competishun — Power of real gurus

MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-14

Permutation & Combination

ANSWER KEY OF CAPS-14

1.	(A)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(D)
6.	(C)	**7.**	(B)	**8.**	(A)	**9.**	(A)	**10.**	(ABD)
11.	(ABC)	**12.**	(BC)	**13.**	(6)	**14.**	(7)	**15.**	(9)
16.	(0)	**17.**	(48)	**18.**	(2)	**19.**	(42)	**20.**	(B)
21.	(A)	**22.**	(A)	**23.**	(A)				
24.	(i) $^{23}C_3$	(ii) $^{19}C_3$	(iii) $^{19}C_3 - 4.^9C_3$	(iv) $^{11}C_8$		**25.**	(6890)		

SCQ (Single Correct Type) :

1. The number of selections of four letters from the letters of the word ASSASSINATION is-

(A) 72 (B) 71 (C) 66 (D) 52

Ans. **[A]**

Sol. The word ASSASSINATION consists of 3 A's, 4 S's, 2 I's and 2N's, T and O.

The number of selections of four letters

= coefficient of x^4 in

$$\underset{\text{for 3A's}}{\underset{\downarrow}{(1+x+x^2+x^3)}}\ \underset{\text{for 4S's}}{\underset{\downarrow}{(1+x+x^2+x^3+x^4)}}\ \underset{\text{for 2I's}}{\underset{\downarrow}{(1+x+x^2)^2}}\ \underset{\text{for T and O}}{\underset{\downarrow}{(1+x)^2}}$$

= coefficient of x^4 in $\left[\frac{1-x^4}{1-x}\right]\left[\frac{1-x^5}{1-x}\right]$

$$\left[\frac{1-x^3}{1-x}\right]^2\left[\frac{1-x^2}{1-x}\right]^2$$

= coefficient of x^4 in $(1 - x^4)(1 - x^5)(1 - 2x^3 + x^6)(1 - 2x^4 + x^4)(1 - x)^{-6}$

= coefficient of x^4 in $(1 - 2x^2 - 2x^3 + 3x^5 + \ldots.)$

$(1 + {}^6C_1x + {}^7C_2x^2 + {}^8C_3x^3 + {}^9C_4x^4 + \ldots..)$

$= {}^9C_4 - 2.{}^7C_2 - 2.\,{}^6C_1 = 126 - 42 - 12 = 72$

2. The maximum value of p such that 3^p divides $99 \times 97 \times 95 \times \ldots \times 51$ is -

(A) 11 (B) 14 (C) 13 (D) 12

Ans. **[B]**

Sol. $99 \times 97 \times 95 \times \ldots \times 51$

$= \frac{100\,!}{(100 \times 98 \times 96 \times \ldots \times 52)} \times \frac{1}{50\,!}$

$= \frac{100\,! \times 25\,!}{2^{25} \times 50! \times 50\,!}$

maximum power of 3 in 100 !

$$= \left[\frac{100}{3}\right] + \left[\frac{100}{9}\right] + \left[\frac{100}{27}\right] + \left[\frac{100}{81}\right]$$

= 33 + 11 + 3 + 1 = 48

maximum power of 3 in 50 !

$$= \left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right] \quad = 16 + 5 + 1 = 22$$

maximum power of 3 in 25 !

$$= \left[\frac{25}{3}\right] + \left[\frac{25}{9}\right] \quad = 8 + 2 = 10$$

$\therefore$ exponent of 3 = 48 + 10 – (22 × 2) = 14

3. There are 8 events that can be scheduled in a week, then The total number of ways in which the events can be scheduled is

(A) 7^8 (B) 8^7 (C) 7! (D) 8

Ans. **(B)**

Sol. For 1st event there are 7 ways

2nd event there are 7 ways

2nd event there are 7 ways

2nd event there are 7 ways

8th event there are 7 ways

$\Rightarrow$ Total no. of ways = 7^8

4. Total number of integers 'n ' such that $2 \le n \le 2000$ and H.C.F of 'n ' and 36 is one, is equal to

(A) 666 (B) 667 (C) 665 (D) 668

Ans. **(A)**

Sol. $36 = 2^2 \times 3^3$

From 2 to 2000 number of multiples of 2 are $\left[\frac{2000}{2}\right] = 1000$

From 2 to 2000 number of multiples of 3 are $\left[\frac{2000}{3}\right] = 666$

From 2 to 2000 number of multiples of 6 are $\left[\frac{2000}{6}\right] = 333$

$\therefore$ Number of possible 'n' are = 1999− [1000 + 666−333]

5. Statement 1: A = { x : x is a prime number, x < 30 } then number of distinct rational numbers whose numerator and denominator belong to ' A' is 93

Statement 2: $\frac{p}{q} \in Q \forall\ q \ne 0$ and $p, q \in I$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Ans. **(D)**

Sol. A={2,3,5,7,11,13,17,19,23,29}

Two different numbers for numbers and denominator from these can be obtained in

$^{10}P_2 = 10.9 = 90$ ways and if $\frac{p}{p} = \frac{q}{q} = 1$

So, number of ways (if numerator and denominator are same) = 90 + 1 = 91

6. Six cards are drawn one by one from a set of unlimited number of cards, each card is marked with numbers -1, 0 or 1. Number of different ways in which they can be drawn if the sum of the numbers shown by them vanishes, is:

(A) 111 (B) 121 (C) 141 (D) none

Ans. **(C)**

Sol. Here the sum of the numbers are vanishes of six cards i.e

Case I : If selected 3 cards each of number –1 or 1 i.e

The number of arrangement = $\frac{6!}{3!\,3!}$ = 20

Case II : If selected 2 cards each of no. –1, 0 or 1 i.e

number of arrangement = $\frac{6!}{2!\,2!\,2!}$ = 90

Case III : If selected one card each of number –1 and 1 and 4 cards of no. 0.

so no. of arrangement is $\frac{6!}{1!\,1!\,4!}$ = 30

Case IV : If all cards selected fram the no. 0

So no. of arrangement is $\frac{6!}{6!}$ = 1

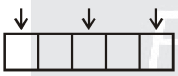
Hence total no. of arrangement is 20 + 90 + 30 + 1 = 141

7. A five letter word is to be formed such that the letters appearing in the odd numbered positions are taken from the letters which appear without repetition in the word "MATHEMATICS". Further the letters appearing in the even numbered positions are taken from the letters which appear with repetition in the same word "MATHEMATICS". The number of ways in which the five letter word can be formed is:

(A) 720 (B) 540 (C) 360 (D) none

Ans. **(B)**

Sol.



There are 2M, 2T, 2A and 1 H, E, I, C, S

First find the number of ways if odd's no. position place be filled is $^5p_3$ = 60

Now Case I If even place words is same i.e no. of ways = 3

Case II If even place words is different i.e no. of ways = $^3c_2 \times 2!$ = 6

Hence total no. of arragment is 60 × (3 + 6) = 540

8. In a shooting competition a man can score 5, 4, 3, 2, or 0 points for each shot. Then the number of different ways in which he can score 30 in seven shots is-

(A) 420 (B) 421 (C) 422 (D) None of these

Ans. [A]

Sol. The number of ways of making 30 in 7 shots is the coefficient of x^{30} in

$(x^0 + x^2 + x^3 + x^4 + x^5)^7$

$\Rightarrow (x^0 + x^2 + x^3 + x^4 + x^5)^7 = [x^4 (x+1) + (x^3 + x^2 + 1)]^7$

$\Rightarrow x^{28} (x + 1)^7 + 7C_1\, x^{24} (x+1)^6 (x^3 + x^2 + 1) + 7C_2\, x^{20}(x+1)^5 (x^3 + x^2 + 1)^2 + \ldots$

so, coefficient of x^{30} is $7C_5 + 7C_1 ({}^6C_3 + {}^6C_2 + 1) + 7C_2 ({}^5C_1 + 2)$

$= 21 + 7 (20 + 15 + 1) + 21 (5 + 2) = 420$

9. The maximum number of points of intersection of 7 straight lines and 5 circles when 3 straight lines are parallel and 2 circles are concentric, is/are-

(A) 106 (B) 96 (C) 90 (D) None of these

Ans. [A]

Sol. Points of intersection due to

7 straight lines = ${}^7C_2 - 3 = 18$

Two concentric circles can intersect these 7 lines at

maximum = 14 + 14 = 28 points

Third circle can intersect the given system at

maximum = 14 + 2 + 2 = 18

Fourth circle can intersect the system at maximum

= 14 + 2 + 2 + 2 = 20 points

for fifth circle = 14 + 2 + 2 + 2 + 2 = 22

maximum no. of points of intersection

= 18 + 28 + 18 + 20 + 22 = 106

MCQ (One or more than one correct) :

10. If x be the number of 5 digit numbers, sum of whose digit is even and y be the number of 5 digit numbers, sum of whose digits is odd, then

(A) $x = y$ (B) $x + y = 90000$ (C) $x < y$ (D) $x = 45000$

Ans. [A,B, D]

Sol. Total no. of 5 digits nos = 9.10^4

= 90000

but $x = y$

So $x = y = 45000$

11. The number of words which can be made from letters of the word INTERMEDIATE is-

(A) 907200 if word starts with I and end with E

(B) 21600 if vowels and consonants occupy their original places

(C) 43200 if vowels and consonants occur alternatively

(D) 302400if all the vowels occur together

Ans. [A, B, C]

Sol. INTERMEDIATE has 3E's, 2I's and 2 T's and A, D, M, N, R one each, thus total of 12 letters. If words start with I and end with E i.e. I x x x x x x x x x x E; the ten places (shown by cross) has to be filled with 2 E's and 2 T's and 6 distinct letters.

$\therefore$ Number of words = $\frac{10!}{2!\,2!}$ = 907200

If vowels and consonants occupy their original places, then 6 vowels (E, E, E, I, I, A) can be arranged in $\frac{6!}{3!\,2!}$ at their original places.

6 consonant (N, T, T, M, D, R) can be arranged in $\frac{6!}{2!}$ at their original place.

$\therefore$ Number of words = $\frac{6!}{3!\,2!} \times \frac{6!}{2!}$ = 21600

If vowels and consonants occur together, then 6 vowels can be arranged in 6 places in $\frac{6!}{3!\,2!}$. E × E × E × I × I × A

6 consonants can be arranged in 5 places shown be crosses and one place either extreme left or extreme right in $\frac{6!}{2!} \times 2!$.

$\therefore$ Number of words = $\frac{6!}{3!\,2!} \times \frac{6!}{3!} \times 2!$ = 43200

If all the vowels occur together, then group of 6 vowels taken together will be considered one and 6 consonants can be arranged in $\frac{7!}{3!}$ ways.

[There are two T's]. But 6 vowels can be arranged in $\frac{6!}{3!\,2!}$ ways.

$\therefore$ Number of words = $\frac{7!}{2!} \times \frac{6!}{3!\,2!}$ = 151200

12. Identify the correct statement(s).

(A) Number of zeroes standing at the end of 125 ! is 30.

(B) A telegraph has 10 arms and each arm is capable of 9 distinct positions excluding the position of rest. The number of signals that can be transmitted is $10^{10} - 1$.

(C) Number of numbers greater than 4 lacs which can be formed by using only the digits 0, 2, 2, 4, 4 and 5 is 90.

(D) In a table tennis tournament, every player plays with every other player. If the number of games played is 5050 then the number of players in the tournament is 100.

Ans. **(BC)**

Sol. First find no. of '2' at the end of (125)! is

(A) $\left[\frac{125}{2}\right]+\left[\frac{125}{2^2}\right]+\left[\frac{125}{2^3}\right]+\left[\frac{125}{2^4}\right]+\left[\frac{125}{2^5}\right]+\left[\frac{125}{2^6}\right]+\left[\frac{125}{2^7}\right]+$

$= 62 + 31 + 15 + 7 + 3 + 1 + 0 = 119$

Find the number of '5' at the end of (125)! is $\left[\frac{125}{5}\right]+\left[\frac{125}{5^2}\right]+\left[\frac{125}{5^3}\right]+ \ldots\ldots = 25 + 5 + 1 = 31$

Hence no. of zero is 31

(B) Total no. of singals can made by each arm = 10 so total number of different signals can be formed $= 10^{10} - 1$

(here – 1 is because if all arms are at the poisition of rest, then no signal will pass away)

(C) $\boxed{4}\,\square\square\square\square\square$ $\boxed{5}\,\square\square\square\square\square$

$\frac{5!}{2!} = 60$ $\frac{5!}{2!.2!} = 30$

Total number of arrangement = 90

(D) Let number of player is n

then total number of games is ${}^nc_2 = 5050 \Rightarrow$ n = 101

Numerical based Questions :

13. If n is the number of ways in which 15 identical blankets can be distributed among six beggars such that everyone gets atleast one blanket and two particular beggars get equal blankets and another three particular beggars get equal blankets, then the value of n/2 is

Ans. (6)

Sol. The number of ways of distributing blankets is equal to the number of solutions of the equation 3a + 2b + c =15

b,c $\geq$1, which is equal to the coefficient of t^{15} in

$\left(t^3+t^6+t^9+t^{12}.....\right)\left(t^2+t^4+.....\right)\left(t+t^2+.....\right)$

= coefficient of t^9 in $(1+t^2 +t^2 +t^4 +t^5 + 2t^6 +t^7 + 2t^8 + 2t^9)(1+t +t^2 +...+t^9)$

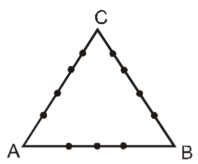
=1+1+1+1+1+ 2 +1+ 2 + 2 =12

14. The sides AB, BC & CA of a triangle ABC have 3, 4 & 5 interior points respectively on them. If the number of triangles that can be constructed using these interior points as vertices is k, then sum of digits in the number k equals.

Ans. 7

Sol. Total number of triangle = Two points taken from AB and one point either BC or CA + similarly BC + similarly $\angle$A + one point each sides.

$= {}^3C_2\,[{}^4C_1 + {}^5C_1] + {}^4C_2\,[{}^5C_1 + {}^3C_1] + {}^5C_2\,[{}^3C_1 + {}^4C_1] + {}^3C_1{}^4C_1{}^5C_1 = 205$



Total – (collinear points used)

$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 220 - 15 = 205$

Alternate

Total – Collinear points used $= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 220 - 15 = 205$

15. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is k, then sum of the digits in number of k equals.

Ans. **9**

Sol. Coin dividing in any are possible i.e.

1, 2, 4

so the number of ways is

$^7C_1 \,.\, ^6C_2 \,.\, ^4C_4 \,.\, 3! = 7 \times 15 \times 6 = 630$

16. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12 is N where 'N' is a four digit number of the form abcd then (a – c) equals.

Ans. **0**

Sol. Let number be $x_1\, x_2\, x_3\, x_4\, x_5\, x_6$

But Here $x_1 + x_2 + ... x_6 = 12$

so coefficient of x^{12} in expansion $(1 + x + x^2 + ... + x^9)^6 = (1 - x^{10})^6 \,.\, (1 - x)^{-6}$

$\Rightarrow$ $^{17}C_{12} - {}^6C_1 \,.\, ^7C_2 = 6188 - 126 = 6062$

17. The number of ways in which 8 non-identical apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is a four digit number of the form pqrs then p.q.r equals.

Ans. **48**

Sol. 8 - non identical

number of ways =	B_1	B_2	B_3
	1	3	4
	2	3	3
	2	2	4

Here required number of ways $= 3!\,\{^8C_1 \,.\, ^7C_3 \,.\, ^4C_4 + \dfrac{^8C_2 \,.\, ^6C_3 \,.\, ^3C_3}{2!} + \dfrac{^8C_2 \,.\, ^6C_2 \,.\, ^4C_4}{2!}\,\}$

$$= 6\left[\frac{8!}{3!.4!} + \frac{8!}{2!.6!} \times \frac{6!}{3!.2!.2!} + \frac{8!}{2!.6!} \times \frac{6!}{2!.4!.2!}\right] = 6[280 + 280 + 210] = 6 \times 770 = 4620$$

18. In a shooting competition a man can score 0, 2 or 4 points for each shot. Then the number of different ways in which he can score 14 points in 5 shots, is N then number of digits in 'N' equals

Ans. **2**

Sol. Find the coefficient of x^{14} in the expansion of

$$(x^0 + x^2 + x^4)^5 = (1 + x^2 + x^4)^5 = \left(\frac{1 - x^6}{1 - x^2}\right)^5 = (1 - x^6)^5\,(1 - x^2)^{-5}$$

$$= (1 - 5x^6 + 10x^{12}............)\left(1 + {}^5C_1x^2 + {}^6C_2x^4 + {}^7C_3x^6 + ...\right) = {}^{11}C_7 - 5\,.\,{}^8C_4 + 10.5$$

$$= \frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4} - 5.\frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} + 50 = 330 - 350 + 50 = 30$$

19. The number of permutations which can be formed out of the letters of the word "SERIES" taking three letters together, is:

Ans. **42**

Sol. SERIES

S - 2, E - 2 , R, I

case-I when all letter distinct is

$^4C_3 \times 3! = 4 \times 6 = 24$

case-II when 2 letters are same the

$^2C_1 \cdot {}^3C_1 \times \frac{3!}{2!} = 2 \cdot 3 \cdot 3 = 18$ total number is 24 + 18 = 42

Comprehension Type Question:

Comprehension # 1

Consider the letters of the word MATHEMATICS there are eleven letters some of them are identical. Letters are classified as repeating and non-repeating letters. Set of repeating letters = {M, A, T}. Set of non-repeating letters = {H, E, I, C, S}

20. Possible number of words taking all letters at a time such that atleast one repeating letter is at odd position in each word

(A) $\frac{9!}{2!\,2!\,2!}$ (B) $\frac{11!}{2!\,2!\,2!}$ (C) $\frac{11!}{2!\,2!\,2!} - \frac{9!}{2!\,2!}$ (D) $\frac{9!}{2!\,2!}$

And. **[B]**

Sol. Since there are 5 even places and 3 pairs of repeated letters therefore at least one of these must be at an odd place.

$\therefore$ the number of ways = $\frac{11!}{2!\,2!\,2!}$

21. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A are not together-

(A) $7!.\ {}^8C_2$ (B) $\frac{11!}{2!\,2!\,2!} - \frac{10!}{2!\,2!}$ (C) $\frac{6!\,4!}{2!\,2!}$ (D) $\frac{9!}{2!\,2!\,2!}$

Ans. **[A]**

Sol. Make a bundle of both M's and another bundle of T's. Then except A's we have 5 letters remaining so M's, T's and the letters except A's can be arranged in 7! ways

$\therefore$ total number of arrangements = $7! \times {}^8C_2$

22. Find the number of words in which no vowel is together

(A) $\frac{7!}{2!\,2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$ (B) $\frac{7!}{2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$ (C) $7!.\ {}^8C_4 \cdot \frac{4!}{2!}$ (D) $\frac{7!}{2!\,2!\,2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$

Ans. **[A]**

Sol. Non vowels can be placed in $\frac{7!}{2!\,2!}$ ways

Then there are 8 places 4 vowels

$\therefore$ Number of ways = $\frac{7!}{2!\,2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$

Matrix Match Type :

23. Match the following

Column-I	**Column-I**
(A) Number of ways in which all the letters of word "RESONANCE" can be arranged such that vowels and consonants occurs alternately is	(p) 720
(B) If probability that product of four whole numbers ends in '5' is q then 10^4q is les than	(q) 210
(C) The variance of a binomial random variable is 180. The minimum numbers of trials possible for the experiment are	(r) 370
(D) If sum of roots of equation $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$ A, then value of $\left(\frac{9}{7}A\right)!$ is	(s) 360

(A) A-p; B-p,r; C-p; D-p
(B) A-r; B-s; C-r; D-q
(C) A-s; B-p; C-r; D-q
(D) A-r; B-q; C-r; D-q

Ans. **(A)**

Sol. (A) There are 4 vowels e, e, o, a and 5 consonants R, S, N, N, C.

If vowels and consonant should came alternately is that consonants occupy odd and vowels occupy even places in the '9' letter word

Hence number of ways is $\frac{5!}{2!} \times \frac{4!}{2!} = 720$

(B) x + y + z +w = 20 where $1 \leq$ x, y, z,w $\leq$ 6
Let 1 2 x = 6 $-t_1$, y = 6 $-t_2$ and so on

Equation becomes, $t_1 + t_2 + t_3 + t_4 = 4$

$0 \leq t_1, t_2, t_3, t_4 \leq 4$

Number of solutions is ${}^7C_3$

Hence $p = \frac{{}^7C_3}{6^4} = \frac{35}{6^4}$

(C) $w_1w_{2w3}w_4$ ends in 5

Pr (Product is odd) $= \left(\frac{1}{2}\right)^4$

Pr (product ends in 1, 3, 7, 9) = $\left(\frac{4}{10}\right)^4 = \left(\frac{2}{5}\right)^4$

Hence $q = \left(\frac{1}{16} - \frac{16}{625}\right) = \left(\frac{369}{10000}\right)$

(D) npq =180 p + q =1

n is minimum when pq is maximum which is at p = q = $\frac{1}{2}$

Hence minimum value of 'n' is 720

(E) $x_1 + x_2 + x_3 + x_4 = \frac{56}{12} = \frac{14}{3} = A$

Subjective Based Questions :

24. Find the number of positive integral solutions of $x + y + z + w = 20$ under the following conditions:

(i) x, y, z, w are whole number

(ii) x, y, z, w are natural number

(iii) $x, y, z, w \in \{1, 2, 3, ..., 10\}$

(iv) x, y, z, w are odd natural number

Ans. (i) ${}^{23}C_3$ (ii) ${}^{19}C_3$ (iii) ${}^{19}C_3 - 4.{}^{9}C_3$ (iv) ${}^{11}C_8$

Sol. (i) If zero value are include i.e. $x, y, z, w \geq 0$

So Required no of solution $= {}^{20+4-1}C_{4-1} = {}^{23}C_3$ **Ans.**

(ii) If zero value are exclude i.e. $x, y, z, w \geq 1$

$x + y + z + w = 20 \Rightarrow y_1 + y_2 + y_3 + y_4 = 16$ $\{\because y_1, y_2, y_3, y_4 \geq 0\}$

So Required number of solution $= {}^{16+4-1}C_{4-1} = {}^{19}C_3$ **Ans.**

(iii) $x + y + z + w = 10$

$1 \leq x, y, z, w \leq 10$ = coefficient of x^{10} in $(x + x^2 + ...+ x^7)^4$

= coefficient of x^6 in $(1 + x + x^2 + ...+ x^6)^4$

$= {}^{4+6-1}C_6 = {}^{9}C_6$

Alter

First find any one is exceed 10

i.e. $10 + x_1 + y + z + w = 20$ Hence $x_1, y, z, w \geq 1$

So $y_1, y_2, y_3, y_4 \geq 0$

$= y_1 + y_2 + y_3 + y_4 = 6$

number of solution $= {}^{9}C_3$

Hence number of solution if all variable may exceed 10, zero values exclude is

${}^{19}C_3 - 4.{}^{9}C_3$

(iv) Let the number be $x = 2x_1 + 1$

$y = 2x_2 + 1$

$z = 2x_3 + 1$

$w = 2x_4 + 1$

So A/Q $(2x_1 + 1) + (2x_2 + 1) + (2x_3 + 1) + (2x_4 + 1) = 20$

$\Rightarrow 2(x_1 + x_2 + x_3 + x_4) = 16 \Rightarrow x_1 + x_2 + x_3 + x_4 = 8$

So Required no of solution $= {}^{11}C_3$ **Ans.**

25. Find the number of words of 5 letters that can be made with the letters of the word **"PROPOSITION"**.

Ans. **6890**

Sol. Word is PROPOSITION

Here P's = 2, O's = 3, I's = 2 and T, R, N, S = 1

We have to made 5 letters words

Case I : When All 5 are different $= {}^{7}p_5 = \dfrac{7!}{2!} = 2520$

Case II : When 2 alike ,3 different = ${}^{3}C_{1} \times {}^{6}C_{3} \times \frac{5!}{2!} = 3600$

Case III : When 2 alike , 2 alike, 1 different = ${}^{3}C_{2} \times {}^{5}C_{1} \times \frac{5!}{2!2!} = 450$

Case IV : When 3 alike, 2 different = ${}^{1}C_{1} \times {}^{6}C_{2} \times \frac{5!}{3!} = 300$

Case V : When 3 alike , 2 alike = ${}^{1}C_{1} \times {}^{2}C_{1} \times \frac{5!}{3!2!} = 20$

So total required number = 6890 Ans.