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# MATHEMATICS

TARGET : JEE- Advanced 2023

# CAPS-4

## Inverse Trigonometric Function & Solution of Triangle

**SCQ (Single Correct Type) :**

**1.** The value of

$\sin^{-1}(\cos 2) - \cos^{-1}(\sin 2) + \tan^{-1}(\cot 4) - \cot^{-1}(\tan 4) + \sec^{-1}(\text{cosec}\,6) - \text{cosec}^{-1}(\sec 6)$ is

(A) 0 (B) $3\pi$ (C) $8 - 3\pi$ (D) $5\pi - 16$

**2.** Let $\alpha = \cot^{-1}\left(\frac{\pi}{3}\right)$, $\beta = \sin^{-1}\left(\frac{\pi}{4}\right)$ and $\gamma = \sec^{-1}\left(\frac{2\pi}{3}\right)$, then the correct order sequence is

(A) $\alpha < \gamma < \beta$ (B) $\beta < \alpha < \gamma$ (C) $\gamma < \beta < \alpha$ (D) $\alpha < \beta < \gamma$

**3.** Let $f(x) = \frac{2}{\pi}\left(\sin^{-1}[x] + \tan^{-1}[x] + \cot^{-1}[x]\right)$ where [x] denotes greatest integer less than or equal to x. If A and B denote the domain and range of f (x) respectively, then the number of integers in $(A \cup B)$, is

(A) 1 (B) 2 (C) 3 (D) 4

**4.** The radii of the escribed circles of $\Delta ABC$ are $r_a$, $r_b$ and $r_c$ respectively.

If $r_a + r_b = 3R$ and $r_b + r_c = 2R$, then the smallest angle of the triangle is

(A) $\tan^{-1}\left(\sqrt{2}-1\right)$ (B) $\frac{1}{2}\tan^{-1}\left(\sqrt{3}\right)$ (C) $\frac{1}{2}\tan^{-1}\left(\sqrt{2}+1\right)$ (D) $\tan^{-1}\left(2-\sqrt{3}\right)$

**5.** If $\cos^{-1}\frac{x}{a} - \sin^{-1}\frac{y}{b} = \theta$ (a, b $\neq$ 0), then the maximum value of $b^2x^2 + a^2y^2 + 2ab\,xy\sin\theta$ equals

(A) ab (B) $(a + b)^2$ (C) $2(a + b)^2$ (D) $a^2b^2$

**6.** In $\Delta ABC$, the bisector of the angle A meets the side BC at D and the circumscribed circle at E, then DE equals

(A) $\frac{a^2\sec\frac{A}{2}}{2(b+c)}$ (B) $\frac{a^2\sin\frac{A}{2}}{2(b+c)}$ (C) $\frac{a^2\cos\frac{A}{2}}{2(b+c)}$ (D) $\frac{a^2\,\text{cosec}\frac{A}{2}}{2(b+c)}$

**7.** If $\left[\cos^{-1}x\right]+\left[\cot^{-1}x\right]=0$, where [.] denotes the greatest integer function, then the complete set of values of *x* is

(A) (cos 1, 1] (B) (cos 1, cot 1) (C) (cot 1, 1] (D) [0, cot 1)

**8.** f : R→(0,π / 2] and f(x) = $\cot^{-1}(x^2 - 2ax + a + 1)$ is sujective than a∈

(A) $\left\{\frac{1-\sqrt{5}}{2},\frac{1+\sqrt{5}}{2}\right\}$ (B) $\left\{\frac{1-\sqrt{5}}{2},\frac{1-\sqrt{5}}{2}\right\}$

(C) $\left(-\infty,\frac{1-\sqrt{5}}{2}\right)\cup\left(\frac{1+\sqrt{5}}{2},\infty\right)$ (D) None of these

**9.** In a ΔABC, if $b^2 + c^2 = 1999a^2$, then $\frac{\cot B+\cot C}{\cot A}$ is equal to____.

(A) $\frac{1}{999}$ (B) $\frac{1}{1999}$ (C) 999 (D) 1999

**10.** If 2a cos B = c in a ΔABC with sides a, b, c then $\tan\frac{A}{2}\left(\tan\frac{A}{2}+2\tan\frac{C}{2}\right)$ is equal to_____.

(A) 1 (B) 21 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

**11.** In ΔABC, if BC = 1, $\sin\frac{A}{2}=x_1$, $\sin\frac{B}{2}=x_2$, $\cos\frac{A}{2}=x_3$, and $\cos\frac{B}{2}=x_4$ with

$\left(\frac{x_1}{x_2}\right)^{2010}-\left(\frac{x_3}{x_4}\right)^{2009}=0$, then length of AC is equal to_____.

(A) > 1 (B) < 1 (C) $\frac{1}{2}$ (D) 1

**MCQ (One or more than one correct) :**

**12.** Let f : R → R defined by f (x) = $\cos^{-1}(-\{-x\})$

where {x} is fractional part function. Then which of the following is/are correct?

(A) f is many one but not even function. (B) Range of f contains two prime numbers.

(C) f is aperiodic. (D) Graph of f does not lie below x–axis.

**13.** Which of the following statement(s) is/are **TRUE**?

(A) Domain of $y = \cos^{-1}(e^x)$ is same as range of $y = -\sqrt{-x}$.

(B) Number of elements common in the range of function $y = \tan^{-1}(\text{sgn } x)$ and $y = \cot^{-1}(\text{sgn } x)$ is only 1 (where sgn x denotes signum function of x.)

(C) The function $y = \text{sgn}(\cot^{-1}x)$ and y = 1 are identical functions.

(D) Number of integers in the solution set of $1 < \log_2(\tan^{-1}x) < 2$ is 4.

**14.** Let a function $f : A \to B$ be defined as $f(x) = \sin^{-1}(\tan x) - \text{cosec}^{-1}(\cot x)$.

Which of the following statement(s) is/are **TRUE** for the function $f(x)$?

(A) $f(x)$ is periodic with fundamental period $\pi$.

(B) The function $f(x)$ is non-invertible.

(C) The composite function $f(f(x))$ is not defined.

(D) The function $f(x)$ is an even function.

**15.** Which of the following expression(s) have their value equal to four times the area of the triangle ABC? (All symbols used have their usual meaning in a triangle)

(A) $rs + r_1(s - a) + r_2(s - b) + r_3(s - c)$

(B) $\dfrac{(a+b+c)^2}{\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}}$

(C) $(a^2 + b^2 - c^2)\tan B$

(D) $b^2 \sin 2C + c^2 \sin 2B$

**16.** Following the usual notations, in a triangle ABC, if $\left(\sqrt{3}-1\right)$ a = 2b and A = 3B, then C cannot be equal to______.

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{6}$

**17.** Let ABC be a triangle with $\angle BAC = 120°$ and AB.AC = 1. Also, let AD be the length of the angle bisector of angle A of the triangle. Then _____.

(A) Minimum value of AD is $\frac{1}{2}$

(B) Maximum value of AD is $\frac{1}{2}$

(C) AD is minimum when $\Delta$ABC is isosceles

(D) AD is maximum when $\Delta$ABC is isosceles

**Comprehension Type Question:**

Let $\alpha, \beta$ and $\gamma$ are the roots of the equation $x^3 + 6x + 3 = 0$

and $A = \cos^{-1}\left(\sin\left((\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}\right)\right)$

$$B = \cos\left(\tan^{-1}\left(\sin\left(\frac{\alpha+\beta+\gamma}{2}\right)\right)\right)$$

$C = \sec^{-1}\left(\text{cosec}\left((1-\alpha)(1-\beta)(1-\gamma)\right)\right)$.

**18.** If the range of the quadratic trinomial $g(x) = x^2 - 2Bx + k$ is $[0, \infty)$, then range of k equals

(A) $[1, \infty)$ (B) $(1, \infty)$ (C) $\{1\}$ (D) $(-\infty, 1]$

**19.** The value of $(5A + B - C)$ is equal to

(A) 1 (B) 10 (C) 5 (D) 0

**20.** Range of the function $f(x) = \dfrac{(5A - C)x^5 + 6Bx^2}{x^4 + (B-1)x^3 + 1}$, is

(A) $[3, \infty)$ (B) $[0, 3]$ (C) $[-3, 3]$ (D) $(-\infty, \infty)$

**Paragraph for question nos. 21 to 23**

Let $f : A \to B$ be an onto function defined as $f(x) = \dfrac{\sin^{-1} x + \tan^{-1} x}{\cos^{-1} x + \cot^{-1} x}$.

**21.** If the minimum and maximum value of $f(x)$ be $m$ and $M$ respectively then the value of $\dfrac{M}{m}$ is

(A) – 1 (B) – 4 (C) – 7 (D) Infinite

**22.** Let $g : B \to A$ be a function such that $g(f(x)) = x \ \forall x \in A$ and $f(g(x)) = x \ \forall x \in B$, then the value of g(3) is

(A) – 1 (B) 0 (C) 0.5 (D) 1

**23.** The number of solutions of the equation $f(x^3 + 14x^2 + 13x - 5) = f(1 - x^2 + x^3)$ is

(A) 0 (B) 1 (C) 2 (D) 3

**Numerical based Questions :**

**24.** If $\sum_{r=1}^{10} \tan^{-1}\left(\dfrac{3}{9r^2 + 3r - 1}\right) = \cot^{-1}\left(\dfrac{m}{n}\right)$ (where m and n are coprime), then find (2m + n).

**25.** Perpendiculars are drawn from the angles A, B, C of an acute-angled triangle on opposite sides and produced to meet the circumscribing circle. If these produced parts are $\alpha$, $\beta$, $\gamma$ respectively, then the value of $\dfrac{(a/\alpha) + (b/\beta) + (c/\gamma)}{\tan A + \tan B + \tan C}$ is _____.

**Matrix Match Type :**

**26.** An function is defined on R or subset of R. Match the function given in column-I with their properties given in column-II. (Where [x] denotes the greatest integer less than or equal to x.)

| **Column-I** | | **Column-II** | |
|---|---|---|---|
| (A) | $\sec^{-1}x$ | (P) | aperiodic |
| (B) | $\sqrt{\lvert x \rvert}$ | (Q) | into |
| (C) | $\cos^{-1}(\cos x)$ | (R) | odd or even |
| | | (S) | neither odd nor even |

**27.** Match the following :

| **Column-I** | | **Column-II** | |
|---|---|---|---|
| A | The number of solutions to the equation $\frac{\pi}{2}\left(\left\lvert \sin^{-1}(\sin x)\right\rvert\right) = \left\lvert x^2 - \pi\lvert x\rvert\right\rvert$ are | p | 4 |
| B | The least value of the expression $2\log_{10} x - \log_x 0.01$ for x > 1 is | q | 2 |
| C | The number of common terms in the series 1, 4, 7, 10, 13 .....91 and 4, 8, 16, 32 ..... 1024 are | r | 5 |
| D | Let A be a real number, and let $A = \frac{\sqrt{\lvert x \rvert - 2} + \sqrt{2 - \lvert x \rvert}}{\lvert 2 - x \rvert}$ then the unit digit of $[A]^{2013}$ is (where [.] denote greatest integer function) | s | 3 |
| | | t | 0 |

**Code :**

(A) A–p; B–r; C–q; D–s

(B) A–r; B–p; C–s; D–t

(C) A–s; B–q; C–p; D–r

(D) A–r; B–p; C–q; D–r

**28.** **Match the folloiwng :**

| | Column-I | | Column-II |
|---|---|---|---|
| A | If p2 – 2 p cos x = 673 and tan $\frac{x}{2} = 7$, the integral value of p is | p | 8 |
| B | If sin θ + cos θ = m then the maximum value of $m^2$ is | q | 9 |
| C | $r_1$, $r_2$, $r_3$ are the radii of the circle drawn on the altitudes PD, PE and PF of ΔPBC, ΔPCA, ΔPAB respectively as diameter where P is the circumcentre of the acute angled ΔABC. The minimum value of $\frac{1}{18}\left[\frac{a^2}{r_1^2}+\frac{b^2}{r_2^2}+\frac{c^2}{r_3^2}\right]$ is (a, b, c are sides of ΔABC) | r | 2 |
| D | In ΔABC, a = 6, b = 3 and $\cos(A-B)=\frac{4}{5}$ then the area of the ΔABC is | s | 25 |

**Code :**

(A) A→ (s); B→ (r); C → (p); D → (q)

(B) A→ (s); B→ (p); C → (p); D → (q)

(C) A→ (r); B→ (r); C → (p); D → (q)

(D) A→ (s); B→ (q); C → (p); D → (r)