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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-5

Sequence & Series

SCQ (Single Correct Type) :

1. The sequence a_1, a_2, a_3 ,....... satisfies $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$, n = 1, 2, 3, ... the value of $\frac{2^{2009}}{2011} \cdot a_{2012}$ is

(A) ${}^{2010}C_{1005}$ (B) ${}^{2011}C_{1006}$ (C) ${}^{2011}C_{1005}$ (D) ${}^{2012}C_{1006}$

2. Let $a_1, a_2, a_3, ...a_n$ be in G.P. If the area bounded by the curves $y^2 = 4a_nx$ and $y^2 = 4a_n(a_n - x)$ be A_n, then the sequence $A_1, A_2, A_3, ..., A_n$ are in

(A) A. P. (B) G. P. (C) H. P. (D) None of these

3. The sum of first 'n ' terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ is

(A) 2^{n-1} (B) $1 - 2^{-n}$ (C) $2^{-n} - n + 1$ (D) $2^{-n} + n - 1$

4. Find the value of $\sum_{r=1}^{n}\sum_{s=1}^{n} \delta_{rs} 2^r 3^s$ where $\begin{cases} \delta_{rs} = 0, \text{ if } r \neq S \\ \delta_{rs} = 1, \text{ if } r = S \end{cases}$

(A) $\frac{6}{5}(6^n - 1)$ (B) $6^n - 1$ (C) $\frac{1}{5}(6^n - 1)$ (D) None of these

5. Value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)(r+3)} =$ ___

(A) 1 (B) 1/2 (C) 1/18 (D) 0

6. If α, γ are the roots of $t_1 x^2 - 4x + 1 = 0$ and β, δ are the roots of $t_2x^2 - 6x + 1 = 0$ and $\alpha, \beta, \gamma, \delta$ are in H.P. then

(A) $-t_1 + t_2 = 5$ (B) $t_1 + t_2 = 12$ (C) $t_1 = 8$ (D) $t_2 = 5$

7. If $1^2 + 2^2 + 3^2 + ... + 2003^2 = (2003)(4007)(334)$ and
(1) (2003) + (2) (2002) + (3) (2001) + ... + (2003) (1) = (2003) (334) (x)., then x equals

(A) 2005 (B) 2004 (C) 2003 (D) 2001

MCQ (One or more than one correct) :

8. Consider the sequence a_n given by $a_1 = \frac{1}{2}, a_{n+1} = a_n^2 + a_n$,

Let $S_n = \frac{1}{a_1+1} + \frac{1}{a_2+1} + ... + \frac{1}{a_n+1}$ then find the value of $[S_{2012}]$, where [.] denotes greatest integer function.

(A) 1 (B) [e / 2] (C) [e] (D) $[\pi - 1]$

9. Given that x + y + z = 15 when a, x, y, z, b are in A. P. and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$ when a, x, y, z, b are in H. P. Then

(A) G. M. of a and b is 3 (B) one possible value of (a + 2b) is 11

(C) A. M. of a and b is 5 (D) H. M. of a and b is $\frac{9}{5}$

10. The product of two positive real numbers a and b is 192. The quotient of A.M. by H.M. of their G.C.D and L.C.M is $\frac{169}{48}$. The smaller of a and b can be

(A) 2 (B) 4 (C) 6 (D) 12

11. Let a, b, c are distinct real numbers such that expression $ax^2 + bx + c$, $bx^2 + cx + a$ and $cx^2 + ax + b$ are always positive then possible value(s) of $\frac{a^2+b^2+c^2}{ab+bc+ca}$ may be:

(A) 1 (B) 2 (C) 3 (D) 4

12. For ΔABC, if $81+144a^4+16b^4+9c^4=144abc$, (where notations have their usual meaning), then

(A) a > b > c (B) A < B < C

(C) Area of $\Delta ABC = \frac{3\sqrt{3}}{8}$ (D) Traiangle ABC is right handled

13. Let x, y, z $\in \left(0, \frac{\pi}{2}\right)$ are first three consecutive terms of an arithmetic progression such that cos x + cos y + cos z = 1 and sin x + sin y + sin z = $\frac{1}{\sqrt{2}}$, then which of the following is/are correct?

(A) $\cot y = \sqrt{2}$ (B) $\cos(x-y) = \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$

(C) $\tan 2y = \frac{2\sqrt{2}}{3}$ (D) sin(x− y) + sin(y − z) = 0

14. a_1, a_2 .... are distinct terms of an A.P. We call (p, q, r) an increasing triad if a_p, a_q, a_r are in G.P. where $p,q,r \in \mathbb{N}$ such that $p < q < r$. If (5, 9, 16) is an increasing triad, then which of the following option is/are correct

(A) If a_1 is a multiple of 4 then every term of the A.P. is an integer

(B) (85, 149, 261) is an increasing triad

(C) If the common difference of the A.P. is $\frac{1}{4}$, then its first term is $\frac{1}{3}$

(D) Ratio of the $(4k + 1)^{th}$ and $4k^{th}$ term can be 4

Numerical based Questions :

15. If $\frac{25}{k} = 1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \ldots\ldots\ldots \infty$, then find the value of k

16. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid he dies leaving a third of the debt unpaid. Find the value of the first installment.

17. If x > 0, and $\log_2 x + \log_2\left(\sqrt{x}\right) + \log_2\left(\sqrt[4]{x}\right) + \log_2\left(\sqrt[8]{x}\right) + \log_2\left(\sqrt[16]{x}\right) + \ldots\ldots = 4$, then find x.

18. If $x_i > 0$, i = 1, 2, ..., 50 and $x_1 + x_2 + .. + x_{50} = 50$, then find the minimum value of $\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_{50}}$.

19. The number of terms in an A.P. is even ; the sum of the odd terms is 24, sum of the even terms is 30, and the last term exceeds the first by 10½; find the number of terms.

20. If a, b, c are in GP, a – b, c – a, b – c are in HP, then the value of a + 4b + c is

21. If $S = \frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \ldots$ up to ∞, then find the value of 36S.

22. If $S = \frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots\ldots\ldots\infty$, then find the value of 14S.

Matrix Match Type :

23. **Match Column I with Column II:**

	Column-I		Column-II
A	$\lim_{n\to\infty}\sum_{k-1}^{n}\frac{1}{(k+1)\sqrt{k}+k\sqrt{k+1}}$ is equal to	p	1
B	$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$ is equal to	q	2
C	$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2-\frac{1}{2}}{k^4+\frac{1}{4}}$ is equal to	r	3
D	$x_1=\frac{1}{2}$ and $x_{k+1}=x_k^2+x_{k^-}T=\sum_{k=1}^{n}\frac{1}{x_i+1}$ then [T] is equal to (where [.] denotes G.I.F.)	s	–1
		t	0

Code :

(A) A-p; B-r; C-q; D-s

(B) A-r; B-p; C-s; D-r

(C) A-p; B-q; C-p; D-p

(D) A-r; B-p; C-q; D-r

Subjective Type Questions :

24. In an A.P. of which 'a' is the Ist term, if the sum of the Ist 'p' terms is equal to zero, show that the sum of the next 'q' terms is $-\frac{a\,(p+q)\,q}{p-1}$.

25. The sum of first p-terms of an A.P. is q and the sum of first q terms is p, find the sum of first (p + q) terms.