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MATHEMATICS

TARGET : JEE- Advanced 2022

CAPS-1

Fundamental of Mathematics & Trigonometry

ANSWERKEY (CAPS-1)

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(C)	**5.**	(D)	**6.**	(BC)	**7.**	(ABD)
8.	(ACD)	**9.**	(AD)	**10.**	(ACD)	**11.**	(AC)	**12.**	(BD)	**13.**	(ABCD)	**14.**	11.00
15.	0.00	**16.**	4.00	**17.**	18.00	**18.**	34	**19.**	9	**20.**	1024	**21.**	(C)
22.	(A)	**23.**	(B)	**24.**	(C)	**25.**	(D)						

SCQ (Single Correct Type) :

1. The number of solutions of the system of equations

$x+\sqrt{x^2+1}=10^{y-x}$

$y+\sqrt{y^2+1}=10^{z-y}$

$z+\sqrt{z^2+1}=10^{x-z}$ are

(A) 4 (B) 3 (C) 2 (D) 1

Ans. **(D)**

Sol. Clearly x = y = z = 0 is a solution

If x > 0 $\Rightarrow x+\left(\sqrt{x^2+1}\right)>1$

$\Rightarrow$ $10^{y-x}>1$ $\Rightarrow$ $y > x$

i.e. $0 < x < y$ $\Rightarrow$ $y > 0$

$\Rightarrow$ $y+\sqrt{y^2+1}>1$ $\Rightarrow$ $10^{z-y}>1$

$\Rightarrow$ $10 > y$

i.e. $0 < x < y < z$

$\Rightarrow$ $10 > 0$ $\Rightarrow$ $z+\sqrt{z^2+1}>1$

$\Rightarrow$ $10^{x-z}>1$ $\Rightarrow$ $10 > z$

i.e. $0 < x < y < z < x$

a contradiction hence not greater than 0

If x < 0 we obtain

$x < z < y < x < 0$, a contradiction hence x is not less than 0

$\therefore$ Only one solution

2. Total number of values of x, of the form $\frac{1}{n}, n\in N$ in the interval $x\in\left[\frac{1}{25},\frac{1}{10}\right]$, which satisfy the equation $\{x\}+\{2x\}+....+\{12x\}=78x$, (where $\{\cdot\}$ represents fractional part function)

(A) 12 (B) 13 (C) 14 (D) 15

Ans. **(B)**

Sol. $(x-\{x\}) + (2x-\{2x\}) + ...+ (12x-\{12x\}) = 0$

$\Rightarrow [x] + [2x] + ...+ [12x] = 0$

$\therefore$ $0 \le 12x < 1$

$\Rightarrow$ $0 \le x < \frac{1}{12} \;\forall\; \frac{1}{25}\le x\le\frac{1}{10}$

$\therefore$ $\frac{1}{25}\le x<\frac{1}{12}$

$\Rightarrow$ total values of x are 13

3. The value of $\sum_{K=1}^{7} \tan^2\left(\frac{K\pi}{16}\right)$ **is__**

(A) $\frac{3}{2}$ (B) 105 (C) 35 (D) 21

Ans. **(C)**

Sol. $\sum_{K=1}^{7} \tan^2\left(\frac{K\pi}{16}\right)$ = $\left(\tan\frac{\pi}{16}+\cot\frac{\pi}{16}\right)^2+\left(\tan\frac{\pi}{8}+\cot\frac{\pi}{8}\right)^2+\left(\tan\frac{3\pi}{16}+\cot\frac{3\pi}{16}\right)^2+1-6$

$=\left(\cos es^2\frac{\pi}{8}+\sec^2\frac{\pi}{8}\right)=3+4\left(2+\cot^2\frac{\pi}{8}+\tan^2\frac{\pi}{8}\right)$

$\left(\cos es^2\frac{\pi}{8}+\sec^2\frac{\pi}{8}\right)=3+4\left(2+\cot^2\frac{\pi}{8}+\tan^2\frac{\pi}{8}\right)$

$= 11+4\left(\left(\sqrt{2}+1\right)^2+\left(\sqrt{2}-1\right)^2\right)=35$

4. If $\frac{x^2-2|x|-3}{x^2-4|x|-5}>-1$, then x equal to

(A) $(-\infty, -5) \cup (5, \infty)$ (B) $(-4, 4)$

(C) $(-\infty, -5) \cup (-4, 4) \cup (5, \infty)$ (D) R

Ans. **(C)**

Sol. Let $|x| = t$

$\therefore \quad \frac{t^2-2t-3}{(t-2)^2-9}>-1$

Case I : If $(t-2)^2 - 9 > 0$

$\Rightarrow \quad t > 5 \quad \Rightarrow \quad x \in R - [-5, 5]$

Then $t^2 - 3t - 4 > 0$

$\Rightarrow \quad (t-4)(t+1) > 0 \quad \Rightarrow \quad t < -1 \text{ or } t > 4$

$\Rightarrow \quad -4 < x < 4$

$\therefore \quad x \in (-\infty, -5) \cup (5, \infty)$

Case II : If $(t-2)^2 - 9 < 0$

$\Rightarrow \quad t < 5$

$-5 < x < 5$

Then $t^2 - 3t - 4 < 0$

$\Rightarrow \quad (t-4)(t+1) < 0 \quad \Rightarrow \quad t < 4$

$\Rightarrow \quad -4 < x < 4$

$\therefore \quad x \in (-4, 4) \quad \therefore \quad x \in (-\infty, -5) \cup (-4, 4) \cup (5, \infty)$

5. In ΔABC, if $\sin 2A + \sin 2B + \sin 2C = 4 - 4\cos A \cos B$, then triangle is

(A) equilateral (B) right angled but not isosceles

(C) isosceles but not right angled (D) isosceles and right angled

Ans. **(D)**

Sol. We know that in ΔABC, $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

$\therefore \sin A \sin B \sin C = 1 - \cos A \cos B$

$\sin C=\frac{1-\cos A\cos B}{\sin A\sin B}\le 1$

$\cos(A - B) \geq 1$ i.e. $A = B$

Hence $\angle C=\frac{\pi}{2}, \angle A=\angle B=\frac{\pi}{4}$

MCQ (One or more than one correct) :

6. If $f(\theta) = \sum_{n=1}^{6} \text{cosec}\left(\theta + \frac{(n-1)\pi}{4}\right)\text{cosec}\left(\theta + \frac{n\pi}{4}\right)$, where $0 < \theta < \frac{\pi}{2}$,

then minimum value of f

(A) lies between 3 and 4

(B) lies between 2 and 3

(C) occures when $\theta = \frac{\pi}{4}$

(D) occures when $\theta = \frac{\pi}{6}$

Ans. (BC)

Sol. We have

$$f(\theta) = \sqrt{2}\sum_{n=1}^{6} \frac{\sin\left[\left(\theta+\frac{n\pi}{4}\right)-\left(\theta+(n-1)\frac{\pi}{4}\right)\right]}{\sin\left(\theta+(n-1)\frac{\pi}{4}\right)\sin\left(\theta+\frac{n\pi}{4}\right)} = \sqrt{2}\sum_{n=1}^{6}\left[\cot\left(\theta+(n-1)\frac{\pi}{4}\right)-\cot\left(\theta+\frac{n\pi}{4}\right)\right]$$

$$= \sqrt{2}\left(\cot\theta - \cot\left(\theta+\frac{3\pi}{2}\right)\right) = \sqrt{2}\ (\cot\theta + \tan\theta) = \sqrt{2}\left[\left(\sqrt{\tan\theta}-\sqrt{\cot\theta}\right)^2+2\right]$$

$$\therefore \quad f_{min.}\left(\theta = \frac{\pi}{4}\right) = 2\sqrt{2}$$

7. Let $E = \cos^2\frac{\pi}{7} + \cos^2\frac{2\pi}{7} + \cos^2\frac{3\pi}{7}$. Then which of the following alternative(s) is/are incorrect?

(A) $\frac{1}{2} < E < \frac{3}{4}$

(B) $\frac{3}{4} < E < 1$

(C) $1 < E < \frac{3}{2}$

(D) $\frac{3}{2} < E < \frac{7}{4}$

Ans. (ABD)

Sol. We have $E = \cos^2\frac{\pi}{7} + \cos^2\frac{2\pi}{7} + \cos^2\frac{3\pi}{7}$

$$= \frac{1+\cos\frac{2\pi}{7}}{2} + \frac{1+\cos\frac{4\pi}{7}}{2} + \frac{1+\cos\frac{6\pi}{7}}{2} = \frac{3}{2}+\frac{1}{2}\underbrace{\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)}_{S}$$

Now, $S = \cos 2\theta + \cos 4\theta + \cos 6\theta$, where $\theta = \frac{\pi}{7}$

$\Rightarrow \quad 2S\sin\theta = \sin 3\theta - \sin\theta + \sin 5\theta - \sin 3\theta + \sin 7\theta - \sin 5\theta = \underbrace{\sin 7\theta}_{\text{zero}} - \sin\theta$

$\therefore \quad S = -\frac{1}{2} \quad \Rightarrow \quad E = \frac{3}{2} - \frac{1}{4} = \frac{5}{4}$

Clearly $1 < E < \frac{3}{2}$

8. Consider the equation $\sin\left(\pi\log_3\left(\frac{1}{x}\right)\right) = 0$ Then which of the following statements is/are true ?

(A) there exists, infinitely many solutions for the equation in $(0, 2\pi)$

(B) there exists, finitely many solutions for the equation in $(0, 2\pi)$

(C) $x = 3$ is one of the solution for the given equation in $(0, 2\pi)$

(D) sum of all the solutions of the given equation in $(0, 2\pi)$ is $\frac{9}{2}$

Ans. (ACD)

Sol. $\sin\left(\pi\log_3\dfrac{1}{x}\right)=0$

$\Rightarrow \quad \pi\log_3\dfrac{1}{x} = n\pi,\ n\in Z \qquad \Rightarrow \quad x=\dfrac{1}{3^n}$

For, n = –1, 0, 1, 2 ...$x\in(2, 2\pi)$

And sum of all the possible values of x is $3 + 1 + \dfrac{1}{3}+\dfrac{1}{3^2}+\ldots=\dfrac{9}{2}$

9. If $S(n) = \sum_{r=1}^{n}\dfrac{\cos(2\times 3^{r-1}\alpha)}{\sin(3^r\alpha)}, \alpha=18°$ and $P(n) = \sum_{r=1}^{n}\dfrac{\cos(3\cdot 2^{r-1}\beta)}{\cos(2^r\beta)}, \beta=36°$, then

(A) S(2012) + P(2012) = 2010 (B) S(2012) + P(2012) = 2012

(C) P(2011) – S(2011) = 2009 (D) P(2011) – S(2011) = 2008

Ans. **(AD)**

Sol. $2\alpha + 3\alpha = 90° \Rightarrow 2\alpha = 90° – 3\alpha$

$2 \times 3^{r–1}\alpha = 90° \times 3^{r–1} – 3^r\alpha$

$\cos(2 \times 3^{r–1}\alpha) = \pm \sin(3^r\alpha)$

$\therefore \dfrac{\cos(2\times 3^{r-1}\alpha)}{\sin(3^r\alpha)}=\begin{cases}1, & r \text{ is odd}\\ -1, & r\text{ is even}\end{cases}$

Similarly, $\cos(2^r\beta)=\begin{cases}-\cos(3\times 2^{r-1}\beta), & r=1\\ \cos(3\times 2^{r-1}\beta), & r\ge 2\end{cases}$

10. If sinx + cosx + tanx+ cotx + secx + cosec = 7 and sin 2x = a – b$\sqrt{7}$, then a – b + 14 is divisible by _____.

(A) 2 (B) 3 (C) 4 (D) 7

Ans. **(ACD)**

Sol. $\sin x+\cos x+\dfrac{1}{\sin x\cos x}+\dfrac{\sin x+\cos x}{\sin x\cos x}=7$

$\Rightarrow \quad (\sin x+\cos x)\left(1+\dfrac{2}{\sin 2X}\right)=7-\dfrac{2}{\sin 2x}$

$\Rightarrow \quad (1+\sin 2x)\left(1+\dfrac{2}{\sin 2x}\right)^2=49+\dfrac{4}{\sin^2 2x}-\dfrac{28}{\sin 2x}$

$\Rightarrow \quad t^2 – 44t + 36 = 0$, where t = sin2x

Solving, we get sin 2x = 22 – 8$\sqrt{7}$

$\therefore \quad$ a = 32, b = 8

11. Let $\dfrac{b\cos x}{2\cos 2x-1}=\dfrac{b+\sin x}{(\cos^2x-3\sin^2x)\tan x}, b\in R$. Which of the following are correct?

(A) The given equation has a solution if $b\in\left(-\infty,\dfrac{1}{2}\right)-\left\{-1,0,\dfrac{1}{3}\right\}$

(B) The given equation has a solution if $b\in\left(\dfrac{1}{2},\infty\right)-\left\{1,\dfrac{1}{3}\right\}$

(C) The given equation has no solution if b = 10

(D) The given equation has no solution if b = – 10

Ans. **(AC)**

Sol. (1) $2\cos 2x - 1 \neq 0 \quad \Rightarrow x \neq n\pi \pm \dfrac{\pi}{6}$

(2) $\tan x \neq 0 \quad \Rightarrow x \neq \pm \dfrac{n\pi}{2}$

(3) $\cos^2 x - 3\sin^2 x \neq 0 \qquad \Rightarrow x \neq n\pi \pm \dfrac{\pi}{6}$

Also, $2\cos 2x - 1 = 2(\cos^2 x - \sin^2 x) - (\cos^2 x + \sin^2 x) = \cos^2 x - 3\sin^2 x$

Now, the given equation reduces to $b \sin x = b + \sin x \Rightarrow \sin x = \dfrac{b}{b-1}$

$\because -1 \leq \sin x \leq 1 \qquad \Rightarrow 1 \leq \dfrac{b}{b-1} \leq 1 \Rightarrow \dfrac{b}{b-1} 1 \geq 0$ and $\dfrac{b}{b-1} 1 \leq 0$

$\Rightarrow \dfrac{2b-1}{b-1} \geq 0$ and $\dfrac{1}{b-1} \leq 0 \quad \Rightarrow b \in \left(-\infty, \dfrac{1}{2}\right] \cup [1, \infty)$ and $b \in (-\infty, 1]$

When $b = \dfrac{1}{2} \quad \Rightarrow \sin x = 1$ which is not possible

$\therefore b \in \left(-\infty, \dfrac{1}{2}\right)$

12. If $\left(\sin 2x + \sqrt{3}\cos 2x\right)^2 - 5 = \cos\left(2x - \dfrac{\pi}{6}\right)$ then the value of cos x can be

(A) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (B) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (C) $-\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)$ (D) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

Ans. (BD)

Sol. The given equation is $4\left(\dfrac{\sin 2x}{2} + \dfrac{\sqrt{3}}{2}\cos 2x\right)^2 = 5 + \cos\left(2x - \dfrac{\pi}{6}\right)$

or $\quad 4\left(\cos 2x \cos\dfrac{\pi}{6} + \sin^2 x \cdot \sin\dfrac{\pi}{6}\right)^2 = 5 + \cos\left(2x - \dfrac{\pi}{6}\right)$

or $\quad 4\cos^2\left(2x - \dfrac{\pi}{6}\right) - \cos\left(2x - \dfrac{\pi}{6}\right) - 5 = 0$

Let $\quad \cos\left(2x - \dfrac{\pi}{6}\right) = t$

$4t^2 - t - 5 = 0 \Rightarrow (4t - 5)(t + 1) = 0$

$\therefore \quad t \neq \dfrac{5}{4}$ hence $t = -1$

$\cos\left(2x - \dfrac{\pi}{6}\right) = \cos(2n-1)\pi$

$2x - \dfrac{\pi}{6} = (2n-1)\pi \qquad \Rightarrow x = (2n-1)\dfrac{\pi}{2} + \dfrac{\pi}{6} \qquad \Rightarrow x = n\pi - \dfrac{5\pi}{12}, n \in I$

Now verify all the alternatives.

13. Possible integral value(s) of m for the equation $\sin x - \sqrt{3}\cos x = \dfrac{4m-6}{4-m}$ can be valid for some $x \in [0, 2\pi]$ is

(A) –1 (B) 0 (C) 1 (D) 2

Ans. (ABCD)

Sol. $-2 \le \sin x - \sqrt{3}\cos x \le 2$

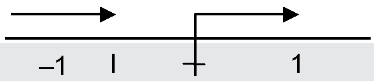
hence $-2 \le \dfrac{4m-6}{4-m} \le 2$

$-1 \le \dfrac{2m-3}{4-m} \le 1$

now if $\dfrac{2m-3}{4-m} \le 1$

$\dfrac{(2m-3)-(4-m)}{4-m} \le 0$

$\dfrac{3m-7}{m-4} \ge 0$



Again $-1 \le \dfrac{2m-3}{4-m} \propto \dfrac{(m-4)-(2m-3)}{4-m} \le 0 \propto \dfrac{(2m-3)-(m-4)}{4-m} \le 0$

$\dfrac{m+1}{m-4} \le 0$

hence $m \in \left[-1, \dfrac{7}{3}\right] \Rightarrow m \in \{-1, 0, 1, 2\}$

Numerical based Questions :

14. If complete solution set of inequality $\sqrt{x^2 - 2x} > 4 - x$, is $\left(\dfrac{m}{n}, \infty\right)$, m, n being co-prime numbers, then value of m + n is

Ans. **11.00**

Sol. Case-I:

$4 - x \le 0$

$x \ge 4$

$x \in [4, \infty])$

Case-II:

$4 - x > 0$

$x \in (-\infty, 4)$ ....(i)

$x^2 - 2x \ge 0$

$\Rightarrow x \in (-\infty, 0] \cup [2, \infty)$ .....(ii)

SBS $\Rightarrow x^2 - 2x > 16 - 8x + x^2$

$\Rightarrow x > \dfrac{8}{3}$ ....(iii)

$\Rightarrow x \in \left(\dfrac{8}{3}, 4\right)$

Hence complete solution is $x \in \left(\dfrac{8}{3}, \infty\right)$ m + n = 8 = 3 = 11

15. The number of solutions of equation $8[x^2 - x] + 4[x] = 13 + 12[\sin x]$ is (here [.] represents greatest integer less than or equal to 'x')

Ans. **0.00**

Sol. $\because$ RHS is always odd

While LHS is always even

16. If $3\,p\sin x - (p + \sin x)(p^2 - p\sin x + \sin^2 x) = 1$ has a solution for x, then the number of integral values of p are _________

Ans. **4.00**

Sol. $3p\sin x - (p^3 + \sin^3 x) = 1$

$\Rightarrow p^3 + \sin^3 x + 1 = 3\,p\sin x$

$\Rightarrow \sin x + p + 1 = 0$ or $\sin x = p = 1$

So, four such values

17. If α and β are roots of equation $(\log_2 x)^2 + 4(\log_2 x) - 1 = 0$, then value of $\log_\beta \alpha + \log_\alpha \beta$ equals

Ans. **18.00**

Sol. Let $\log_2 x = t$

$\log_2 \alpha\ - t_1$ and $\log_2 \beta = t_2$ then $\frac{t_1}{t_2} + \frac{t_2}{t_1} = -18$

18. Let $x = (\log_{1/3} 5)(\log_{125} 343)(\log_{49} 729)$ and $y = 25^{3\log_{289} 11\ \log_{28}\sqrt{17}\ \log_{1331} 784}$, then find the value of $x^2 + y^2$ is

Ans. **34**

Sol. $x = (-\log_3 5)\left(\log_{5^3}(7^3)\right)\left(\log_{7^2}(3^6)\right)$

$x = -\frac{3}{3}.\frac{6}{2}\log_3 5\ .\ \log_5 7\ .\ \log_7 3$ and $y = 25^{\left(3\log_{17^2} 11\right)\left(\log_{28} 17^{1/2}\right)\left(\log_{(11)^3}(28)^2\right)}$

$y = 25^{\frac{3}{2}.\frac{1}{2}.\frac{2}{3}(\log_{17} 11).(\log_{28} 17)(\log_{11} 28)}$

$y = 5$

$\therefore \quad x^2 + y^2 = (-3)^2 + 5^2 = 34$

19. Let $f(x) = |x - 2|$ and $g(x) = |3 - x|$ and

A be the number of real solutions of the equation $f(x) = g(x)$

B be the minimum value of $h(x) = f(x) + g(x)$

C be the area of triangle formed by $f(x) = |x - 2|$, $g(x) = |3 - x|$ and x-axis and $\alpha < \gamma < \beta < \delta$ where $\alpha < \beta$ are the roots of $f(x) = 4$ and $\gamma < \delta$ are the roots of $g(x) = 4$, then find the value of sum of digits of .

$$\frac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC}$$

Ans. 9

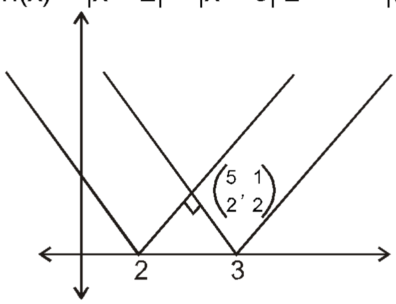
Sol. $f(x) = |x - 2|$, $g(x) = |x - 3|$

$f(x) = g(x) \quad \Rightarrow \quad |x - 2|^2 = |x - 3|^2 \quad \Rightarrow \quad x^2 - 4x + 4 = x^2 - 6x + 9$

$\Rightarrow \quad x = \frac{5}{2}$

Hence A = 1

$h(x) = |x - 2| + |x - 3| \geq \quad |(x - 2) - (x - 3)| = 1 \quad \Rightarrow \quad B = 1$



hence $C = \frac{1}{4}$

$|x - 2| = 4 \quad \Rightarrow \quad x = 2 \pm 4 \quad \Rightarrow \quad \alpha = -2$ and $\beta = 6$

$|x - 3| = 4 \quad \Rightarrow \quad x = 3 \pm 4 \quad \Rightarrow \quad \gamma = -1$ and $\delta = 7$

$\frac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC} = 4(4 + 36 + 1 + 49) = 360$

Sum of digits = 9

20. The reciprocal of the value of the product

$\sin\left(\frac{5\pi}{2^{11}}\right) \cos\left(\frac{5\pi}{2^{11}}\right) \cos\left(\frac{5\pi}{2^{10}}\right) \cos\left(\frac{5\pi}{2^{9}}\right) \ldots\ldots\ldots \cos\left(\frac{5\pi}{2^{3}}\right) \cos\left(\frac{5\pi}{2^{2}}\right)$ equals.

Ans. 1024

Sol. We have

$$\frac{2^{10}\sin\left(\frac{5\pi}{2^{11}}\right)\cos\left(\frac{5\pi}{2^{11}}\right)\cos\left(\frac{5\pi}{2^{10}}\right)\cos\left(\frac{5\pi}{2^{9}}\right)\ldots\ldots\cos\left(\frac{5\pi}{2^{3}}\right)\cos\left(\frac{5\pi}{2^{2}}\right)}{2^{10}}$$

$$= \frac{\sin\frac{5\pi}{2}}{2^{10}} = \frac{\sin 450°}{2^{10}} = \frac{1}{2^{10}} = \frac{1}{1024}$$

$\Rightarrow$ Reciprocal of the value of the product = 1024 **Ans.**

Comprehension Type Question:

Paragraph for question nos. 21 to 23

Let A denotes the sum of the roots of the equation $\frac{1}{5 - 4\log_4 x} + \frac{4}{1 + \log_4 x} = 3$.

B denotes the value of the product of m and n, if $2^m = 3$ and $3^n = 4$.

C denotes the sum of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right) + (\log_3 x)^2 = 1$.

21. The value of A + B equals

(A) 10 (B) 6 (C) 8 (D) 4

Ans. **(C)**

22. The value of B + C equals

(A) 6 (B) 2 (C) 4 (D) 8

Ans. **(A)**

23. The value of A + C – B equals

(A) 5 (B) 8 (C) 7 (D) 4

Ans. **(B)**

Sol. (A) We have

$$\frac{1}{5 - 4\log_4 x} + \frac{4}{1 + \log_4 x} = 3$$

Put $\log_4 x = y$

$\Rightarrow \quad \frac{1}{5 - 4y} + \frac{4}{1 + y} = 3$

$\Rightarrow \quad (1 + y) + 4(5 - 4y) = 3(5 - 4y)(1 + y)$

$\Rightarrow \quad 2y^2 - 3y + 1 = 0$

$\therefore \quad y = 1, \frac{1}{2}$

$\Rightarrow \quad \log_4 x = 1 \;\; \& \;\; \log_4 x = \frac{1}{2}$

$\therefore \quad x = 4 \;\; \& \;\; x = 2$

$\Rightarrow$ sum of the roots of the equation = 4 + 2 = 6

(B) We have $2^m = 3$ and $3^n = 4$

$\Rightarrow \quad m = \frac{\log 3}{\log 2}$ and $n = \frac{\log 4}{\log 3}$

$\therefore \quad mn = \frac{\log 3}{\log 2} \cdot \frac{\log 4}{\log 3} = \log_2 4 = 2$

$\therefore \quad mn = 2$

(C) Now, we have clearly $0 < x \neq \frac{1}{3}$

$\log_{3x} 3 - \log_{3x} x + (\log_3 x)^2 = 1$

$\Rightarrow \quad \frac{1}{\log_3(3x)} - \frac{1}{\log_x(3x)} + (\log_3 x)^2 = 1$

$\Rightarrow \quad \frac{1}{1+\log_3 x} - \frac{1}{\log_x 3 + 1} + (\log_3 x)^2 = 1$

Put $\log_3 x = y$, we get

$\Rightarrow \quad \frac{1}{y+1} - \frac{1}{\frac{1}{y}+1} + y^2 = 1$

$\Rightarrow \quad \frac{(1-y)}{y+1} + y^2 = 1 \qquad \Rightarrow \quad (1 - y) + y^2 (y + 1) = y + 1$

$\Rightarrow \quad 1 - y + y^3 + y^2 - y - 1 = 0 \qquad \Rightarrow \quad y^3 + y^2 - 2y = 0$

$\therefore \quad y(y^2 + y - 2) = 0$

or $\quad y(y + 2)(y - 1) = 0 \qquad \therefore \quad y = 0, -2, 1$

But $\log_3 x = y$ gives $n = 1, \frac{1}{9}, 3 \qquad \therefore$ Sum of integral roots = 1 + 3 = 4

Matrix Match Type :

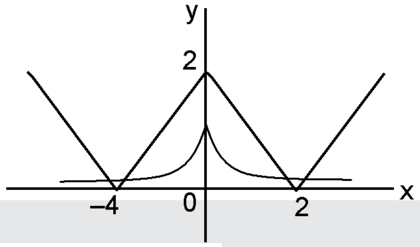
24. Math the following :

	Column-I		Column-II
A	The number of real roots of the equation $3^{\lvert x\rvert}\,\lvert 2 - \lvert x\rvert\rvert = 1$ is /are	p	3
B	The number of real roots of the equation $3\cos^{-1}x - \pi x - \frac{\pi}{2} = 0$ is / are	q	4
C	The number of real roots of sin x cos y = 1 $\forall x, y \in [0, 2\pi]$ is / are	r	2
D	The least degree of polynomial with integer coefficient one of the roots may be cos 12° is/are	s	1

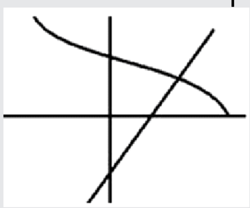
Code :
(A) A → (q); B → (p); C → (p); D → (q)
(B) A → (q); B → (s); C → (q); D → (q)
(C) A → (q); B → (s); C → (p); D → (q)
(D) A → (p); B → (s); C → (p); D → (q)

Ans. **(C)**

Sol. **(A)**



(B)



(C) $(x, y) \to \left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 2\pi\right), \left(\frac{3\pi}{2}, \pi\right)$

(D) Put $\theta = 12°$

$\cos 60° = \cos (2\theta + 3\theta)$

$\frac{1}{2} = \cos 2\theta \cos 3\theta - \sin 2\theta \sin 3\theta$

Put $\cos \theta =$ t and express in terms of t we get

$32t^5 - 40t^3 + 10t - 1 = 0$

But $t \neq \frac{1}{2}$, so we get $16^4 + 8t^3 - 16t^2 - 8t + 1 = 0$

So, its degree is 4

25. Math the following :

	Column-I		**Column-II**
A	If $\alpha, \beta (\alpha < \beta)$ are the two roots of the equation $\frac{1 - 8(\log_{10} x)^2}{\log_{10} x - 2(\log_{10} x)^2} = 1$ then the value of $\frac{(\beta^3 + 1)}{\alpha^2 \beta^3}$	P	5
B	Number of solution(s) of equation $\log_2 (x^2 + 3) = \frac{1}{2} \log_{1/3} \left(x + \frac{1}{x}\right), x > 0$ is	Q	11
C	Number of integers satisfying the equation $\lvert x \rvert + \left\lvert \frac{4 - x^2}{x} \right\rvert = \left\lvert \frac{4}{x} \right\rvert$ is	R	0
D	If $bog_c\, 2 \log_b 125 = \log_{10} 8 \log_c 10$ where $c > 0, c \neq 1, b > 0, b \neq 1$, then b is	S	4

Code:
(A) A - P; B - S; C - R; D - Q
(B) A - Q; B - S; C - R; D - P
(C) A - P; B - R; C - S; D - Q
(D) A - Q; B - R; C - S; D - P

Ans. **(D)**

Sol. **(A)** Let $\log_{10} x = y$

$\Rightarrow \quad \frac{1-8y^2}{y-2y^2} = 1 \qquad \Rightarrow \quad 1 - 8y^2 = y - 2y^2$

$\Rightarrow \quad 6y^2 + y - 1 = 0 \qquad \Rightarrow \quad 6y^2 + 3y - 2y - 1 = 0$

$\Rightarrow \quad (3y - 1)(2y + 1) = 0 \qquad \Rightarrow \quad y = \frac{1}{3}, -\frac{1}{2}$

$\Rightarrow \quad \log_{10} x = \frac{1}{3}, -\frac{1}{2} \qquad \Rightarrow \quad x = 10^{\frac{1}{3}}, 10^{-\frac{1}{2}}$

$\Rightarrow \quad \alpha = 10^{-\frac{1}{2}}, \beta = 10^{\frac{1}{3}}$

So, $\quad \frac{\beta^3 + 1}{\alpha^2\beta^3} = \frac{10+1}{\frac{1}{10} \times 10} = 11$

(B) $\quad x > 0 \Rightarrow x^2 + 3 > 3 \qquad \Rightarrow \quad \log_2(x^2+3) > \underbrace{\log_2 3}_{+ve}$

$\Rightarrow \quad \log_2 (x^2 + 3)$ is always positive

Also, $x > 0 \quad \Rightarrow x + \frac{1}{x} \geq 2 \qquad \Rightarrow \quad \log_{\frac{1}{3}}\left(x + \frac{1}{x}\right) \leq \log_{\frac{1}{3}} 2$

$\Rightarrow \quad \frac{1}{2}\log_{\frac{1}{3}}\left(x + \frac{1}{x}\right) \leq \underbrace{\frac{1}{2}\log_{\frac{1}{3}} 2}_{-ve} \qquad \Rightarrow \quad \frac{1}{2}\log_{\frac{1}{3}}\left(x + \frac{1}{x}\right)$ is always negative

Hence, there is no values of x for which $\log_2(x^2+3) = \frac{1}{2}\log_{\frac{1}{3}}\left(x + \frac{1}{x}\right)$

(C) $\quad \left|\frac{4-x^2}{x}\right| = \frac{4}{|x|} - |x|$

$\Rightarrow \quad \frac{|4-x^2|}{|x|} = \frac{4-x^2}{|x|} \qquad \Rightarrow \quad |4 - x| = 4x^2 \quad ; x \neq 0$

$\Rightarrow \quad 4 - x \geq 0 \quad ; x \neq 0 \qquad \Rightarrow \quad (x + 2)(x - 2) \leq 0 \quad ; x \neq 0$

$\Rightarrow \quad x \in [-2, 2] - \{0\}$

So, integer values of x are –2, –1, 1, 2

So, number of integral values of x is 4.

(D) $\quad \log_c 2 \log_b 125 = \log_{10} 8 \log_c 10$

$\Rightarrow \quad \log_b 125 = \log_{10} 8 \frac{\log_c 10}{\log_c 2} = \log_{10} 8 \log_2 10$

$\Rightarrow \quad \log_b 125 = \log_2 8 = 3 \qquad \Rightarrow \quad 125 = b^3$

$\Rightarrow \quad b^3 = 5^3 \qquad \Rightarrow \quad b = 5$