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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-9

BINOMIAL THEOREM

SCQ (Single Correct Type) :

1. If a, b, c, d be four consecutive coefficients in the binomial expansion of $(1 + x)^n$, then value of the expression $\left(\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right)$ (where x > 0 and $n \in N$) is

(A) Positive (B) negative (C) zero (D) depend on n

2. The coefficient of x^{28} in the expansion of $(2 - x^3 + x^6)^{30}$ is

(A) 0 (B) 1 (C) 14 (D) 28

3. If each coefficient in the expansion of the expression $x(1+x)^n \ (n \in \mathbb{N})$ in powers of x is divided by the exponent of corresponding power, then the sum of the values thus obtained is equal to _____.

(A) $\frac{2^n}{n+1}$ (B) $\frac{2^n-1}{n+1}$ (C) $\frac{2^n+1}{n+1}$ (D) $\frac{2^{n+1}-1}{n+1}$

4. The number of distinct terms in the expansion of $(x+y^2)^{13} + (x^2+y)^{14}$ is _____.

(A) 27 (B) 29 (C) 28 (D) 25

5. If $f(x) = \sum_{r=1}^{n}\left[r\left(n \cdot {}^{n-1}C_{r-1} - r.\ {}^{n}C_{r-1}\right) + (2r+1)\,{}^{n}C_r\right]$ then ______.

(A) $f(n) = n^2 - 1$ (B) $f(n) = (n+1)^2 - 1$ (C) $f(n) = (n+1)^2 + 1$ (D) $\sum_{n=1}^{10} f(n) = 374$

6. If $(1+px+x^2)^n = 1 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, where $n \in \mathbb{N}$, $p \in \mathbb{R}$. If p = −3 and n is even number, then the value of $a_1 + 3a_2 + 5a_3 + 7a_4 + ... + (4n-1)a_{2n}$ is _____.

(A) n (B) 2n −1 (C) 2n – 2 (D) 2n

7. If $6^{83} + 8^{83}$ is divided by 49, then the remainder is

(A) 35 (B) 5 (C) 1 (D) 0

8. The sum of the series $(1^2 + 1).1! + (2^2 + 1).2! + (3^2 + 1).3! + \ldots\ldots + (n^2 + 1)\ n!$

(A) (n + 1). (n + 2)! (B) n.(n + 1)! (C) (n + 1) (n + 1)! (D) none of these

MCQ (One or more than one correct) :

9. If $J_m = \sum_{r=0}^{m-3} {}^mC_r \, {}^mC_{r+3}$ and m_1, m_2 are two values of m satisfying $5\ J_M = 3J_{M+1}$, then correct statement is/are (where [.] denotes greatest integer function

(A) $m_1 + m_2 = \frac{-8}{7}$ (B) $m_1 m_2 = \frac{46}{7}$

(C) $[(1-m_1)(1-m_2)] = 8$ (D) $[(1+m_1)(1+m_2)] = 8$

10. For natural number m, n if $(1 - y)^m (1 + y)^n = 1 + a_1 y + a^2y^2 + ...$ and $a_1 = a_2 = 10$ then

(A) m < n (B) m > n (C) m + n = 80 (D) m – n = 20

11. Which of the following is/are true

(A) $5^6 - {}^5C_1.4^6 + {}^5C_2.3^6 - {}^5C_3.2^6 + {}^5C_4.1^6 = {}^6C_2.\underline{|5}$

(B) $6^5 - {}^6C_1.5^5 + {}^6C_2.4^5 - {}^6C_3.3^5 + {}^6C_4.2^5 - {}^6C_1.1^5 = 0$

(C) $6^6 - {}^6C_1.5^6 + {}^6C_2.4^6 - {}^6C_3.3^6 + {}^6C_4.2^6 - {}^6C_5.1^6 = 720$

(D) $6^5 - {}^6C_1.5^5 + {}^6C_2.4^5 - {}^6C_3.3^5 + {}^6C_4.2^5 - {}^6C_5.1^5 = {}^5C_2.\underline{|6}$

12. In the expansion of $\left(x^3 + 3.2^{-\log_{\sqrt{2}} \sqrt{x^3}}\right)^{11}$

(A) there appears a term with the power x^2

(B) there does not appear a term with the power x^2

(C) there appears a term with the power x^{-3}

(D) the ratio of the co-efficient of x^3 to that of x^{-3} is 1/3

13. If $(9+\sqrt{80})^n = I + f$ where I, n are integers and 0 < f < 1, then

(A) I is an odd integer (B) I is an even integer

(C) (I + f) (1 – f) = 1 (D) $1 - f = (9-\sqrt{80})^n$

Comprehension Type Question:

Comprehension # 1

If $(1 + x + x^2)^{2n} = 1 + a_1x + a_2x^2 + ... a_{4n}x^{4n}$, then

14. The value of $\sum_{r=0}^{n-1} a_{2r}$, is

(A) $\frac{9^n + 1 - 2a_{2n}}{4}$ (B) $\frac{9^n + 1 + 2a_{2n}}{4}$ (C) $\frac{9^n + 1 - 2a_n}{4}$ (D) $\frac{9^n - 1 - 2a_{2n}}{4}$

15. What value a_2 takes ?

(A) ${}^{2n}C_2$ (B) ${}^{2n+1}C_2$ (C) ${}^{2n-1}C_2$ (D) ${}^nC_2$

Numerical based Questions :

16. If the coefficient of x^5 in $(1 + x)^{21} + (1 + x)^{22} + \ldots.+(1+x)^{30}$ is $\lambda\ {}^{n}C_r + \mu\ {}^{m}C_r$, λ, μ, n, r, m being integers and ${}^{n}C_r$, ${}^{m}C_r$ are binomial coefficients, then the value of $\lambda(n + r) + \mu(m + r)$ is

17. The value of $\sum_{r=0}^{9} \frac{{}^{10}C_r}{{}^{10}C_r + {}^{10}C_{r+1}}$ is equal to

18. When the terms in the binomial expansion of $\left(\sqrt{x} + \frac{1}{2x^{1/4}}\right)^n$ are arranged in decreasing powers of x, the coefficients of the first three terms are in arithmetic progression. The number of terms in the expansion with integer powers of x is _____.

19. Find the coefficient of x^7 in $(1 - x + 2x^3)^{10}$.

20. Find the coeffcient of x^{49} in the polynomial

$$\left(x - \frac{C_1}{C_0}\right)\left(x - 2^2 \cdot \frac{C_2}{C_1}\right)\left(x - 3^2 \cdot \frac{C_3}{C_2}\right)\ldots\ldots\ldots\ldots\left(x - 50^2 \cdot \frac{C_{50}}{C_{49}}\right) \text{ where } C_r = {}^{50}C_r .$$

Matrix Match Type :

21. Match the following:

Column – I	**Column – II**
(A) Number of distinct terms in the expansion of $(x+y-z)^{16}$ is	(p) 2^{12}
(B) Number of terms in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$ is	(q) 97
(C) The number of irrational terms in $\left(\sqrt[8]{5} + \sqrt[6]{2}\right)^{100}$ is	(r) 4
(D) The sum of numerical coefficients in the expansion of $\left(1 + \frac{x}{3} + \frac{2y}{3}\right)^{12}$ is	(s) 153

Options:

(A) A → (s); B → (r); C → (q); D → (p)

(B) A → (s); B → (r); C → (p); D → (q)

(C) A → (r); B → (s); C → (q); D → (p)

(D) A → (s); B → (q); C → (r); D → (p)

22. Match the following:

Column – I	Column – II
(A) The minimum value of a + b + c + d if $\log_3(a+b)+\log_3(c+d)\geq 4$ is	(p) 18
(B) The number of distinct terms in the expansion of $\left(x+y+\frac{1}{x}+\frac{1}{y}\right)^{14}$ is	(q) 225
(C) The remainder when $(23)^{86}$ is divided by 100 is	(r) 89
(D) If the third term in the expansion of $\left(\frac{1}{x}+x^{\log_{10} x}\right)^5$ is 1000 and x > 1, then the value of x is	(s) 100

(A) A → (p); B → (q); C → (s); D → (r)

(B) A → (p); B → (r); C → (q); D → (s)

(C) A → (p); B → (q); C → (r); D → (s)

(D) A → (q); B → (p); C → (r); D → (s)

Subjective based Questions:

23. (a) Find the index of n of the binomial $\left(\frac{x}{5}+\frac{2}{5}\right)^n$ if the 9 the term of the expansion has numerically greatest coefficient ($n \in N$).

(b) For what positive value of x is the fourth term in the expansion of $(5 + 3x)^9$ is the greatest.

24. Show that $(1\cdot 2)\,{}^nC_2 + (2\cdot 3)\,{}^nC_3 + \ldots\ldots + (n-1)\cdot n\,{}^nC_n = n(n-1)2^{n-2}$

25. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in A.P. and S_n is the sum of first n term's, show that

$$\sum_{k=0}^{n} {}^nC_k S_k = 2^{n-2}(na_1 + S_n)$$

26. Show that $S_1 = \sum_{r=1}^{2n} {}^{4n}C_{2r-1}(-1)^{r-1} = 0$

27. In the expansion of $(x + y)^n$, if the sum of odd term's be p and sum of even term's be q, then prove that

(i) $p^2 - q^2 = \left(x^2 - y^2\right)^n$

(ii) $4pq = (x + y)^{2n} - (x - y)^{2n}$