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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-7

Determinants and System of Equations

SCQ (Single Correct Type) :

1. The number of positive integral solutions (x, y, z) of the equation

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11 \text{ is\_\_}$$

(A) 0 (B) 3 (C) 6 (D) 12

2. If x = a, y = b, z = c is a solution of the system of linear equations x + 8y + 7z = 0, 9x + 2y + 3z= 0, x + y + z = 0 such that the point (a, b, c) lies on the plane x + 2y + z = 6, then 2a + b + c equals :

(A) 1 (B) 0 (C) 1 (D) 2

3. Let $A = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ such that |A| = 0. If a, b, c are distinct, then the sum of the coordinates of the fixed point through which the line ax + by + c = 0 passes is

(A) 2 (B) 3 (C) 4 (D) 5

4. If a system of the equation $(\alpha + 1)^3 x + (\alpha + 2)^3 y \quad (\alpha + 3)^3 = 0$ and $(\alpha + 1) x + (\alpha + 2) y \quad (\alpha + 3) = 0$, $x + y \quad 1 = 0$ is consistent, then the value(s) of α is/are

(A) 1 (B) 0 (C) 3 (D) 2

5. If a, b, c are complex numbers and $z = \begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix}$ is

(A) purely real (B) purely imaginary (C) 0 (D) none of these

6. If f(x) = $\log_{10}x$ and g(x) = $e^{i\pi x}$ and $h(x) = \begin{vmatrix} f(x)g(x) & [f(x)]^{g(x)} & 1 \\ f(x^2)g(x^2) & [f(x^2)]^{g(x^2)} & 0 \\ f(x^3)g(x^3) & [f(x^3)]^{g(x^3)} & 1 \end{vmatrix}$, then the value of h(10) is

(A) 0 (B) 2 (C) 1 (D) 4

7. s

ax+2y+5z=1

2x+y+3z=1

3y+7z=1

is consistent. Then the set S is :

(A) equal to R (B) equal to R {1} (C) equal to {1} (D) an empty set

8. Let α, β, γ are the real roots of the equation $x^3 + ax^2 + bx + c = 0$ ($a, b, c \in R$ and $a \neq 0$). If the system of equations (in u, v and w) given by

$$\alpha u + \beta v + \gamma w = 0$$
$$\beta u + \gamma v + \alpha w = 0$$
$$\gamma u + \alpha v + \beta w = 0$$

has non-trivial solutions, then a^2 equals

(A) b (B) 2b (C) 3b (D) 4b

9. For a unique value of p and q, the system of equations given by

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$2x + 5y + pz = q$$

has infinitely many solutions, then the value of (p+q) is equal to

(A) 14 (B) 24 (C) 34 (D) 44

MCQ (One or more than one correct) :

10. Consider the system of equations

$$ax_1 + x_2 + x_3 = 1$$
$$x_1 + ax_2 + x_3 = 1$$
$$x_1 + x_2 + ax_3 = 1$$

then :

(A) if a = 2, then the system has unique solution.

(B) if a = 1, then the system has infinite solution.

(C) if a =   2, then the system has no solution.

(D) if a = 2, then the system has infinite solution.

Comprehension Type Question:

Comprehension # 1

For $\alpha, \beta, \gamma, \theta \in R$. Let

$$A_\theta(\alpha, \beta, \gamma) = \begin{vmatrix} \cos(\alpha+\theta) & \sin(\alpha+\theta) & 1 \\ \cos(\beta+\theta) & \sin(\beta+\theta) & 1 \\ \cos(\gamma+\theta) & \sin(\gamma+\theta) & 1 \end{vmatrix}$$

11. If $a = A_{\pi/2}(\alpha, \beta, \gamma)$, $b = A_{\pi/3}(\alpha, \beta, \gamma)$. Which of the following is true

(A) a = b (B) a < b (C) a > b (D) 2a = b

12. $A_\theta^2 + A_\phi^2 \quad 2(A_{\theta+\phi})^2$ equals

(A) $\quad 2A_\theta A_\phi$ (B) $A_\theta + A_\phi$ (C) $A_\theta \quad A_\phi$ (D) None of these

13. If α, β, γ are fixed, then y = A_X (α, β, γ) represents

(A) a straight line parallel to x-axis

(B) a straight line through the origin

(C) a parabola with vertex at origin

(D) None of these

Comprehension # 2

Consider the system of equations :

ax + 4y + z = 0

2y + 3z   1 = 0

3x   bz + 2 = 0

Then

14. The given system of equations will have a unique solution if

(A) ab = 15 (B) ab $\neq$ 15 (C) ab = 5 (D) a $\neq$ 5

15. The system of equations will have infinite solutions if

(A) a = 3, b = 2 (B) a = 3, b = 4 (C) a = 5, b = 3 (D) a = 3, b = 5

16. The given system of the equations will have no solution if

(A) ab = 15, a $\neq$ 3 (B) ab $\neq$ 15, a $\neq$ 3 (C) ab $\neq$ 15, a = 3 (D) None of these

Numerical based Questions :

17. If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 3$ then the value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ is ___.

18. If $D = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ = then find the units digit of $\frac{D}{(10!)^3}$

19. Total number of 2 × 2 determinants whose entries are from the set { 1, 0, 1} has value equal to   1 is N. Then Sum of digits of N is :

20. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+2x & 1+x & 1+x \\ 1+2y & 1+3y & 1+y \\ 1+2z & 1+2z & 1+4z \end{vmatrix} = 0$, then the value of $x^{-1} + y^{-1} + z^{-1} + 9$ is

21. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \geq 0$, where a, b, c $\in$ R^+ {0}, then $\frac{a+b}{c}$ is

22. If a_1, a_2, a_3 , 5, 4, a_6, a_7, a_8, a_9 are in H.P. and D = $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$, then the value of 21D is

(Where [.] represents, the greatest integer function)

23. The absolute value of a for which system of equations, $a^3x + (a + 1)^3y + (a + 2)^3z = 0$, $ax + (a + 1)\,y + (a + 2)\,z = 0$, $x + y + z = 0$, has a non–zero solution is:

24. Consider the system of equations

$x + y + z = 4$

$2x + y + 3z = 6$

$x + 2y + pz = q$

Let L denotes the value of p if the system of equations has no solution. and M denotes the value of q if the system of equations has infinite solutions.

Find the unit digit of $(L^2 + M^2)$.

25. If 3^n is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+3}C_1 & {}^{n+6}C_1 \\ {}^{n}C_2 & {}^{n+3}C_2 & {}^{n+6}C_2 \end{vmatrix}$ then the maximum value of n is...........

26. Let the matrix A and B be defined as A = $\begin{bmatrix} 3 & 2 \\ 2 & \alpha \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$. If $\det(2A^9 B^{-1})$ = 2, then find the number of distinct possible real values of α .

27. Let A_n and B_n be square matrices of order 3, which are defined as

$A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \dfrac{2i + j}{3^{2n}}$ and $b_{ij} = \dfrac{3i - j}{2^{2n}}$ for all i and j, $1 \le i, j \le 3$.

If $\quad l = \lim\limits_{n \to \infty} \text{Tr.}\left(3A_1 + 3^2A_2 + 3^3A_3 + ... + 3^nA_n\right)$

and $\quad m = \lim\limits_{n \to \infty} \text{Tr.}\left(2B_1 + 2^2B_2 + 2^3B_3 + ... + 2^nB_n\right)$,

then find the value of $(l + m)$.

[**Note :** Tr. (P) denotes the trace of matrix P.]

Matrix Match Type :

28. Consider a square matrix A of order 2 which has its elements as 0,1,2 and 4. Let N denote the number of such matrices.

Column - A	**Column - B**	
(A) Possible non-negative value of det(A) is	(P)	2
(B) Sum of values of determinants corresponding to N matrices is	(Q)	4
(C) If absolute value of (det(A)) is least, then possible value of | adj(adj(adj A)) |	(R)	2
(D) If det (A) is algebraically least, then possible value of det(4A 1) is	(S)	0
	(T)	8

Subjective Type Questions:

29. If the system of equations x = cy + bz, y = az + cx and z = bx + ay has a non-zero solution and at least one of a, b, c is a proper fraction, prove that $a^2 + b^2 + c^2 < 3$ and abc > 1.

30. $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$, Show that minimum value of Δ is $27a^2b^2$. Given ab is constant.

31. If t is real and $\lambda = \dfrac{t^2-3t+4}{t^2+3t+4}$, then find number of solutions of the system of equations 3x y + 4z = 3, x + 2y 3z = 2, 6x + 5y + λz = 3 for a particular value of λ.