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MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-21

3D

ANSWER KEY OF CAPS-21

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(D)
6.	(A)	**7.**	(D)	**8.**	(AC)	**9.**	(AB)	**10.**	(ABC)
11.	(BC)	**12.**	(D)	**13.**	(B)	**14.**	(D)	**15.**	(A)
16.	(B)	**17.**	(6)	**18.**	(9)	**19.**	(198)	**20.**	(8)
21.	(3.26)	**22.**	(61)	**23.**	(25.40)	**24.**	(2)		
25.	A→R; B→Q, C→Q, S, D→P, S								

SCQ (Single Correct Type) :

1. For the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$, which one of the following is incorrect?

(A) it lies in the plane x – 2y + z = 0
(B) it is same as line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$
(C) it passes through (2, 3, 5)
(D) it is parallel to the plane x – 2y + z – 6 = 0

Ans. **(C)**

Sol. On (1, 2, 3) satisfies the plane x – 2y + z = 0 and also $(\hat{i}+2\hat{j}+3\hat{k}).(\hat{i}-2\hat{j}+\hat{k})=0 \Rightarrow$(A)

Since the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ both satisfy t + 1, 2t + 2 and 3t + 3

Hence both are same ⇒ (B). Given line is obviously | | to the plane x – 2y + z = 6 ⇒ (D)

2. Given planes

P_1 : cy + bz = x

P_2 : az + cx = y

P_3 : bx + ay = z

P_1, P_2 and P_3 pass through one line, if

(A) $a^2 + b^2 + c^2 = ab + bc + ca$
(B) $a^2 + b^2 + c^2 + 2abc = 1$
(C) $a^2 + b^2 + c^2 = 1$
(D) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + 2abc = 1$

Ans. **(B)**

Sol. Infinite solution $\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow \vec{n}_1 \times \vec{n}_2 \cdot \vec{n}_3$

note that 3 such planes can meet only at one point i.e. (0, 0, 0) or they may have the same line of intersection i.e. at infinite solution.

3. The equation of the line passing through A(1, 0, 3), intersecting the line $\frac{x}{2}=\frac{x-1}{3}=\frac{z-2}{1}$ and which is parallel to the plane x + y + z = 2 is _____.

(A) $\frac{3x-1}{2}=\frac{2y-3}{3}=\frac{2z-5}{-1}$ (B) $\frac{x-1}{2}=\frac{y-0}{3}=\frac{z-3}{-1}$

(C) $\frac{x-1}{2/3}=\frac{y-1}{3/2}=\frac{z-3}{-1/2}$ (D) $\frac{3x-1}{2}=\frac{2y-3}{-3}=\frac{6z-13}{5}$

Ans. **(D)**

Sol. Any point on $\frac{x}{2}=\frac{x-1}{3}=\frac{z-2}{1}$ is $(2\lambda,1+3\lambda,2+\lambda)$

So, vector from A(1, 0, 3) to this point is $(2\lambda-1,1+3\lambda,\lambda-1)$

Since it is parallel to x + y + z = 2, we have

$(2\lambda-1,1+3\lambda,\lambda-1)\cdot(1,1,1)=0\Rightarrow\lambda=\frac{1}{6}$

So, the point is $\left(\frac{1}{3},1+\frac{1}{2},2+\frac{1}{6}\right)\equiv\left(\frac{1}{3},\frac{3}{2},\frac{13}{6}\right)$

Direction ratios of vector are $\left(\frac{1}{3}-1,\frac{3}{2}-3\right)\equiv\left(-\frac{2}{3},\frac{3}{2},-\frac{5}{6}\right)$

Equation of line is $\frac{x-\left(\frac{1}{3}\right)}{-\left(\frac{2}{3}\right)}=\frac{y-\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)}=\frac{z-\left(\frac{13}{6}\right)}{-\left(\frac{5}{6}\right)}$,

that is $\frac{3x-1}{-2}=\frac{2y-3}{3}=\frac{6z-13}{-5}$ or $\frac{3x-1}{2}=\frac{2y-3}{-3}=\frac{6z-13}{5}$

4. L_1 and L_2 are two lines whose vector equations are given below.

$L_1:\bar{r}=\lambda\left(\left(\cos\theta+\sqrt{3}\right)\hat{i}+\left(\sqrt{2}\sin\theta\right)\hat{j}+\left(\cos\theta-\sqrt{3}\right)\hat{k}\right)$

$L_1:\bar{r}=\mu\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

Here, λ and μ are scalars. If the angle α is the acute angle between the two lines and is independent of θ, then a possible value of α is _____.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans. **(A)**

Sol. $\cos\alpha=\frac{a\left(\cos\theta+\sqrt{3}\right)+b\left(\sqrt{2}\sin\theta\right)+c\left(\cos\theta-\sqrt{3}\right)}{\sqrt{a^2+b^2+c^2}\sqrt{\cos^2\theta+3+2\sqrt{3}\cos\theta+2\sin^2\theta+\cos^2\theta+3-2\sqrt{3}\cos\theta}}$

$=\frac{(a+c)\cos\theta+\sqrt{2}b\sin\theta+\sqrt{3}(a-c)}{\sqrt{a^2+b^2+c^2}\sqrt{8}}$

For α to be independent of θ,

$a + c = 0$ .....(1)

$b = 0$ ....(2)

$\therefore\ \cos\alpha = \dfrac{\sqrt{3}(a-c)}{\sqrt{2c^2}\sqrt{8}} = \dfrac{2\sqrt{3}a}{\sqrt{2}a\sqrt{8}} = \dfrac{\sqrt{3}}{2} \Rightarrow \alpha = \dfrac{\pi}{6}$

5. Let $P_1 \equiv \bar{r}\cdot\bar{r}_1 = d$, $P_2 \equiv \bar{r}\cdot\bar{r}_2 = d_2$, $P_3 \equiv \bar{r}\cdot\bar{r}_3 = d_3$ be three planes where $\bar{r}_1, \bar{r}_2, \bar{r}_3$ are three non-coplanar vectors. Then the lines $P_1 = 0 = P_2$, $P_2 = 0 = P_3$ and $P_3 = 0 = P_1$ are ______.

(A) parallel lines (b) coplanar lines (c) co-incident lines (d) concurrent lines

Ans. **(D)**

Sol. All the 3 are lines of intersection of planes

P_1 and P_2 , P_2 and P_3 , P_3 and P_1

$\therefore\ \bar{b}_1, \bar{b}_2, \bar{b}_3$ are non-coplanar

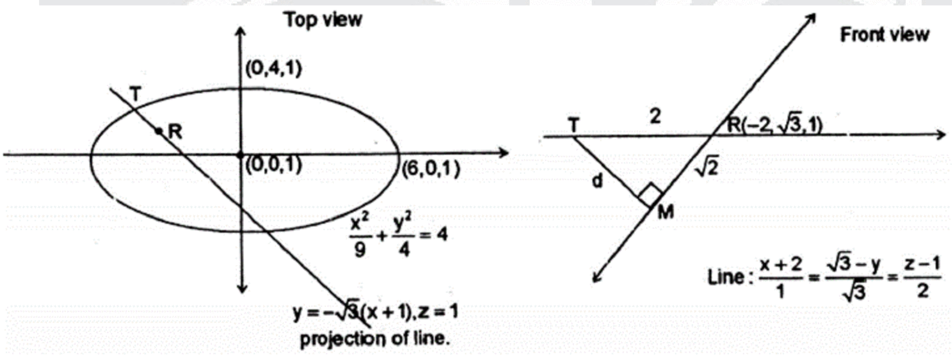
Hence, the lines intersect at a unique point

6. Let P(x,y,1) and Q(x,y,z) be points on the curves $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 4$ and $\dfrac{x+2}{1} + \dfrac{\sqrt{3}-y}{\sqrt{3}} = \dfrac{z-1}{2}$ respectively. Then, the minimum distance between P and Q is ______.

(A) $\sqrt{2}$ (B) $\sqrt{\dfrac{7}{2}}$ (C) 2 (D) none of these

Ans. **(A)**

Sol.



Projection of line on ellipse at z = 1 is given by

$\dfrac{x+2}{1} = \dfrac{\sqrt{3}-y}{\sqrt{3}} \Rightarrow y = -\sqrt{3}(x+1)$ .....(1)

On solving 1 with ellipse we get $T \equiv (-3, 2\sqrt{3}, 1)$

TR = 2

Projection of TR on line is $(\hat{i} - \sqrt{3}\hat{j}) \cdot \left(\dfrac{\hat{i} - \sqrt{3}\hat{j} + 2\hat{k}}{\sqrt{8}}\right) = \sqrt{2}$

Hence shortest distance, d, is $\sqrt{2}$.

7. A line L_1 with direction ratios –3, 2, 4 passes through the point A(7, 6, 2) and a line L_2 with direction ratios 2, 1, 3 passes through the point B(5, 3, 4). A line L_3 with direction ratios 2, –2, –1 intersects L_1 and L_3 at C and D.

The equation of the plane parallel to the line L_1 and containing the line L_2 is equal to _____.

(A) x +3y + 4z = 30 (B) x + 2y + z =15

(C) 2x − y + z =11 (D) 2x +17y −7z = 33

Ans. **(D)**

Sol. Equation of plane parallel to L_1 and containing L_2 is given by

$$a(x-5)+b(y-3)+c(z-4)=0$$

$$\therefore\ 2a+b+3c=0 \text{ and } -3a+2b+4c=0$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{-17}=\frac{c}{7}$$

So, the required plane is $2x+17y-7z=33$

MCQ (One or more than one correct) :

8. Let $\bar{a}=2\hat{i}-\hat{j}+\hat{k},\ \bar{b}=\hat{i}+2\hat{j}-\hat{k},\ \bar{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\bar{b}$ and $\bar{c}$ whose projection on $\bar{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is _____.

(A) $2\hat{i}+3\hat{j}-3\hat{k}$ (B) $2\hat{i}+3\hat{j}+3\hat{k}$ (C) $-2\hat{i}-\hat{j}+5\hat{k}$ (D) $2\hat{i}+\hat{j}+5\hat{k}$

Ans. **(AC)**

Sol. $\bar{r}=\bar{b}+\lambda\bar{c}=(1+\lambda)\hat{i}+(2+\lambda)\hat{j}-(1+2\lambda)\hat{k}$

Projection of $\bar{r}$ on $\bar{a}$ is $\frac{\bar{r}\cdot\bar{a}}{|\bar{a}|}=\pm\sqrt{\frac{2}{3}}$

$$\Rightarrow \frac{2(1+\lambda)-(2+\lambda)-(1+2\lambda)}{\sqrt{6}}=\pm\sqrt{\frac{2}{3}}$$

$$\Rightarrow -\lambda-1\pm 2$$

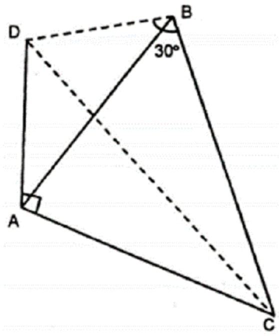
$$\Rightarrow \lambda=1,-3$$

$$\therefore \bar{r}=2\hat{i}+3\hat{j}-3\hat{k},\quad -2\hat{i}-\hat{j}+5\hat{k}$$

9. Let DABC be a tetrahedron such that AD is perpendicular to the base ABC and ∠ABC = 30º. The volume of the tetrahedron is 18 cubic units. If the value of AB + BC + AD is minimum, then the length of AC is _____.

(A) $6\sqrt{2-\sqrt{3}}$ (B) $3\left(\sqrt{6}-\sqrt{2}\right)$ (C) $6\sqrt{2+\sqrt{3}}$ (D) $3\left(\sqrt{6}+\sqrt{2}\right)$

Ans. **(AB)**

Sol. 

Volume $= \frac{1}{3} AD \left(\frac{1}{2} AB \cdot BC \sin 30º \right)$

$= 18 = \frac{1}{12}(AD)(AB)(BC) \Rightarrow (AD)(AB)(BC) = 216$

Maximum value occurs when AB=BC=AD=6

Hence $AC = \sqrt{AB^2 + BC^2 - 2(AB)(BC)\cos 30º} = 6\sqrt{2-\sqrt{3}}$

10. If the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ intersects line $3\beta^2 x + 3(1-2\alpha)y + z = 3 = -\frac{1}{2}\left(6\alpha^2 x + 3(1-2\beta)y + 2z\right)$, then the point $(\alpha, \beta, 1)$ lies on the plane ______.

(A) 2x − y + z = 4 (b) x + y − z = 2 (c) x −2y = 0 (d) 2x − y = 0

Ans. **(ABC)**

Sol. Intersection of line with both the planes are the same

$\Rightarrow \frac{3}{3\beta^2 + 6(1-2\alpha) + 3} = \frac{-6}{6\alpha^2 + 6(1-2\beta) + 6}$

$\Rightarrow 2(\beta - 1)^2 + 3(\alpha - 2)^2 = 0$

$\Rightarrow \alpha = 2, \beta = 1$

11. Consider the planes P_1 : 2x + y + z + 4 = 0, P_2 : y − z + 4 = 0 and P_3 : 3x + 2 y + z +8 = 0. Let L_1 , L_2 , L_3 be the lines of intersection of the planes P_2 and P_3 , P_3 and P_1 , and P_1 and P_2 respectively. Then,

(A) Atleast two of the lines L_1 , L_2 and L_3 are non–parallel

(B) Atleast two of the lines L_1 , L_2 and L_3 are parallel

(C) The three planes intersect in a line

(D) The three planes form a triangular prism

Ans. **(BC)**

Sol. Observe that the line L_1 , L_2 and L_3 are parallel to the vector (1, −1, −1)

Also, $\Delta = 0 = \Delta_1$ and $b_1c_1 - b_2c_1 \neq 0$

$\therefore$ The three planes intersect in a line

Comprehension Type Question:

Comprehension # 1

Consider a plane

$x + y - z = 1$ and the point A(1, 2, –3)

A line L has the equation

$x = 1 + 3r$

$y = 2 - r$

$z = 3 + 4r$

12. The co-ordinate of a point B of line L, such that AB is parallel to the plane, is

(A) 10, –1, 15 (B) –5, 4, –5 (C) 4, 1, 7 (D) –8, 5, –9

Ans. (D)

13. Equation of the plane containing the line L and the point A has the equation

(A) $x - 3y + 5 = 0$ (B) $x + 3y - 7 = 0$ (C) $3x - y - 1 = 0$ (D) $3x + y - 5 = 0$

Ans. (B)

Sol. (i) line $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = r$; A(1, 2, – 3)

Any point say B $\equiv 3r + 1, 2 - r, 3 + 4r$ (on the line L)

$\overline{AB} = 3r, -r, 4r + 6$

Hence, $\overline{AB}$ is parallel to $x + y - z = 1$

hence, $3r - r - 4r - 6 = 0$

$2r = -6 \quad \Rightarrow \quad r = -3$

hence B is –8, 5, –9

(ii) Equation of plane containing the line L is

$A(x - 1) + B(y - 2) + C(z - 3) = 0$, where $3A - B + 4C = 0$ ....(1)

$\therefore$ (1) also contains the point A(1, 2, –3)

hence C = 0; 3A = B

equation of plane $x - 1 + 3(y - 2) = 0$

$x + 3y - 7 = 0$ Ans.

Comprehension # 2

A ray of light emanating from the point source $P(\hat{i} - 3\hat{j} + 2\hat{k})$ and travelling parallel to the line $\frac{x-2}{1} = \frac{y}{3} = \frac{z+1}{2}$ is incident on the plane $x + 3y - 3z = 0$ at the point Q. After reflection from the plane the ray travels along the line QR. It is also known that the incident ray, reflected ray and the normal to the plane at the point of incident are in the same plane.

14. The position vector of Q is _____.

(A) $3\hat{i} + 15\hat{j} + 6\hat{k}$ (B) $3\hat{i} + 6\hat{j} + 3\hat{k}$ (C) $-3\hat{i} - 6\hat{j} - 3\hat{k}$ (D) $-3\hat{i} - 15\hat{j} - 6\hat{k}$

Ans. (D)

Sol. Q is point of intersection of line $\frac{x-2}{1}=\frac{y}{3}=\frac{z+1}{2}$ and plane x +3y −3z = 0

15. The vector equation of line containing QR is _____.

(A) $\bar{r}=\left(12\hat{i}+22\hat{j}+4\hat{k}\right)+\lambda\left(15\hat{i}+37\hat{j}+10\hat{k}\right)$

(B) $\bar{r}=\left(3\hat{i}+15\hat{j}+6\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)$

(C) $\bar{r}=\left(3\hat{i}+6\hat{j}+3\hat{k}\right)+\lambda\left(15\hat{i}+37\hat{j}+10\hat{k}\right)$

(D) $\bar{r}=\left(\hat{i}+\hat{j}+\hat{k}\right)+\lambda\left(-\hat{i}-\hat{j}-\hat{k}\right)$

Ans. **(A)**

16. The equation of the plane in Cartesian form is _____.

(A) 5x + 2y − z +3 = 0 (B) 11x −5y + 2z = 30

(C) 5x − y − z = 6 (D) x − y + z = 6

Ans. **(B)**

Numerical based Questions :

17. The lengths of two opposite edges of a tetrahedron are 3 and 4 units, the shortest distance between them is equal to 6 unit and angle between them is 30 . Then the volume of tetrahedron in cubic units is _____.

Ans. **(6)**

Sol. Since we know that volume of tetrahedron $=\frac{1}{6}abd\sin\theta$

$=\frac{1}{6}(3)(4)(6)\sin 30^\circ = 6$ cubic unit

18. A line L_1 with direction ratios –3, 2, 4 passes through the point A(7, 6, 2) and a line L_2 with direction ratios 2, 1, 3 passes through the point B(5, 3, 4). A line L_3 with direction ratios 2, –2, –1 intersects L_1 and L_3 at C and D.

The length CD is equal to _____.

Ans. **(9)**

Sol. $L_1\equiv\frac{x-7}{-3}=\frac{y-6}{2}=\frac{z-2}{4}$ and $L_2\equiv\frac{x-5}{2}=\frac{y-3}{1}=\frac{z-4}{3}$

$\therefore$ $C(3\lambda+7, 2\lambda+6, 4\lambda+2)$ and $D(3\mu+5, \mu+3, 3\mu+4)$

So, $\frac{2-3\lambda-2\mu}{2}=\frac{3+2\lambda-\mu}{-2}=\frac{-2+4\lambda-3\mu}{-1}$

$\therefore$ $\lambda = 2$, $\mu = 1$

So, C(1, 10, 10) and D (7, 4, 7) $\Rightarrow$ CD = 9

19. A line L_1 with direction ratios –3, 2, 4 passes through the point A(7, 6, 2) and a line L_2 with direction ratios 2, 1, 3 passes through the point B(5, 3, 4). A line L_3 with direction ratios 2, –2, –1 intersects L_1 and L_3 at C and D.

The volume of the parallelepiped formed by $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$ is equal to _____.

Ans. **(138)**

Sol. Volume $=\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right]=\begin{vmatrix}2 & 3 & -2\\ 6 & -4 & -8\\ 0 & 2 & -5\end{vmatrix}=138$

20. If the length of shortest distance between the two lines $\frac{1}{2}(x-1)=\frac{1}{4}(y-3)=z+2$ and $3x-y-2z+4=0=2x+y+z+1$ is $s\sqrt{14}$, then the value of s is ______.

Ans. **(8)**

Sol. Any plane through the second line is $(3x-y-2z+4)+\lambda(2x+y+z+1)=0$

or $(3+2\lambda)x+(\lambda-1)y+(\lambda-2)z+(4+\lambda)=0$

If this plane is parallel to the first line its normal must be at right angle to first line.

$(3+2\lambda)2+(\lambda-1)4+(\lambda-2)=0\Rightarrow\lambda=0$

Equation of plane through the second line and parallel to the first line is

$3x-y-2z+4=0$ (1)

Now the required shortest distance s = perpendicular distance of a point (1, 3, –2) on the first line to the plane.

21. The perpendicular distance of the point (1,−2,3) to plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

Ans. **(3.26)**

Sol. $\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\times\left(4\hat{i}+2\hat{j}+3\hat{k}\right)=8\hat{i}-\hat{j}-10\hat{k}$ is parallel to the required plane

D.R.s of the normal to the plane is given by

$\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\left(8\hat{i}-\hat{j}-10\hat{k}\right)=26\left(\hat{i}-2\hat{j}+\hat{k}\right)$

Equation of the plane is (x −0)−2(y −0)+1(z −0) = 0

22. Let the equations of two straight lines L_1, L_2 be respectively be respectively be $x-5=\frac{y-3}{5}=\frac{z-15}{2}$ and $\frac{x}{2}=\frac{y+1}{5}=\frac{z+6}{3}$. A,B are two distinct points on the x – axis such that two straight lines l_1 , l_2 both perpendicular to the x – axis (l_1 through A, l_2 through B) are drawn so as to intersect both L_1, L_2 .

If θ is the acute angle between the lines l_1 , l_2 and $\cos\theta=\frac{\lambda}{5\sqrt{794}}$ = then λ =

Ans. **(61)**

Sol. Let (t, 0, 0) be a point on the x-axis through which a straight line L is drawn perpendicular to the x-axis and intersecting both the lines L_1 , L_2 D.R's of L may be taken as (0, 1, λ)

L and L_1 are coplanar $\Rightarrow \begin{vmatrix} 5-t & 3 & 15 \\ 1 & 5 & 2 \\ 0 & 1 & \lambda \end{vmatrix} = 0$

L and L_1 are coplanar $\Rightarrow \begin{vmatrix} t & 1 & 6 \\ 2 & 5 & 3 \\ 0 & 1 & \lambda \end{vmatrix} = 0$

Solving we get $(\lambda, t) = \left(\frac{-3}{4}, 2\right)$ or $\left(\frac{13}{25}, \frac{137}{5}\right)$

23. Let the equations of two straight lines L_1, L_2 be respectively be respectively be $x-5 = \frac{y-3}{5} = \frac{z-15}{2}$ and $\frac{x}{2} = \frac{y+1}{5} = \frac{z+6}{3}$. A,B are two distinct points on the x – axis such that two straight lines l_1 , l_2 both perpendicular to the x – axis (l_1 through A, l_2 through B) are drawn so as to intersect both L_1, L_2 .

The shortest distance between the lines l_1, l_2 is

Ans. **(25.40)**

Sol. Let (t, 0, 0) be a point on the x-axis through which a straight line L is drawn perpendicular to the x-axis and intersecting both the lines L_1 , L_2 D.R's of L may be taken as (0, 1, λ)

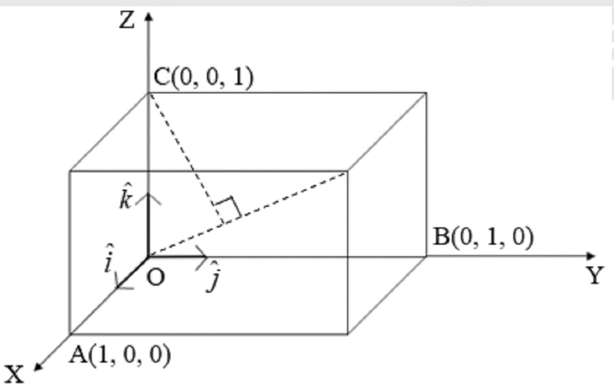
L and L_1 are coplanar $\Rightarrow \begin{vmatrix} 5-t & 3 & 15 \\ 1 & 5 & 2 \\ 0 & 1 & \lambda \end{vmatrix} = 0$

L and L_1 are coplanar $\Rightarrow \begin{vmatrix} t & 1 & 6 \\ 2 & 5 & 3 \\ 0 & 1 & \lambda \end{vmatrix} = 0$

Solving we get $(\lambda, t) = \left(\frac{-3}{4}, 2\right)$ or $\left(\frac{13}{25}, \frac{137}{5}\right)$

24. If the perpendicular distance of a corner of a unit cube from a diagonal not passing through it is d, then the value of $3d^2$ is

Ans. **(2)**



Sol.

Let the edges OA, OB and OC of the unit cube be along OX, OY and OZ respectively

Since OA = OB = OC = 1 unit, $\overline{OA} = \hat{i}$, $\overline{OB} = \hat{j}$, $\overline{OC} = \hat{k}$

Let CM be the perpendicular from the corner C on the diagonal OP

The vector equation of OP is $\vec{r} - = \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$

Therefore, OM = projection of $\overrightarrow{OC}$ on $\overrightarrow{OP} = \overrightarrow{OC} \cdot \overrightarrow{OP}$

$$= \hat{k} \cdot \frac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Now $OC^2 = OM^2 + CM^2$

$$\Rightarrow CM^2 = \left|\overline{OC}\right|^2 - OM^2 = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow CM = \sqrt{\frac{2}{3}}$$

Matrix Match Type :

25. Consider the following four pairs of lines in **column-I** and match them with one or more entries in **column-II**.

	Column-I		Column-II
(A)	$L_1 : x = 1 + t,\ y = t,\ z = 2 - 5t$ $L_2 : \vec{r} = (2, 1, -3) + \lambda(2, 2, -10)$	(P)	non coplanar lines
(B)	$L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$ $L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$	(Q)	lines lie in a unique plane
(C)	$L_1 : x = -6t,\ y = 1 + 9t,\ z = -3t$ $L_2 : x = 1 + 2s,\ y = 4 - 3s,\ z = s$	(R)	infinite planes containing both the lines
(D)	$L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ $L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$	(S)	lines do not intersect

Ans. **A→R; B→Q, C→Q, S, D→P, S**

Sol. (A) $L_1 : \frac{x-1}{1} = \frac{y-0}{1} = \frac{z-2}{-5}$; $\vec{V}_1 = \hat{i} + \hat{j} - 5\hat{k}$

$L_2 : \frac{x-2}{2} = \frac{y-1}{2} = \frac{z+3}{-10}$; $\vec{V}_2 = 2(\hat{i} + \hat{j} - 5\hat{k})$

Hence lines are parallel and both contains the points (1, 0, 2) and (2, 1, – 3) $\Rightarrow$ coincident line both L_1 and L_2 may lie in an infinite number of planes hence $\Rightarrow$ **(R)**

(B) $\left.\begin{array}{l}\vec{V}_1 = 2\hat{i} + 2\hat{j} - \hat{k} \\ \vec{V}_2 = \hat{i} - 2\hat{j} + 3\hat{k}\end{array}\right] \Rightarrow$ lines not parallel

Also both intersect at (3, 5, 1)

Hence lines are intersecting hence they lie on a unique plane $\Rightarrow$ **(P)**

(C) $L_1 : \frac{x-0}{-6} = \frac{y-1}{9} = \frac{z-0}{-3} = t$

$L_2 : \frac{x-1}{2} = \frac{y-4}{-3} = \frac{z-0}{1} = s$

$\left.\begin{array}{l}L_1 \text{ is parallel to } -6\hat{i} + 9\hat{j} - 3\hat{k} \\ L_2 \text{ is parallel to } 2\hat{i} - 3\hat{j} + \hat{k}\end{array}\right| \Rightarrow$ lines parallel but not conicident

as (0, 1, 0) does not line on L_2, not intersecting

Hence L_1, L_2 lies in a unique planes $\Rightarrow$ (Q), (S)

(D) Lines are skew can be verified $\Rightarrow$ (P), (S)