MATHEMATICS TARGET : JEE- Advanced 2023	CAPS-6 Matrices & Determinant

SCQ (Single Correct Type) :

1. Let $a = \lim_{x\to1} \frac{x}{\ln x} - \frac{1}{x\ln x}$; $b = \lim_{x\to0} \frac{x^3-16x}{4x+x^2}$; $c = \lim_{x\to0} \frac{\ln(1+\sin x)}{x}$ and

$d = \lim_{x\to-1} \frac{(x+1)^3}{3\left(\sin(x+1)-(x+1)\right)}$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(A) Idempotent (B) Involutary (C) Non singular (D) Nilpotent

2. A is a 2 × 2 matrix such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find the sum of the elements of A.

(A) –1 (B) 0 (C) 2 (D) 5

3. Matrix A satisfies $A^2 = 2A - I$ where I is the identity matrix then for $n \geq 2$, A^n is equal to $(n \in N)$

(A) $nA - I$ (B) $2^{n-1}A - (n-1)I$ (C) $nA - (n-1)I$ (D) $2^{n-1}A - I$

4. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then

(A) a = 1, c = – 1 (B) $a = 2, c = -\frac{1}{2}$ (C) a = – 1, c = 1 (D) $a = \frac{1}{2}, c = \frac{1}{2}$

5. The number of solution of the matrix equation $X^2 = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ is

(A) more than 2 (B) 2 (C) 1 (D) 0

6. 'A' is a 3 × 3 matrix with entries from the set $\{-1,0,1\}$. The probability that 'A' is neither symmetric nor skew symmetric is

(A) $\frac{3^9-3^6-3^3+1}{3^9}$ (B) $\frac{3^9-3^6-3^3}{3^9}$ (C) $\frac{3^9-1}{3^{10}}$ (D) $\frac{3^9-3^3+1}{3^9}$

7. $A=\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and MA = A^{2m}, $m \in N$ for some matrix M, then which one of the following is correct?

(A) $M=\begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

(B) $M=(a^2+b^2)^m\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(C) $M=(a^m+b^m)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $M=(a^2+b^2)^{m-1}\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

8. Let 'A' and 'B' be two given matrices such that AB = A and BA = B, then $A^2 B^2$ equals

(A) A (B) B (C) I (D) 0

MCQ (One or more than one correct) :

9. P is a non-singular matrix and A, B are two matrices such that $B = P^{-1} AP$ then the true statements among the following are

(A) A is invertible iff B is invertible

(B) $B^n = P^{-1} A^n P \forall n \in N$

(C) $\forall \lambda \in R$, $B - \lambda I = P^{-1} (A - \lambda I)P$ (I is the identity matrix)

(D) A,B are both singular matrices

10. If A and B are respectively a symmetric and a skew symmetric matrix such that AB = BA then

(A) $(A - B)^{-1} (A + B)$ is orthogonal matrix when (A – B) is non-singular

(B) $(A + B)^{-1} (A - B)$ is orthogonal matrix when (A + B) is non-singular

(C) $\det [(A - B)^{-1} (A + B)] = 1$ and $\det [(A + B)^{-1} (A - B)] = -1$

(D) $\det [(A - B)^{-1} (A + B)] = -1$ and $\det [(A + B)^{-1} (A - B)] = 1$

11. If $A=\begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix}$ (where $\alpha_2 \neq \beta_1$ and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are non-zero) satisfies the equation $x^2 + k = 0$, then

(A) trace A = 0 (B) $\alpha_1 \beta_2 < 0$ (C) det A = k (D) det A = – k

12. Let A is a n × n matrix in which diagonal elements are 1,2,3,.....,n (i.e., $a_{11} = 1$, $a_{22} = 2$, $a_{33} = 3$, ...$a_{ii} = i$,...$a_{nn} = n$) and all other elements are equal to 'n ' then

(A) A_n is singular for all 'n '

(B) A_n is non-singular for all 'n '

(C) det .A_5 =120

(D) det . $A_n = 0$

13. If the matrix A and B are of 3 × 3 and $(I - AB)^{-1}A$ is invertible, then which of the following statements is/are correct?

(A) $I - BA$ is not invertible

(B) $I - BA$ is invertible

(C) $I - BA$ has for its inverse $I + B(I - AB)^{-1}A$

(D) $I - BA$ has for its inverse $I + A(I - BA)^{-1}B$

14. If A, B are two square matrices of same order such that A + B = AB and I is identity matrix of order same as that of A, B, then

(A) AB = BA (B) $|A - I| = 0$ (C) $|B - I| \neq 0$ (D) $|A - B| = 0$

Comprehension Type Question:

Omprehension # 1

A Pythagorean triple is triplet of positive integers (a, b, c) such that $a^2 + b^2 = c^2$. Define the matrices A, B and C by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

15. If we write Pythagorean triples (a, b, c) in matrix form as [a, b, c] then which of the following matrix product is not a Pythagorean triplet?

(A) [3,4,5]A (B) [3,4,5]B (C) [3,4,5]C (D) None of these

16. Which one of the following does not hold good?

(A) A^{-1} = adj. A (B) $(AB)^{-1}$ = adj. (AB)

(C) $(BC)^{-1}$ = adj. (BC) (D) $(ABC)^{-1} \neq$ adj. (ABC)

17. Tr $(A + B^T + 3C)$ equals

(A) 17 (B) 15 (C) 19 (D) 18

Omprehension # 2

Consider the matrix function

$$A(x) = \begin{bmatrix} \cos^{-1}x & \sin^{-1}x & \operatorname{cosec}^{-1}x \\ \sin^{-1}x & \sec^{-1}x & \tan^{-1}x \\ \operatorname{cosec}^{-1}x & \tan^{-1}x & \cot^{-1}x \end{bmatrix}$$

and $B = A^{-1}$. Also, det.$(A(x))$ denotes the determinant of square matrix A(x).

18. Which of the following statement(s) is(are) correct?

(A) $A(-x) = A(x)$.

(B) $A(x) + A(-x) = \pi I_3$

(C) $A(-x) = -A(x)$.

(D) $A(x) + A(-x) = -\pi I_3$

[**Note :** I_3 denotes an identity matrix of order 3.]

19. Which of the following statement(s) is (are) correct?

(A) A(x) is a symmetric matrix.

(B) A(x) is a skew symmetric matrix.

(C) Maximum value of det.(A(x)) equals $\frac{\pi^3}{8}$

(D) Minimum value of det.(A(x)) equals $\frac{\pi^3}{16}$.

20. Which of the following statement(s) is(are) correct?

(A) det.(A(x)) is a continuous function in its domain but not differentiable in its domain.

(B) det.(A(x)) is a continuous and differentiable function in its domain.

(C) det.(A(x)) is a bounded function.

(D) det.(A(x)) is one-one and odd function.

21. Which of the following statement(s) is(are) correct?

(A) If $a = \det.(B) + \det.(B^2) + \det.(B^3) + ... \infty$, then minimum value of a equals $\frac{8}{\pi^3 - 8}$.

(B) If $b = \det.\,\text{adj}.(B) + \det.\,\text{adj}.(B^2) + \det.\,\text{adj}.(B^3) + ... \infty$, then maximum value of b is $\frac{256}{\pi^6 - 256}$.

(C) If $a = \det.(B) + \det.(B^2) + \det.(B^3) + ... \infty$, then maximum value of a equals $\frac{16}{\pi^3 - 16}$.

(D) If $b = \det.\,\text{adj}.(B) + \det.\,\text{adj}.(B^2) + \det.\,\text{adj}.(B^3) + ... \infty$, then minimum value of b is $\frac{64}{\pi^6 - 64}$.

Numerical based Questions :

22. If $(\alpha\beta)(\delta\beta) = \gamma\gamma\gamma$ such that $\alpha, \beta, \delta, \gamma$ represents a number from 1 to 9 & $\alpha, \beta, \delta, \gamma$ are all different digits & $\alpha\beta, \delta\beta$ are two digit numbers & $\gamma\gamma\gamma$ is a three digit number , $\alpha > \delta$, and the trace of the matrix

$$A = \begin{bmatrix} \alpha & 1 & 2 & 0 \\ 0 & \beta & 1 & 1 \\ 0 & 0 & \gamma & 3 \\ 1 & 1 & 0 & \gamma \end{bmatrix}$$ is a, then $\frac{a}{7}$ is equal to

23. Let A be the set of all 3×3 symmetric matrices all of whose entries either 0 or 1. Five of these entries are 1 and four of them are 0. If n is the number of such matrices, then n/2 is

Matrix Match Type :

24. **Match the following:**

For 3×3 matrix, a_{ij} represents the elements of i^{th} row and j^{th} column.

Column-I		**Column-II**	
A	$a_{ij} + a_{jk} + a_{ki} = 0$, then	p	This represents a symmetric matrix
B	$a_{ij} - a_{jk} - a_{ki} = 0$, then	q	This represents a skew symmetric matrix
C	$(-1)^{i+j} a_{ij} + (-1)^{j+k} a_{jk} + (-1)^{k+i} a_{ki} = 0$, then	r	This represents a matrix whose all diagonal elements are zeros
		s	Value of determinant is zero

Code :

(A) A → (q, r, s); B → (p, q, r, s); C → (q, r, s)

(B) A → (q, s); B → (p, q, r, s); C → (q, r, s)

(C) A → (q, r, s); B → (p, q, s); C → (q, r, s)

(D) A → (q, r, s); B → (p, q, r, s); C → (q, s)

25. **Match the following:**

Column-I		**Column-II**	
A	If $\|A\| = 2$ then $\|2A^{-1}\| =$ (where A is a matrix of order 3)	p	1
B	If $\|A\| = \frac{1}{8}$ then $\|\text{adj}(\text{adj}\, 2A)\| =$ (where A is matrix of order 3)	q	4
C	If $(A + B)^2 = A^2 + B^2$ and $\|A\| = 2$ & $\|B\| =$ (where A and B are matrices of odd order)	r	Does not exist
D	$\|A_{2\times2}\| = 2$, $\|B_{3\times3}\| = 3$ and $\|C_{4\times4}\| = 4$ & $\|ABC\| =$	s	0

Code:

(A) A → q; B → p; C → s; D → r

(B) A → q; B → r; C → p; D → s

(C) A → q; B → r; C → r; D → s

(D) A → q; B → q; C → s; D → r