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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-8

Complex Number

SCQ (Single Correct Type) :

1. The least value of $|z-3-4i|^2+|z+2-7i|^2+|z-5+2i|^2$ occurs when

(A) 1 + 3i (B) 3 + 3i (C) 3 + 4i (D) none of these

2. $|z-3i+4|+|z-ki+4|=k$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2

3. If $|z|^2+\bar{A}z^2+A\bar{z}^2+B\bar{z}+\bar{B}z+c=0$ represents a pair of intersecting lines with angle of intersection θ then the value of |A| is

(A) tan θ (B) cos θ (C) sec θ (D) $\frac{\sec\theta}{2}$

4. If $|z-z_1| = |z_1|$ and $|z-z_2| = |z_2|$ be the of two circles if the two circles touch each other then

(A) Re $(z_1 z_2) = 0$ (B) Re $\left(\frac{z_1}{z_2}\right) = 0$ (C) $I_m (z_1 z_2) = 0$ (D) $I_m \left(\frac{z_1}{z_2}\right) = 0$

5. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(A) 24 (B) 48 (C) 72 (D) 96

6. If z = x + i y then the equation of a straight line Ax + By + C = 0 where A, B, C $\in$ R, can be written on the complex plane in the form $\bar{a}z+a\bar{z}+2C=0$ where 'a' is equal to :

(A) $\frac{(A+iB)}{2}$ (B) $\frac{A-iB}{2}$ (C) A + i B (D) none

MCQ (One or more than one correct) :

7. If from a point P, representing the complex number z_1, on the curve $|z|=2$, two tangents are drawn to the curve $|z|=1$, meeting the curve at points $Q(z_2)$ and $Q(z_3)$, then _____.

(A) the complex number $\frac{z_1+z_2+z_3}{3}$ will lie on the curve $|z|=1$

(B) $\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)\left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)=9$ (C) $\arg\left(\frac{z_2}{z_3}\right)=\frac{2\pi}{3}$

(D) orthocenter and circumcenter of ΔPQR will coincide

8. If z_1, z_2, z_3 are any three roots of the equation $z^6 = (z+1)^6$, , then arg $\left(\frac{z_1 - z_3}{z_2 - z_3}\right)$ can be equal to _____.

(A) 0 (B) π (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

9. Z_1, Z_2 are two complex numbers satisfying $i|z_1|^2 z_2 - |z_2|^2 z_1 = z_1 - iz_2$. Then which of the following is/are correct.

(A) $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$ (B) $I_m\left(\frac{z_1}{z_2}\right) = 0$

(C) $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$ (D) $|z_1||z_2| = 1$ or $|z_1| = |z_2|$

10. The curve represented by $z = \frac{3}{2 + \cos\theta + i\sin\theta}$, $\theta \in [0, 2\pi)$

(A) never meets the imaginary axis (B) meets the real axis in exactly two points

(C) has maximum value of |z| as 3 (D) has minimum value of |z| as 1

Numerical based Questions :

11. Let z_1, z_2, z_3 be three complex number such that $|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_1 z_3} + \frac{z_3^2}{z_1 z_2} + 1 = 0$, then sum of possible values of $|z_1 + z_2 + z_3|$ is _____.

12. If the vertices of a triangle ABC, A (z_1), B(z_2) and C(z_3) lie on the circle $|z - 3| = 16$, such that $z_1 + z_2 + z_3 = 9$, then the value of $\frac{z_1 \tan A + z_2 \tan B + z_3 \tan C}{\tan A \tan B \tan C}$ is equal to _____.

13. If ω is a non-real complex root of $z^{28} = 1$ and such that $|\omega + 1|$ is maximum and $x = \frac{1}{2}\left|\omega - \frac{1}{\omega}\right|$, then $8x^4 + 4x^3 - 8x^2 - 3x + 4$ is equal to _____.

14. Let z_1, z_2, z_3 be complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3|$ then $z_1^2 + z_2^2 + z_3^2$ is _____.

15. Let 1 $\alpha_1, \alpha_2, \alpha_3, ...\alpha_{10}$ be the eleven 11th roots of unity. Let $\lambda = \sum_{r=1}^{10} r\left(\alpha_r + \alpha_{11-r}\right)$. The value of $\frac{\lambda + 11}{11}$ equals

16. If a and b are positive integer such that $N = (a + ib)^3 - 107i$ is a positive integer then find the value of $\frac{N}{2}$

17. Let z,w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. If Re(z) < 0 and principal arg z = $\frac{a\pi}{b}$ then find the value of a + b. (where a & b are co-prime natural numbers)

18. Let z is a complex number satisfying the equation, $z^3 - (3 + i)z + m + 2i = 0$, where $m \in R$. Suppose the equation has a real root α, then find the value of $\alpha^4 + m^4$

19. If $x = 9^{1/3}\, 9^{1/9}\, 9^{1/27} \ldots\infty$, $y = 4^{1/3}\, 4^{-1/9}\, 4^{1/27} \ldots \infty$, and $z = \sum_{r=1}^{\infty}(1+i)^{-r}$ and principal argument of P = (x + yz) is $-\tan^{-1}\left(\frac{\sqrt{a}}{b}\right)$ then determine $a^2 + b^2$. (where a & b are co-prime natural numbers)

20. $z_1, z_2 \in c$ and $z_1^2 + z_2^2 \in R$,

$z_1(z_1^2 - 3z_2^2) = 2$, $z_2(3z_1^2 - z_2^2) = 11$

If $z_1^2 + z_2^2 = \lambda$ then determine λ^2

21. Let |z| = 2 and $w = \frac{z+1}{z-1}$ where z, w$\in$C (where C is the set of complex numbers), then find product of maximum and minimum value of |w|.

22. If ω and ω^2 are the non-real cube roots of unity and a, b, c $\in$ R such that $\frac{1}{a+\omega} + \frac{1}{b+\omega} + \frac{1}{c+\omega} = 2\omega^2$ and $\frac{1}{a+\omega^2} + \frac{1}{b+\omega^2} + \frac{1}{c+\omega^2} = 2\omega$. If $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} = \lambda$ then determine λ^4

Matrix Match Type :

23. **Match the following:**

Consider the two circles in the complex plane

$C_1 : \left|\frac{z+i}{z-1}\right| = 2$

$C_2 : z\bar{z} - (3+4i)z - (3-4i)\bar{z} + 9 = 0$

Let (α_1, β_1) be the centre of C_1 and (α_2, β_2) be that of C_2

Column-I	**Column-II**
(A) $2\alpha_1 + \beta_1$ equals	(p) 11
(B) $\alpha_2 - 2\beta_2$ euqals	(q) 3
(C) The radius of circle C_1 equals	(r) 4
(D) The radius of circle C_2 equals	(s) $\frac{2\sqrt{2}}{3}$
	(t) 2

Options:

(A) A $\to$ q; B $\to$ p; C $\to$ s; D $\to$ r;

(B) A $\to$ q; B $\to$ r; C $\to$ s; D $\to$ p;

(C) A $\to$ q; B $\to$ s; C $\to$ p; D $\to$ r;

(D) A $\to$ p; B $\to$ q; C $\to$ s; D $\to$ r;

Subjective based Questions :

24. If z_1 and z_2 are the two complex numbers satisfying $|z - 3 - 4i| = 8$ and $\text{Arg}\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then find the range of the values of $|z_1 - z_2|$.

25. Let a, b, c be distinct complex numbers such that $\frac{a}{1-b} = \frac{b}{1-c} = \frac{c}{1-a} = k$, (a, b, c $\neq$ 1). Find the value of k.