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MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-19

Differential Equation

ANSWER KEY OF CAPS-19

1.	(B)	**2.**	(A)	**3.**	(C)	**4.**	(B)	**5.**	(B)
6.	(A)	**7.**	(A)	**8.**	(AB)	**9.**	(ABC)	**10.**	(AC)
11.	(C)	**12.**	(B)	**13.**	(C)	**14.**	(A)	**15.**	(C)
16.	(C)	**17.**	(0)	**18.**	(2)	**19.**	(2)	**20.**	(4)
21.	(62)	**22.**	(A)	**23.**	(C)	**24.**	$\frac{1}{8}\left(\frac{3\pi}{16}\right)^4$	**25.**	$y = \frac{c}{x^3}e^{-1/x}$

SCQ (Single Correct Type) :

1. If $\left(1+x^2\right)\frac{dy}{dx} = x(1-y), y(0) = \frac{4}{3}$, then the value of $y\left(\sqrt{8}\right)+\frac{8}{9}$ is :

(A) 4 (B) 2 (C) 3 (D) 5

Ans. **(B)**

Sol. $\frac{dy}{dx}+\frac{x}{1+x^2}y = \frac{x}{1+x^2}$

This is linear equation with $IF = \sqrt{1+x^2}$,

so $y\sqrt{1+x^2} = \int\frac{x}{\sqrt{1+x^2}}dx+C$

$y\sqrt{1+x^2} = \frac{1}{2}\times 2\sqrt{1+x^2}+C$

$y(0) = \frac{4}{3} \Rightarrow C = \frac{1}{3}$

$\therefore y\left(\sqrt{8}\right)+\frac{8}{9} = 2$

2. Solution of the differential $y' = \frac{3yx^2}{x^3+2y^4}$ is

(A) $x^3y^{-1} = \frac{2}{3}y^3+c$ (B) $x^2y^{-1} = \frac{2}{3}y^3+c$ (C) $xy^{-1} = \frac{2}{3}y^3+c$ (D) None of these

Ans. **(A)**

Sol. $\left(3yx^2\right)dx+\left(-x^3-2y^4\right)dy = 0$

$x^3y^{-1} = \frac{2}{3}y^3+c$

3. If $ydx-dy = e^{-x}y^4dy$ and y = 1 at x = –ln3, then at y = 3, x = lnk, k is

(A) 21 (B) 24 (C) 27 (D) 81

Ans. **(C)**

Sol. $\frac{ye^x dx - e^x dy}{y^2} = y^2 dy$

$\Rightarrow d\left(\frac{e^x}{y}\right) = y^2 dy \qquad \Rightarrow \frac{e^x}{y} = \frac{y^3}{3} + c$

Put $y = 1, x = \ln\left(\frac{1}{3}\right), c = 0$

Put y = 3, x = ln27

4. The solution of the differential equation $\left(x\cot y + \ln(\cos x)\right)dy + \left(\ln(\sin y) - y\tan x\right)dx = 0$ is _____.

(A) $(\sin x)^y (\cos y)^x = c$ (B) $(\sin y)^x (\cos x)^y = c$

(C) $(\sin x)^y (\cos y)^x = c$ (D) $(\cot x)^y (\cot y)^x = c$

Ans. **(B)**

Sol. $\left(x\cot y dy + \ln(\sin y)dx\right) + \left(\ln(\cos x)dy - y\tan x dx\right) = 0$

$d\left(x\ln(\sin y)\right) + d\left(y\ln(\cos x)\right) = 0$

$x\ln(\sin y) + y\ln(\cos x) = \ln(k)$

$(\sin y)^x (\cos x)^y = c$

5. If x and y are related, Also $\frac{d^2x}{dy^2}\cdot\left(\frac{dy}{dx}\right)^3 + k\frac{d^2y}{dx^2} = 0$. Then value of k is _____.

(A) –1 (B) 1 (C) –2 (D) 2

Ans. **(B)**

Sol. $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy} = \left(\frac{dy}{dx}\left(\frac{dx}{dy}\right)^2\right)$

$= \frac{d}{dy}\left(\frac{dy}{dx}\right)\times\left(\frac{dx}{dy}\right)^2 + \frac{dy}{dx}\times 2\left(\frac{dx}{dy}\right)\times\frac{d^2x}{dy^2}$

$= \frac{d^2y}{dx^2}\times\left(\frac{dx}{dy}\right)^3 + \frac{2d^2x}{dy^2} - \frac{d^2x}{dy^2} = \frac{d^2y}{dx^2}\left(\frac{dx}{dy}\right)^3$

or, $= \frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = 0 \Rightarrow k = 1$

6. Solution of the differential equation $y\left(xy + 2x^2y^2\right)dx + x\left(xy - x^2y^2\right)dy = 0$ is given by

(A) $2\log|x| - \log|y| - \frac{1}{xy} = C$ (B) $2\log|y| - \log|x| - \frac{1}{xy} = C$

(C) $2\log|x| + \log|y| + \frac{1}{xy} = C$ (D) $2\log|y| + \log|x| + \frac{1}{xy} = C$

Ans. **(A)**

Sol. Given differential equation is $\left(xy^2+2x^2y^2\right)dx+\left(x^2y-x^3y^2\right)dy=0$

dividing by x^3y^3

$$\Rightarrow\left(\frac{1}{x^2y}+\frac{2}{x}\right)dx+\left(\frac{1}{xy^2}-\frac{1}{y}\right)dy=0$$

$$\Rightarrow\left(\frac{ydy+xdy}{x^2y^2}\right)+\frac{2}{x}dx-\frac{1}{y}dy=0$$

$\therefore$ Solution is

$$-\frac{1}{xy}+2\log|x|-\log|y|=C \text{ or } 2\log|x|-\log|y|-\frac{1}{xy}=C$$

7. The solution of the differential equation $\frac{dy}{dx}=\frac{1}{xy\left(x^2\sin y^2+1\right)}$ is, ('c' being an arbitrary constant).

(A) $x^2\left(\cos y^2-\sin y^2-2ce^{-y^2}\right)=2$

(B) $x^2\left(\cos y^2-\sin y^2-2ce^{-y^2}\right)=4c$

(C) $y^2\left(\cos x^2-\sin x^2-2ce^{-y^2}\right)=2$

(D) $y^2\left(\cos x^2-\sin x^2-ce^{-y^2}\right)=4c$

Ans. **(A)**

Sol. Put U = y^2 then equation will be transformed into the linear equation

$$\frac{d}{du}\left(-\frac{1}{2x^2}\right)-\frac{1}{2x^2}=\frac{\sin U}{2} \quad IF=e^U$$

$\therefore$ Solution is $e^U\left(-\frac{1}{2x^2}\right)=\int\frac{e^U\sin U}{2}dU$ give the result A.

MCQ (One or more than one correct) :

8. The solution of $\left(\frac{dy}{dx}\right)^2-2\left(x+\frac{1}{4x}\right)\frac{dy}{dx}+1=0$

(A) $y=x^2+c$

(B) $y=\frac{1}{2}\ln(x)+c, x>0$

(C) $y=\frac{x}{2}+c$

(D) $y=\frac{x^2}{2}+c$

Ans. **(AB)**

Sol. $\left(\frac{dy}{dx}-2x\right)\left(\frac{dy}{dx}-\frac{1}{2x}\right)=0$

$\therefore$ $dy=2xdx$ or $dy=\frac{1}{2x}dx$

$\Rightarrow y=x^2+c$ and $y=\frac{1}{2}\ln|x|+c$ are solutions

9. A function $y = f(x)$ satisfying the differential equation $\frac{dy}{dx} . \sin x - y \cos x + \frac{\sin^2 x}{x^2} = 0$ is such that, $y \to 0$ as $x \to \infty$ then the statement which is correct is

(A) $\underset{x \to 0}{\text{Limit}} \; f(x) = 1$

(B) $\int_0^{\pi/2} f(x)\,dx$ is less than $\frac{\pi}{2}$

(C) $\int_0^{\pi/2} f(x)\,dx$ is greater than unity

(D) $f(x)$ is an odd function

Ans. **(ABC)**

Sol. $\frac{dy}{dx} \sin x - y \cos y = -\frac{\sin^2 x}{x^2}$

$\frac{dy}{dx} - y\left(\frac{\cos y}{\sin y}\right) = -\frac{\sin x}{x^2}$

Now, I.F. $= e^{\int -\frac{\cos x}{\sin x}.dx} \Rightarrow e^{-\ln \sin x} \Rightarrow \frac{1}{\sin x}$

$\frac{1}{\sin x}.\frac{dy}{dx} - y.\left(\frac{\cos x}{\sin^2 x}\right) = -\frac{1}{x^2}$

$\Rightarrow \frac{d}{dx}\left(\frac{y}{\sin x}\right) = -\frac{1}{x^2}$

Integrate both sides

$\frac{y}{\sin x} = \frac{1}{x} + c$ ... (1)

at $x \to \infty$, $y \to 0$

$y = \lim_{x\to\infty}\left(\frac{\sin x}{x}\right) + c \sin x$ (vary from [–1, 1])

$\Rightarrow c = 0$

$y = \frac{\sin x}{x}$

So (A) & (B) are correct.

10. Let T be the triangle with vertices (0, 0), (0, c^2) and (c, c^2) and let R be the region between $y = cx$ and $y = x^2$ where $c > 0$ then

(A) Area (R) $= \frac{c^3}{6}$

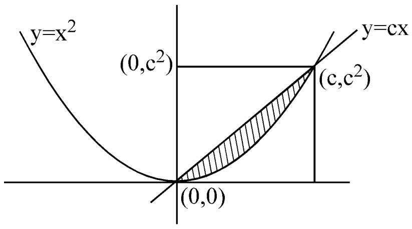
(B) Area of R $= \frac{c^3}{3}$

(C) $\lim_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = 3$

(D) $\lim_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = \frac{3}{2}$

Ans. **(AC)**

Sol. Area (T) = $\frac{c \cdot c^2}{2} = \frac{c^3}{2}$



Area (R) = $\frac{c^3}{2} - \int_0^c x^2\,dx$

$= \frac{c^3}{2} - \frac{c^3}{3} = \frac{c^3}{6}$

$\therefore \quad \lim_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = \lim_{c\to 0^+} \frac{c^3}{2}\cdot\frac{6}{c^3} = 3$

Comprehension Type Question:

Comprehension # 1

Suppose f(x) and g(x) are differentiable functions such that x g(f(x)). f'(g(x)).g'(x)=f(g(x)).g'(f(x)) f'(x) $\forall$ x $\in$ R and f(x) & g(x) are positive for all x $\in$ R. Also $\int_0^x f(g(t))\,dt = \frac{1}{2}(1-e^{-2x})\,\forall x \in R$, g(f(0)) = 1 and h(x) = $\frac{g(f(x))}{f(g(x))}\,\forall x \in R$.

11. The graph of y = h(x) is symmetric with respect to the line:

(A) x = –1 (B) x = 0 (C) x = 1 (D) x = 2

Ans. **(C)**

12. The value of f(g(0)) + g(f(0)) is equal to:

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

13. The largest possible value of h(x) $\forall$ x $\in$ R is

(A) 1 (B) $e^{1/3}$ (C) e (D) e^2

Ans. **(C)**

Sol. **(10 to 13)**

$\int_0^x f\left(g(t)\right)dt = \frac{1}{2}(1 - e^{-2x})$

differentiating both sides w.r.t. x

$f(g(x)) = e^{-2x} \Rightarrow f'(g(x)).g'(x) = -2e^{-2x}$

let g(f(x)) = y

$\therefore\ x.y.(-2).e^{-2x} = e^{-2x}.\frac{dy}{dx} \quad \Rightarrow \frac{dy}{dx} = -2yx \ \Rightarrow \frac{dy}{y} + 2x\,dx = 0$

$\Rightarrow \lambda n\, y + x^2 = c \quad \Rightarrow y = e^{c-x^2} \quad \Rightarrow g(f(x)) = e^{-x^2}$

$\therefore \quad h(x) = \frac{e^{-x^2}}{e^{-2x}} = e^{2x-x^2}$

Comprehension # 2

Let y = f(x) satisfy the equation $f(x) = (e^{-x} + e^{x})\cos x - 2x - \int_0^x (x-t)f'(t)dt$. Then,

14. f(x) satisfies the differential equation

(A) $\frac{dy}{dx} + y = e^{x}(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$

(B) $\frac{dy}{dx} = y + e^{x}(\cos x + \sin x) + e^{-x}(\cos x - \sin x)$

(C) $\frac{dy}{dx} + y = e^{x}(\cos x + \sin x) - e^{-x}(\cos x + \sin x)$

(D) None of these

Ans. (A)

15. $f'(0) + f''(0) =$

(A) –1 (B) 2 (C) 0 (D) 1

Ans. **(C)**

16. f(x) as a function of x equals

(A) $e^{-x}(\cos x - \sin x) + \frac{e^{x}}{5}(3\cos x + \sin x) + \frac{2}{5}e^{-x}$

(B) $e^{-x}(\cos x + \sin x) + \frac{e^{x}}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

(C) $e^{-x}(\cos x - \sin x) + \frac{e^{x}}{5}(3\cos x - \sin x) + \frac{2}{5}e^{-x}$

(D) None of these

Ans. **(C)**

Sol. **(14 to 16)**

$f(0) = 2$ & $f(x) = (e^{x} + e^{-x})\cos x - 2x - \left[x\int_0^x f'(t)dt - \int_0^x tf'(t)dt\right]$

$\Rightarrow \quad f(x) = (e^{x} + e^{-x})\cos x - \int_0^x f(t)\,dt$

or $f'(x) + f(x) = (e^{x} - e^{-x})\cos x - (e^{x} + e^{-x})\sin x$

Also $f'(0) + f''(0) = 0$

Integration factor of DE $\Rightarrow e^{x}$

$\Rightarrow \quad ye^{x} = \int e^{2x}(\cos x - \sin x)dx - \int (\cos x + \sin x)dx$

$\Rightarrow \quad y = e^{x}\left(\frac{3\cos x}{5} - \frac{\sin x}{5}\right) - (\sin x - \cos x)e^{-x} + \frac{2}{5}e^{-x}$

Numerical based Questions :

17. If $ye^{y}dx = \left(y^{3} + 2xe^{y}\right)dy, y(0) = 1$, then the value of x when y = 1 is

Ans. **(0)**

Sol. $\frac{dx}{dy} = \frac{y^2}{e^y} + \frac{2x}{y}$

$\frac{dx}{dy} - \frac{2}{y}x = \frac{y^2}{e^y}$

$I.F = e^{-2\ln y} = \frac{1}{y^2}$

General solution $x\frac{1}{y^2} = \int \frac{y^2}{e^y}\frac{1}{y^2}dy + c$

$\frac{x}{y^2} = e^{-y} + c$

x = 0, y =1

$\Rightarrow$ c = $-e^{-1}$

$\Rightarrow x = y^2\left(e^{-y} - e^{-1}\right)$

If y = 1 then x = 0

18. If particular solution of $p + \cos px \sin y = \sin px \cos y$ where $\left(p = \frac{dy}{dx}\right)$ is $y = a\sqrt{x^2 - b} - \sin^{-1}\frac{\sqrt{x^2 - c}}{x}$ then evaluate $\left|\frac{b+c}{a}\right|$

Ans. **(2)**

Sol. p = sin(px − y)

$\Rightarrow$ px – y $\sin^{-1}$ p ...(i)

Differentiate w.r.t. x

$\Rightarrow x\frac{dp}{dx} = \frac{1}{\sqrt{1-p^2}}\frac{dp}{dx}$

$\Rightarrow \frac{dp}{dx} = 0$ or $\frac{1}{\sqrt{1-p^2}} = x$

$\Rightarrow p = \frac{\sqrt{x^2-1}}{x}$ put in equation

$y = \sqrt{x^2-1} - \sin^{-1}\frac{\sqrt{x^2-1}}{x}$ $a = 1, b = 1, c = 1$

19. The solution of $x^2dy - y^2dx + xy^2(x-y)dy = 0$ is $\ln\left|\frac{1}{x} - \frac{1}{y}\right| = \frac{y^2}{k} + C$, then the value of k is ______.

Ans. **(2)**

Sol. $x^2y^2\left(\frac{dy}{y^2}-\frac{dx}{x^2}\right)+x^2y^3\left(\frac{1}{y}-\frac{1}{x}\right)dy=0$

$\Rightarrow d\left(\frac{1}{x}-\frac{1}{y}\right)+y\left(\frac{1}{y}-\frac{1}{x}\right)dy=0$

$\Rightarrow \frac{d\left(\frac{1}{x}-\frac{1}{y}\right)}{\frac{1}{x}-\frac{1}{y}}=y\,dy$

$\Rightarrow \ln\left|\frac{1}{x}-\frac{1}{y}\right|=\frac{y^2}{2}+c$

$\Rightarrow$ k = 2

20. Suppose a solution of the differential equation $\left(xy^3+x^2y^7\right)\frac{dy}{dx}=1$ satisfies the initial condition $y\left(\frac{1}{4}\right)=1$. Then the value of $-\frac{5}{4}\times\frac{dy}{dx}$ when y = −1, is _____.

Ans. **(4)**

Sol. $\frac{dx}{dy}=xy^2+x^2y^7$

that is $\frac{1}{x^2}\frac{dx}{dy}-\frac{1}{x}y^3=y^7$

that is $\frac{d\left(-\frac{1}{x}\right)}{dy}-\frac{1}{x}y^3=y^7$

$IF=e^{\frac{y^4}{4}}$

Solution is $\frac{-e^{\frac{y^4}{4}}}{x}=y^4e^{\frac{y^4}{4}}-4e^{\frac{y^4}{4}}+c$

Put $x=\frac{1}{4},y=1\quad\Rightarrow c=-e^{\frac{1}{4}}$

$\therefore$ Solution is $\frac{-e^{\frac{y^4}{4}}}{x}=y^4e^{\frac{y^4}{4}}-4e^{\frac{y^4}{4}}-e^{\frac{1}{4}}$

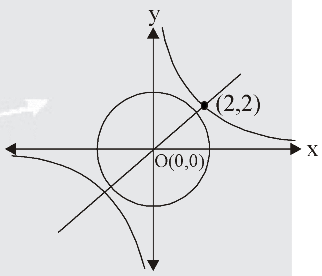
$y=-1\Rightarrow x=\frac{1}{4},\frac{dy}{dx}=\frac{-16}{5}$

21. Let $y = f(x)$ be a curve C_1 passing through (2,2) and $\left(8, \frac{1}{2}\right)$ and satisfying a differential equation $y\left(\frac{d^2y}{dx^2}\right) = 2\left(\frac{dy}{dx}\right)^2$. Curve C_2 is the director circle of the circle $x^2 + y^2 = 2$. If the shortest distance between the curves C_1 and C_2 is $\left(\sqrt{p} - q\right)$ where $p, q \in N$, then find the value of $(p^2 - q)$.

Ans. **(62)**

Sol. Given $y\left(\frac{d^2y}{dx^2}\right) = 2\left(\frac{dy}{dx}\right)^2 \Rightarrow \frac{y''}{y'} = \frac{2y'}{y} \Rightarrow \ln y' = 2 \ln y + \ln a$

$\Rightarrow \frac{y'}{y^2} = a \Rightarrow \int \frac{dy}{y^2} = adx \Rightarrow \frac{-1}{y} = ax + b$



Since curve is passing through (2, 2) and $\left(8, \frac{1}{2}\right)$, so

$2a + b = \frac{-1}{2}$ ....(1)

$8a + b = -2$ ....(2)

$\therefore$ On solving (1) and (2), we get $a = \frac{-1}{4}$, $b = 0$

Hence $C_1 : xy = 4$ and curve C_2 is $x^2 + y^2 = 4$.

$\therefore$ Shortest distance between C_1 and C_2 is $= 2\sqrt{2} - 2 = \sqrt{8} - 2$

Hence $(p^2 - q) = 64 - 2 = 62$ **Ans.**

Matrix Match Type :

22. Match the following

Column-I	Column-II
(A) Solution of the differential equation $y'' + e^{2y}(y')^3 = 0$ is	(p) $x = \frac{1}{4}e^{2y} + c_1 y + c_2$
(B) Solution of the differential equation $2x(x+y)y' = 3y^2 + 4xy$ at $y(1) = 1$ is	(q) $y^2 + 2xy = 3x^3$
(C) Equation of orthogonal trajectories of the curve $y = \tan x + k$ is	(r) $2x + 4y + \sin 2x = c$
(D) Solution of the differential equation $y' + 4xy + xy^3 = 0$ is	(s) $y^2 = \left(ce^{4x^2} - \frac{1}{4}\right)^{-1}$

(A) A-p; B-q; C-r; D-s (B) A-s; B-p; C-r; D-q

(C) A-p; B-q; C-s; D-r (D) A-p; B-s; C-q; D-r

Ans. **(A)**

Sol. (A) $\dfrac{\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^2} = -e^{2y}\dfrac{dy}{dx}$

$$\Rightarrow \int \frac{\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^2}dx = \int -e^{2y}dy$$

$$\Rightarrow \frac{1}{\dfrac{dy}{dx}} = \frac{e^{2y}}{2} + C_1$$

$$\Rightarrow \left(\frac{e^{2y}}{2} + C_1\right)dy = dx$$

$$\Rightarrow \frac{e^{2y}}{4} + C_1y + C_2 = x$$

(B) Homogeneous DE

(D) Bernoulli's Equation

23. **Column I** **Column II**

(A) If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\dfrac{\dfrac{d^3y}{dx^3} - 13\dfrac{dy}{dx}}{y}$ = K then the value of K/3 is — (P) 3

(B) Number of straight lines which satisfy the differential equation $\dfrac{dy}{dx} + x\left(\dfrac{dy}{dx}\right)^2 - y = 0$ is — (Q) 4

(C) If real value of m for which the subtitution, $y = u^m$ will transform the differential equation, $2x^4y\dfrac{dy}{dx} + y^4 = 4x^6$ into a homogenous equation then the value of 2m is — (R) 2

(D) If the solution of differential equation $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 12y$ is $y = Ax^m + Bx^{-n}$ then $|m + n|$ can be — (S) 7

(A) A(R); B(Q); C(S); D(P) (B) A(P); B(S); C(Q); D(R)

(C) A(Q); B(R); C(P); D(S) (D) A(S); B(P); C(Q); D(R)

Ans. **(C)**

Sol. (A) $y = e^{4x} + 2e^{-x}$; $y_1 = 4e^{4x} - 2e^{-x}$; $y_2 = 16e^{4x} + 2e^{-x}$;

$y_3 = 64e^{4x} - 2e^{-x}$

Now, $y_3 - 13y_1 = (64e^{4x} - 2e^{-x}) - 13(4e^{4x} - 2e^{-x}) = 12e^{4x} + 24e^{-x}$.

$y_3 - 13y = 12(e^{4x} + 2e^{-x}) = 12y$

$\therefore$ K = 12 and K/3 = 4.

(B) Let $y = mx + c \Rightarrow \dfrac{dy}{dx} = m$

$\Rightarrow \quad mx + c^\circ m + xm^2 \quad \Rightarrow \quad m = m^2$ and $c = m \quad \Rightarrow \quad m = 0$ or $m = 1$

(C) $y = u^m \Rightarrow \dfrac{dy}{dx} = m\, u^{m-1} \dfrac{du}{dx}$

Substituting the value of y and $\dfrac{dy}{dx}$ in $2x^4 y \dfrac{dy}{dx} + y^4 = 4x^6$

we have $2x^4 u^m u^{m-1} \dfrac{du}{dx} + u^{4m} = 4x^6$.

$$\Rightarrow \quad \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2m\, x^4 u^{2m-1}}$$

For homogeneous $4m = 6 \Rightarrow m = \dfrac{3}{2}$ and $2m - 1 = 2 \Rightarrow m = \dfrac{3}{2}$.

(D) $y = Ax^m + Bx^{-n}$

$$\Rightarrow \quad \frac{dy}{dx} = Amx^{m-1} - nBx^{-n-1} \quad \Rightarrow \quad \frac{d^2y}{dx^2} = Am(m-1)\, x^{m-2} + n(n+1)Bx^{-n-2}$$

Putting these values in $x^2 \dfrac{d^2y}{dx^2} + 2x \dfrac{dy}{dx} = 12y$

We have $= m(m + 1)\, Ax^m + n(n - 1)\, Bx^{-n} = 12(A x^m + Bx^{-n})$

$\Rightarrow \quad m(m + 1) = 12$ or $n(n - 1) = 12$

$\Rightarrow \quad m = 3, -4$ or $n = 4, -3$.

Subjective based Questions :

24. Let the curve $y = f(x)$ passes through $(4, -2)$ satisfy the differential equation,

$$y(x + y^3)\, dx = x(y^3 - x)\, dy \ \&\ \text{let } y = g(x) = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt,$$

$0 \le x \le \dfrac{\pi}{2}$. Then find the area of the region bounded by curves $y = f(x)$, $y = g(x)$ and $x = 0$.

Ans. $\dfrac{1}{8}\left(\dfrac{3\pi}{16}\right)^4$

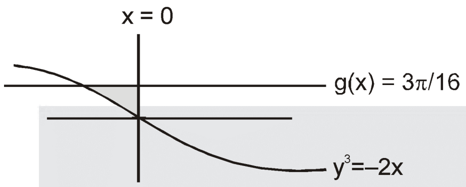
Sol. $y(x + y^3)\,dx = x(y^3 - x)\,dy$

$$\Rightarrow -\frac{y}{x}d\left(\frac{y}{x}\right) + \frac{d(xy)}{(xy)^2} = 0 \qquad \Rightarrow \qquad y^3 + 2x + 2cx^2y = 0$$

It passes through (4, –2)

$-8 + 8 + 2c\,(16)\,(-2) = 0 \;\Rightarrow c = 0$

$\therefore\; y^3 = -2x$



$g'(x) = 2\sin x\cos x\sin^{-1}\sin x + 2\cos x\,(-\sin x)\cos^{-1}(\cos x) = 0$

so g(x) is constant

$$g(\pi/4) = g(x) = \int_{1/8}^{1/2} (\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t})\,dt$$

$$\Rightarrow g(x) = \frac{\pi}{2}\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{3\pi}{16}$$

$$A = \left| -\frac{1}{2}\int_0^{\frac{3\pi}{16}} y^3\,dy \right| \quad \Rightarrow \quad A = \frac{1}{2x4}\left(\frac{3\pi}{16}\right)^4$$

25. Solve the equation $x\int_0^x y(t)\,dt = (x+1)\int_0^x t\,y(t)\,dt$, $x > 0$

Ans. $y = \dfrac{c}{x^3}e^{-1/x}$

Sol. Differentiate the equation with respect to x, we get

$$\int_0^x y(t)\,dt = x^2y(x) + \int_0^x t\,y(t)\,dt$$

Differentiage again with respect to x, we get

$$y(x) = x^2\,y'(x) + 2xy(x) + xy(x) \qquad \Rightarrow (1 - 3x)\,y(x) = \frac{x^2dy(x)}{dx}$$

$$\frac{(1-3x)dx}{x^2} = \frac{dy}{y}$$

$$y = \frac{c}{x^3}e^{-1/x}$$