	(A)								

SCQ (Single Correct Type) :

1. Let A(4, −1), B and C be the vertices of a triangle. Let the internal angular bisectors of angles B and C be x – 1 = 0 and x – y –1= 0 respectively. Let D, E and F be the points of contact of the sides BC, CA and AB respectively with the incircle of triangle ABC. The slope of BC is ______.

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) 12

Ans. **(B)**



Sol.

image of A (4, –1) in bisector of B and C lies on side BC

⇒ image of A (4, 1) with respect to X – Y -1 = 1 0 is

$$\frac{x-4}{1}=\frac{y+1}{-0}=-\frac{2(4+1-1)}{2}\Rightarrow(0,3)$$

image of A(4, –1) with respect to x –1 is

$$\frac{x-4}{1}=\frac{y+1}{0}=-\frac{4-1}{1}\Rightarrow(-2,-1)$$

Hence slope of BC is $m=\left(\frac{3+1}{0+2}\right)=2$

2. Let A(4, −1) , B and C be the vertices of a triangle. Let the internal angular bisectors of angles B and C be x – 1 = 0 and x – y –1= 0 respectively. Let D, E and F be the points of contact of the sides BC, CA and AB respectively with the incircle of triangle ABC. If D′, E′, F′ are the images of D, E, F in the internal angular bisectors of angles A, B, C respectively, then the equation of the circumcircle of $\Delta D'E'F'$ is _____.

(A) $(x-1)^2 + y^2 = 5$ (B) $x^2 + (y-1)^2 = 25$

(C) $(x-1)^2 + (y-1)^2 = 5$ (D) $x^2 + y^2 = 25$

Ans. **(A)**

Sol.

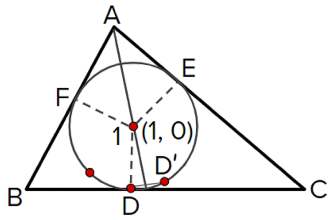


Image of a point P on circumcircle with respective to diameter lies on circle again

$\Rightarrow r = \frac{5}{\sqrt{5}} = \sqrt{5} \Rightarrow$ circle $(x-1)^2 + (y-0)^3 = (\sqrt{5})^2$

3. ABC is a triangle right angled at A with vertices A,B,C in the anti-clockwise sense in that order. A= (1,2), B = (−3,1) and vertex C lies on the X – axis. BCEF is a square with vertices B,C,E,F in the clockwise sense in that order. ACD is an equilateral triangle with vertices A,C,D in the anti-clockwise sense in that order. The abscissa of centroid of ΔBCE is

(A) –1 (B) $\frac{-1}{2}$ (C) $\frac{-1}{3}$ (D) $\frac{-2}{3}$

Ans. **(C)**

Sol. $C = \left(\frac{3}{2}, 0\right)$, $F = \left(-4, \frac{-7}{2}\right)$, $E = \left(\frac{1}{2}, \frac{-9}{2}\right)$, $D = \left(\frac{5+4\sqrt{3}}{4}, \frac{4+\sqrt{3}}{4}\right)$

abscissa of centroid of ΔBCE $= \frac{-3 + \frac{3}{2} + \frac{1}{2}}{3} = -\frac{1}{3}$

4. Statement 1: Consider the point A(0,1) and B(2,0) and 'P' be a point on the line 4x+3y+9 = 0, then coordinates of 'P' such that |PA− PB| is maximum is $\left(\frac{-12}{5}, \frac{17}{5}\right)$

Statement 2: |PA−PB| ≤ |AB|

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

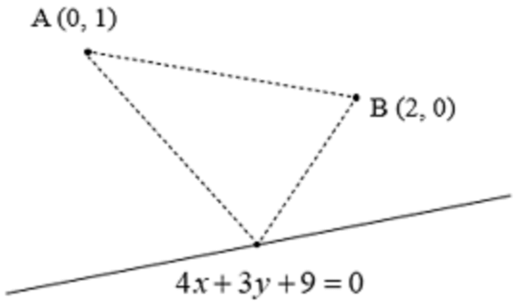
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Ans. **(D)**

Sol.



Statement 2: In ΔAPB, using triangle in equality $|PA-PB| \le |AB|$ so S − 2 is true.

Statement 1: It is false as for PA + PB to be maximum, P should lie at infinite distance on the given line.

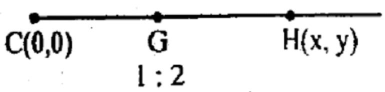
5. The vertices of a triangle are $\left(1,\sqrt{3}\right)$, $\left(2\cos\theta, 2\sin\theta\right)$ and $\left(2\sin\theta, -2\cos\theta\right)$ where $\theta \in R$. The locus of orthocentre of the triangle is :

(A) $\left(x-1\right)^2 + \left(y-\sqrt{3}\right)^2 = 4$ (B) $\left(x-2\right)^2 + \left(y-\sqrt{3}\right)^2 = 4$

(C) $\left(x-1\right)^2 + \left(y-\sqrt{3}\right)^2 = 8$ (D) $\left(x-2\right)^2 + \left(y-\sqrt{3}\right)^2 = 8$

Ans. **(C)**

Sol. $\left(\dfrac{1+2\cos\theta+2\sin\theta}{3}, \dfrac{\sqrt{3}+2\sin\theta-2\cos\theta}{3}\right)$



$\left(x-1\right)^2 + \left(y-\sqrt{3}\right)^2 = 8$

$\dfrac{x}{3} = \dfrac{1+2\cos\theta+2\sin\theta}{3} \Rightarrow x = 1+2\cos\theta+2\sin\theta$

$\dfrac{y}{3} = \dfrac{\sqrt{3}+2\sin\theta-2\cos\theta}{3} \Rightarrow y = \sqrt{3}+2\sin\theta-2\cos\theta$

$\left(x-1\right)^2 + \left(y-\sqrt{3}\right)^2 = 8$

6. The equations $3x+2y+1=0,\ 2x+4y-1=0$ and $3x^2+4xy+4y^2+2x-2y+1+\alpha=0$ will have a unique solution if α equals

(A) $\dfrac{2}{3}$ (B) $\dfrac{4}{5}$ (C) $\dfrac{3}{8}$ (D) $\dfrac{3}{4}$

Ans. **(C)**

Sol. We have $3x^2+4xy+4y^2+2x-2y+1+\alpha=0$

$\Rightarrow \underbrace{x\left(3x+2y+1\right)}_{\text{vanishes}} + \underbrace{y\left(2x+4y-1\right)}_{\text{vanishes}} - y + x + 1 + \alpha = 0$

$\Rightarrow x - y + 1 + \alpha = 0$

$\therefore$ equations are $3x+2y+1=0$

$$2x+4y-1=0 \text{ and } x-y+(1+\alpha)=0$$

So, they will admit a unique solution, if $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 4 & -1 \\ 1 & -1 & 1+\alpha \end{vmatrix} = 0 \Rightarrow \alpha = \frac{3}{8}$

7. If the points (– 2, 0), $\left(-1, \frac{1}{\sqrt{3}}\right)$ and (cos θ, sin θ) are collinear, then the number of values of θ when $0 \le \theta \le \frac{\pi}{2}$.

(A) 0 (B) 1 (C) 2 (D) Infinite

Ans. (A)

Sol. For collinearity of 3 points $\begin{vmatrix} -2 & 0 & 1 \\ -1 & \frac{1}{\sqrt{3}} & 1 \\ \cos\theta & \sin\theta & 1 \end{vmatrix} = 0 \Rightarrow \sqrt{3}\sin\theta - \cos\theta = 2 \Rightarrow \theta = \frac{2\pi}{3}$

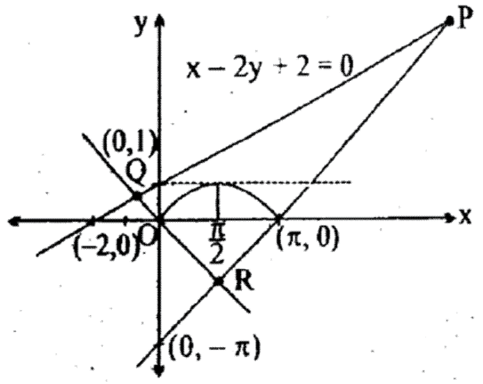
MCQ (One or more than one correct) :

8. The true set of real values of a such that the point M(a, sina) lies inside the triangle formed by the lines $x-2y+2=0$, $x+y=0$ and $x-y-\pi=0$, is

(A) $(0,\pi)$ (B) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (C) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{2},\pi\right)\cup\left(\frac{2\pi}{3},2\pi\right)$-

Ans. (ABC)

Sol.



As point M(a, sin a) lies on y = sin x, so clearly point M lies inside ΔPQR, if $a \in (0,\pi)$

9. Consider the three linear equations, ax + by + c = 0, bx + cy + a = 0, cx + ay + b = 0, where a, b, c$\in \mathbb{R}$. Which of the following is (are) correct?

(A) If a + b + c = 0 and $a^2 + b^2 + c^2 = ab + bc + ca$, then the lines represent the entire XY plane.

(B) If a + b + c = 0 and $a^2 + b^2 + c^2 \neq ab + bc + ca$, then the lines are concurrent.

(C) If a + b + c $\neq$ 0 and $a^2 + b^2 + c^2 = ab + bc + ca$, then the lines are coincident.

(D) If a + b + c $\neq$ 0 and $a^2 + b^2 + c^2 \neq ab + bc + ca$, then the lines are neither coincident nor concurrent.

Ans. (ABCD)

Sol. $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

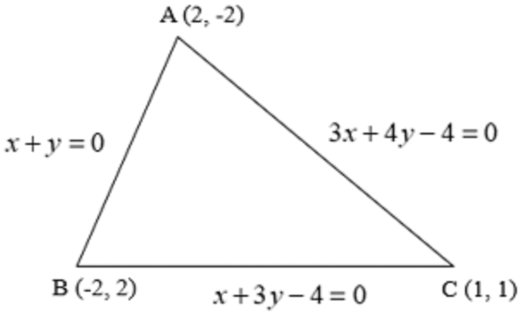
Also, if $a^2+b^2+c^2 = ab+bc+ac,$ then $a=b=c$

10. The triangle formed by the lines x + y = 0, 3x + y −4 = 0 and x +3 y −4 = 0 is

(A) isosceles (B) scalene (C) acute angled (D) obtuse angled

Ans. **(AD)**

Sol. Vertices of triangles can be found by solving equations of lines in pairs



Now

So BC = AC $\Rightarrow$ Iso scales triangles

$AB = 4\sqrt{2}$

$BC = \sqrt{10}$

$AC = \sqrt{10}$

So, checking angle C

$\cos C = \dfrac{AC^2 + BC^2 - AB^2}{2AC \cdot BC}$

$= \dfrac{10+10-32}{2 \cdot 10 \cdot 10} < 0$

So triangle is obtuse angled

11. Consider the straight lines L_1 : x + y = 2, L_2 : 2 x − y + 3 = 0 and a variable point $P(a,a^2)$ where $a \in R$. 'P' lies in the acute angle not containing the origin if 'a ' lies in the interval

(A) $(-4,-3)$ (B) $\left(\dfrac{3}{2},\dfrac{5}{2}\right)$ (C) $\left(-2,\dfrac{-3}{2}\right)$ (D) (5, 7)

Ans. **(AD)**

Sol. Let $L_1 : x+y-2=0$

$L_2 : 2x-y+3=0$

sign given by origin

$L_1(0) = 0+0-2<0$

$L_2(0) = 0-0+3<0$

Sign given by points in acute angle:

$-(a_1a_2 + b_1b_2) = -(2-1) = -1 < 0$

So (a, a^2) should give

(+)ve sign with L_1 and (−)ve sign with L_2 .

So, $2a + a^2 - 2 > 0$

$\Rightarrow a^2 + 2a - a - 2 > 0$

$\Rightarrow (a+2)(a-1) > 0$

$\Rightarrow a \in (-\infty, -2) \cup (1, \infty)$ and $2a - a^2 + 3 < 0$

$a^2 - 2a - 3 > 0$

$(a-3)(a+1) > 0$

Their intersection gives $(-\infty, -2) \cup (3, \infty)$

12. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then :

(A) the lines will pass through a fixed point (B) there will be a set of parallel lines

(C) all the lines intersect the line $x = x_1$ (D) all the lines will be parallel to the line $y = x_1$.

Ans. (BC)

Sol. Line $(y - y_1) = m(x - x_1)$ in passing through a fixed point (x_1, y_1) & having slope m then if m and x_1 are fixed then line will set of parallel lines which are intersects the line $x = x_1$.

13. If $a^2 + 9b^2 - 4c^2 = 6ab$ then the family of lines $ax + by + c = 0$ are concurrent at :

(A) (1/2, 3/2) (B) (– 1/2, – 3/2) (C) (– 1/2, 3/2) (D) (1/2, – 3/2)

Ans. (CD)

Sol. $(a - 3b)^2 - (2c)^2 = (a - 3b + 2c)(a - 3b - 2c) = 0$ **]**

Comprehension Type Question:

Comprehension # 1

Paragraph for question nos. 14 to 16

Let ABCD is a square with sides of unit length. Points E and F are taken on sides AB and AD respectively so that AE = AF. Let P be a point inside the square ABCD.

14. The maximum possible area of quadrilateral CDFE is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

Ans. (C)

15. The value of $(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2$ is equal to

(A) 3 (B) 2 (C) 1 (D) 0

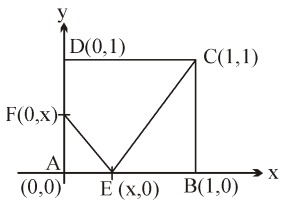
Ans. (D)

16. Let a line passing through point A divides the square ABCD into two parts so that area of one portion is double the other, then the length of portion of line inside the square is

(A) $\frac{\sqrt{10}}{3}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{\sqrt{11}}{3}$ (D) $\frac{2}{\sqrt{3}}$

Ans. **(B)**

Sol. (i) Area of CDFE

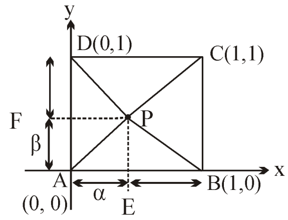


$$A = 1 - \frac{1}{2}x^2 - \frac{1}{2}(1-x)$$

$$= \frac{2-x^2-1+x}{2} = \frac{1+x-x^2}{2}$$

$$A_{max} = \frac{1+\frac{1}{2}-\frac{1}{4}}{2} = \frac{5}{8} \quad \text{at } x = \frac{1}{2}$$

(ii) $(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2$

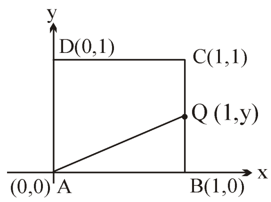


$= \alpha^2 + \beta^2 - [(\alpha-1)^2 + \beta^2] + (\alpha-1)^2 + (\beta-1)^2 - [\alpha^2 + (\beta-1)^2]$

$= \alpha^2 + \beta^2 - (\alpha-1)^2 - \beta^2 + (\alpha-1)^2 + (\beta-1)^2 - \alpha^2 - (\beta-1)^2$

$= 0$

(iii) $\frac{1}{2}y(1) = \frac{1}{3}(1)$



$$y = \frac{2}{3}$$

$$L_{AQ} = \sqrt{(1)^2 + \left(\frac{2}{3}\right)^2} = \frac{\sqrt{13}}{3}$$]

Numerical based Questions :

17. The equation $9x^3 + 9x^2y - 45x^2 = 4y^3 + 4xy^2 - 20y^2$ represents 3 straight lines, two of which pass through the origin. Find the area of the triangle formed by these lines (in sq. units).

Ans. **30**

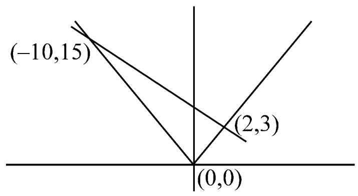
Sol. $9x^2(x + y - 5) = 4y^2(y + x - 5)$

$(x + y - 5)(3x + 2y)(3x - 2y) = 0$

lines are $x + y = 5$

$3x + 2y = 0$

$3x - 2y = 0$



$$A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 3 & 1 \\ -10 & 15 & 1 \end{vmatrix} = \frac{1}{2}(30 + 30) = 30 \text{ sq. units.]}$$

18. If the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a, b, c and $a \neq 1$, $b \neq 1$ and $c \neq 1$, then find the value of abc– (ab + bc + ac)+3 (a+b+c).

Ans. **0**

Sol. Let equation of line is $\ell x + my + n = 0$ ...(i)

given $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear

$\left(\frac{t^3}{t-1}, \frac{t^2-3}{t-1}\right)$ is general point which satisfies line (i)

$\ell\left(\frac{t^3}{t-1}\right) + m\left(\frac{t^2-3}{t-1}\right) + n = 0 \quad \Rightarrow \quad \ell t^3 + m t^2 + nt - (3m + n) = 0$

$a + b + c = -\frac{m}{\ell} \quad \Rightarrow \quad ab + bc + ac = \frac{n}{\ell} \quad \Rightarrow \quad abc = \frac{3m+n}{\ell}$

Now LHS = abc – (ab + bc + ac) + 3 (a + b + c) $= \frac{(3m+n)}{\ell} - \frac{n}{\ell} + 3\left(\frac{-m}{\ell}\right) = 0$

19. If the straight lines joining the origin and the points of intersection of the curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis, then find the value of | k |.

Ans. **1**

Sol. Homogenize $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ by $x + ky = 1$

$5x^2 + 12xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$

it is equally indined with x-axes hence coeff. xy = 0 $\Rightarrow 12 + 4k - 2 + 6k = 0 \Rightarrow k = -1$

20. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so find λ.

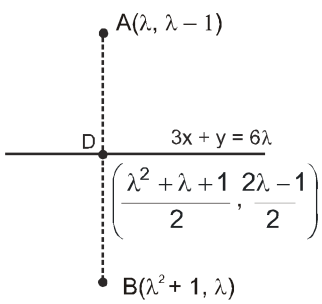
Ans. **(2)**

Sol. D is mid point of AB and lies on the line $3x + y = 6\lambda$

$\Rightarrow \quad 3 \cdot \frac{\lambda^2+\lambda+1}{2} + \frac{2\lambda-1}{2} = 6\lambda$

$3\lambda^2 - 7\lambda + 2 = 0$ ....(1)

$\lambda = \frac{1}{3}, 2$



multiplication of slope of AB & line = –1

$\frac{-1}{\lambda - \lambda^2 - 1}(-3) = -1$

$\lambda^2 - \lambda - 2 = 0$ ....(2)

$\lambda = -1, 2$

$\lambda = 2$ satisfies both (1) & (2)

Matrix Match Type :

21. Let ABC be a triangle such that the coordinates of A are (– 3, 1). Equation of the median through B is $2x + y - 3 = 0$ and equation of the angular bisector of C is $7x - 4y - 1 = 0$. Then match the entries of column-I with their corresponding correct entries of column-II.

	Column-I		**Column-II**
(A)	Equation of the line AB is	(P)	$2x + y - 3 = 0$
(B)	Equation of the line BC is	(Q)	$2x - 3y + 9 = 0$
(C)	Equation of CA is	(R)	$4x + 7y + 5 = 0$
		(S)	$18x - y - 49 = 0$

Ans. (A) A→R; B→S; C→Q (B) A→Q; B→S; C→P

(C) A→R; B→Q; C→S (D) A→R; B→P; C→S

Ans. **(A)**

Sol. BM is the median and CF is the angle bisector.

Since M lies on the median.

Let $M(\alpha, 3 - 2\alpha)$

Now let C be (a, b)

Hence $\frac{a-3}{2} = \alpha \Rightarrow a = 2\alpha + 3$

and $\frac{b+1}{2} = 3 - 2\alpha \Rightarrow b = 6 - 4\alpha - 1 = 5 - 4\alpha$

Hence C is $(2\alpha + 3, 5 - 4\alpha)$

As 'C' lies on angle bisector i.e. $7x - 4y - 1 = 0$

Hence $7(2\alpha + 3) - 4(5 - 4\alpha) - 1 = 0 \Rightarrow (14\alpha + 21) - (20 - 16\alpha) - 1 = 0$

$30\alpha = 0 \Rightarrow \alpha = 0$

Hence C is (3, 4) and $M \equiv (0, 3)$

Equation of AC is

$y - 1 = \frac{4}{6}(x+3) \Rightarrow 3y - 3 = 2x + 6 \Rightarrow 2x - 3y + 9 = 0 \longrightarrow AC \Rightarrow (C) \to (Q)$

Again let the equation of BC is

$2x - 3y + 9 + \lambda(7x - 4y - 1) = 0$ ....(1)

(As any line through the intersecting of AC and FC)

Let (x_1, y_1) be a point on $7x - 4y - 1 = 0$ i.e. $7x_1 - 4y_1 - 1 = 0$

Hence $\left|\frac{2x_1 - 3y_1 + 9}{\sqrt{13}}\right| = \left|\frac{(2x_1 - 3y_1 + 9) + \lambda\overbrace{(7x_1 - 4y_1 + 1)}^{\text{zero}}}{\sqrt{(7\lambda+2)^2 + (4\lambda+3)^2}}\right|$

$\therefore (7\lambda + 2)^2 + (4\lambda + 3)^2 = 13,$ solving $\lambda = -\frac{4}{5}$ or $\lambda = 0$ (not possible)

Hence equation of BC

$$10x - 15y + 45 = 28x - 16y - 4$$

$\Rightarrow \quad 18x - y - 49 = 0 \longrightarrow$ BC

Solving $2x + y - 3 = 0$ and $18x - y - 49 = 0$

Subjective Type Questions:

22. Two vertices of a triangle are (4, –3) and (–2, 5). If the orthocentre of the triangle is at (1, 2), then find the third vertex.

Ans. **(33, 26)**

[Sol. $m_{BC} = \frac{5+3}{-2-4} = \frac{8}{-6} = \frac{-4}{3}$

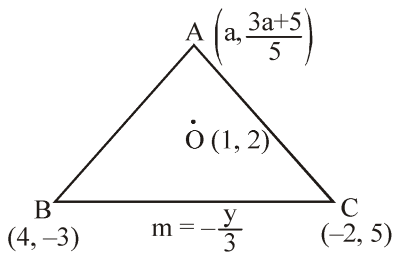
$\therefore \quad m_{AO} = \frac{3}{4}$

$\therefore$ Equation of AO is,

$$y - 2 = \frac{3}{4}(x - 1)$$

$\Rightarrow 4y - 8 = 3x - 3 \Rightarrow 3x - 4y + 5 = 0$

$\therefore$ Let point A be $\left(a, \frac{3a+5}{4}\right)$



$\therefore \quad m_{AB}$ is $\frac{\frac{3a+5}{4}+3}{a-4} = \frac{3a+17}{4(a-4)}$

$m_{OC} = \frac{3}{-3} = -1$

AB $\perp$ OC

$\therefore \quad \frac{3a+17}{4(a-4)} = 1 \Rightarrow 3a + 17 = 4a - 16 \Rightarrow a = 33$

$\therefore \quad A\left(33, \frac{104}{4}\right)$

Point A(33, 26) **Ans.**]

23. A pair of perpendicular straight lines is drawn through the origin and forming with the line 2x + 3y = 6 an isosceles Δ right angled at the origin. Find the equation of the pair of straight lines and area of the Δ.

Ans. **$5x^2 - 24xy - 5y^2 = 0$, $\frac{36}{13}$ sq. unit**

Sol. Then slope of line OA and OB are

$$\tan(45° \pm \theta) \Rightarrow \frac{1 \pm \tan\theta}{1 \mp \tan\theta}$$

when $\tan\theta = -\frac{2}{3}$

$\therefore$ slopes are $\frac{1+\left(-\frac{2}{3}\right)}{1-\left(-\frac{2}{3}\right)} = \frac{\frac{1}{3}}{\frac{5}{3}} = \frac{1}{5}$ and $\frac{1-\left(-\frac{2}{3}\right)}{1+\left(-\frac{2}{3}\right)} = \frac{\frac{5}{3}}{\frac{1}{3}} = 5$

$\therefore$ Equations are $y = \frac{1}{5}x$ $\quad y = 5x$

$\therefore$ Pair of straight lines are $\left(y - \frac{1}{5}x\right)(y - 5x) = 0$

$\Rightarrow$ $5x^2 - 24xy - 5y^2 = 0$

and area of $\Delta OAB = \frac{1}{2} \times P \times 2P = P^2$ (When P = perpendicular distance of line from origin)

$$P = \frac{6}{\sqrt{13}} \quad \Rightarrow \quad \text{Area of } P^2 = \frac{36}{13}$$

24. A straight line passing through O(0, 0) cuts the lines $x = \alpha$, $y = \beta$ and $x + y = 8$ at A, B and C respectively such that $OA \cdot OB \cdot OC = 48\sqrt{2}$ and $f(\alpha, \beta) \le 0$

where $f(x, y) = \left|\frac{y}{x} - \frac{3}{2}\right| + (3x - 2y)^6 + \sqrt{ex + 2y - 2e - 6}$.

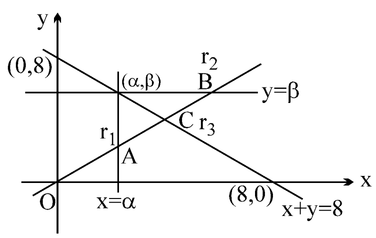
(i) Find the point of intersection of lines $x = \alpha$ and $y = \beta$.

(ii) Find the value of (OA + OB + OC).

(iii) Find the equation of line OA.

Ans. (i) $\alpha = 2$ and $\beta = 3$; (ii) $9\sqrt{2}$; (iii) $x - y = 0$]

Sol. (i) $\because$ $f(\alpha, \beta) \le 0$



$\Rightarrow$ $\left|\frac{\beta}{\alpha} - \frac{3}{2}\right| + (3\alpha - 2\beta)^{3!} + \sqrt{e\alpha + 2\beta - 2e - 6} \le 0$

$\Rightarrow$ $\frac{\beta}{\alpha} - \frac{3}{2} = 0 \quad \Rightarrow \quad \beta = \frac{3\alpha}{2}$

and $e\alpha + 2\beta - 2e - 6 = 0 \Rightarrow \alpha = 2$ and $\beta = 3$

(ii) Let equation of straight line be

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$$

let $OA = r_1$, $OB = r_2$ and $OC = r_3$

then $A(r_1\cos\theta, r_1\sin\theta)$, $B = (r_2\cos\theta, r_2\sin\theta)$ and $C = (r_3\cos\theta, r_3\sin\theta)$

$\because$ A lies on $x = 2 \Rightarrow$ $r_1\cos\theta = 2$ $\Rightarrow$ $r_1 = \frac{2}{\cos\theta}$ ....(1)

$\because$ B lies on y = 3 $\Rightarrow$ $r_1 \sin\theta = 3$ $\Rightarrow$ $r_2 = \frac{3}{\sin\theta}$ ....(2)

and C lines on x + y = 8

$\therefore$ $r_3 \cos\theta + r_3 \sin\theta = 8$

$r_3 = \frac{8}{\cos\theta + \sin\theta}$ ....(3)

$\because$ $r_1 r_2 r_3 = 48\sqrt{2}$

$\Rightarrow$ $\frac{2}{\cos\theta} \cdot \frac{3}{\sin\theta} \cdot \frac{8}{(\cos\theta + \sin\theta)} = 48\sqrt{2}$

$\Rightarrow$ $\sqrt{2}\cos\theta\sin\theta(\cos\theta + \sin\theta) = 1$ $\Rightarrow$ $\sin 2\theta \sin\left(\theta + \frac{\pi}{4}\right) = 1$

both sin 2θ and $\sin\left(\theta + \frac{\pi}{4}\right)$ should be 1

$\therefore$ $\theta = \frac{\pi}{4}$

$\therefore$ from (1), $r_1 = 2\sqrt{2}$

from (2), $r_2 = 3\sqrt{2}$

from (3), $r_3 = \frac{8}{\sqrt{2}} = 4\sqrt{2}$

$\therefore$ $r_1 + r_2 + r_3 =$

(iii) $\therefore$ equation of line OA will be

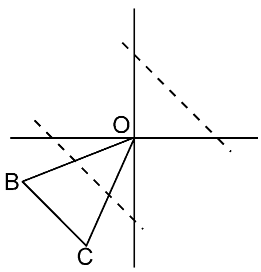
$\frac{x}{\cos(\pi/4)} = \frac{y}{\sin(\pi/4)}$ $\Rightarrow$ x – y = 0

25. The vertices of a triangle OBC are O(0,0) B(–3,–1) and C(–1,–3). Find the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$.

Ans. $\mathbf{x + y + \frac{1}{\sqrt{2}} = 0.}$

Sol. Slope of BC is

$= \frac{-3-(-1)}{-1-(-3)} = \frac{-2}{2} = -1$



$\therefore$ Equation of a line parallel to BC is

y = – x + c

i.e. x + y – c = 0

its distnace from the origin is

$$\left| \frac{c}{\sqrt{2}} \right| = \frac{1}{2}$$

$\therefore$ $c = \pm\frac{1}{\sqrt{2}}$

$\therefore$ Equations of the lines are

$$x + y \pm \frac{1}{\sqrt{2}} = 0$$

Since the required line intersects OB and OC, therefore, it is the line whose y intercept is negative. Hence the required line is $x + y + \frac{1}{\sqrt{2}} = 0$.