competishun
Power of real gurus

MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-3

Function

Subjective Type Questions :

1. Find fundamental period of function $f(x) = 4\cos^4\left(\frac{x-\pi}{4\pi^2}\right) - 2\cos\left(\frac{x-\pi}{2\pi^2}\right)$

2. Consider the function $f(x) = \frac{3x+a}{x^2+3}$ which has the greatest value $\frac{3}{2}$.

Find the minimum value of $f(x)$.

3. Let f be one-one function with domain {x, y, z} and range {1, 2, 3}. It is given that exactly one of the following statements is true and the remaining two are false.

$f(x) = 1$; $f(y) \neq 1$; $f(z) \neq 2$. Determine $f^{-1}(1)$

4 . For what integral value of n, is 3π period of the function $\cos(nx)\sin\left(\frac{5x}{n}\right)$?

5. Find the domain of definition of the function $f(x) = \frac{1}{[|x-1|]+[|7-x|]-6}$, where [.] denotes the greatest integer function

SCQ (Single Correct Type) :

6. Consider the function $f(x) : A \to A$ which satisfy the condition $f(f(x)) = x$, then number of such functions for A = {1, 2, 3, 4, 5} are

(A) 54 (B) 52 (C) 41 (D) 26

7. Let $f(x) = \begin{cases} 2x+a, & x \geq -1 \\ bx^2+3, & x < -1 \end{cases}$ and $g(x) = \begin{cases} x+4, & 0 \leq x \leq 8 \\ -3x-2, & -2 < x < 0 \end{cases}$ g(f(x)) is not defined if

(A) $a \in (10, \infty)$, $b \in (5, \infty)$ (B) $a \in (4,10)$, $b \in (5, \infty)$

(C) $a \in (10, \infty)$, $b \in (0,1)$ (D) $a \in (4,10)$, $b \in (1, 5)$

8. For $x \neq 0, 1$ define $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{1}{1-x}$, $f_5(x) = \frac{x-1}{x}$.

Let h be a function such that $f_3 o h o f_2 = f_4$ then h is equal to

(A) f_5 (B) f_4 (C) f_3 (D) f_2

9. Let $f(x) = \max.\{\sin t : 0 \le t \le x\}$
$g(x) = \min.\{\sin t : 0 \le t \le x\}$
and $h(x) = [f(x) - g(x)]$

where [ ] denotes greatest integer function, then the range of h(x) is

(A) {0, 1} (B) {1, 2} (C) {0, 1, 2} (D) {–3, –2, –1, 0, 1, 2, 3}

MCQ (One or more than one correct) :

10. Let f : X → Y where $3^{(f(x))^2} + 2^{-x} = 4$ and f(x) is one-one and onto function and f(x) ≥ 0 if

(A) $X = (0,\infty), Y = \left(1, \sqrt{\log_3 4}\right)$ (B) $X = \left[-\log_2 3, \infty\right), Y = [0,\infty)$

(C) $X = (0,\infty), Y = \left(0, \sqrt{\log_3 4}\right]$ (D) $X = \left[-\log_2 3, \infty\right), Y = \left[0, \sqrt{\log_3 4}\right)$

11. Let f : R → R be a function defined by $f(x+1) = \frac{f(x)-5}{f(x)-3} \forall x \in R$. Then which of the following statement(s) is/are true_____.

(A) f (x + 2) = f (x) (B) f (x + 4) = f (x) (C) f (x + 6) = f (x) (D) f (x + 8) = f (x)

12. Let $f(x) = a_1 \cos(\alpha_1 + x) + a_2 \cos(\alpha_2 + x) + ...+ a_n \cos(\alpha_n + x)$. If f(x) vanishes for x = 0 and $x = x_1$ (where $x_1 \ne k\pi$, $k \in Z$), then_____.

(A) $a_1 \cos \alpha_1 + a_2 \cos \alpha_2 + ...+a_n \cos \alpha_n = 0$

(B) $a_1 \sin \alpha_1 + a_2 \cos \alpha_2 + ...+a_n \sin \alpha_n = 0$

(C) f(x) = 0 has only two solutions 0, x_1

(D) f(x) is identically zero $\forall x$

13. Which of the following function(s) have no domain?

(A) $f(x) = \log_{x-1}(2 - [x] - [x]^2)$ where [x] denotes the greatest integer function.

(B) $g(x) = \cos^{-1}(2-\{x\})$ where {x} denotes the fractional part function.

(C) h(x) = *l*n *l*n(cosx)

(D) $f(x) = \frac{1}{\sec^{-1}\left(\text{sgn}\left(e^{-x}\right)\right)}$

14. The values of x in $[-2\pi, 2\pi]$, for which the graph of the function $y = \sqrt{\frac{1+\sin x}{1-\sin x}} - \sec x$ and $y = -\sqrt{\frac{1-\sin x}{1+\sin x}} + \sec x$, coincide are

(A) $\left[-2\pi, -\frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ (B) $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $[-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$

15. Let $f(x) = [x]^2 + [x + 1] - 3$, where [x] denotes greatest integer less than or equal to x, then which of the following statement(s) is/are **CORRECT**?

(A) f(x) is many one function.

(B) f(x) vanishes for atleast three values of x.

(C) f(x) is neither even nor odd function.

(D) f(x) is aperiodic.

16. Let $f : I \to R$, defined as $f(x) = 5\cos 4\pi x - 13\sin 7\pi x + 2$. Then which of the following alternative(s) is/are **TRUE**?

(A) Range of f is a singleton set.

(B) f is an even function.

(C) $f\big(f(x)\big) = f(x)\ \forall\ x \in I$

(D) Inverse function of f is non existent.

17. Which of the following pair(s) of function have same graphs?

(A) $f(x) = \dfrac{\sec x}{\cos x} - \dfrac{\tan x}{\cot x}$, $g(x) = \dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\operatorname{cosec} x}$

(B) $f(x) = \operatorname{sgn}(x^2 - 4x + 5)$, $g(x) = \operatorname{sgn}\left(\cos^2 x + \sin^2\left(x + \dfrac{\pi}{3}\right)\right)$ where sgn denotes signum function.

(C) $f(x) = e^{\ln(x^2+3x+3)}$, $g(x) = x^2 + 3x + 3$

(D) $f(x) = \dfrac{\sin x}{\sec x} + \dfrac{\cos x}{\operatorname{cosec} x}$, $g(x) = \dfrac{2\cos^2 x}{\cot x}$

Numerical based Questions :

18. N is the set of all natural number and R is the set of all real numbers. A function $f : N \to R$ is given by:

$f(n) = \dfrac{4n + \sqrt{4n^2 - 1}}{\sqrt{2n-1} + \sqrt{2n+1}}$, then the value of $\left[\dfrac{\sum_{r=1}^{40} f(r)}{40}\right]$ is equal to _____.

(where [·] represents greatest integer function)

19. If $f(x) = \begin{cases} -x + 1, & x \le 0 \\ -(x-1)^2, & x \ge 1 \end{cases}$, then the number of solutions of the equation $f(x) - f^{-1}(x) = 0$ is /are_____.

20. f (x) and g (x) are linear function such that for all x, $f(g(x))$ and $g(f(x))$ are Identity functions. If f (0) = 4 and g (5) = 17, compute f (136).

21. Let f (x) = $\left|x^2-9\right|-\left|x-a\right|$. Find the number of integers in the range of a so that f (x) = 0 has 4 distinct real root.

Comprehension Type Question:

Paragraph for question nos. 22 to 23

Let f (x) = $x^2 - 2x - 1 \ \forall \ x \in R$. Let f : $(-\infty, a] \to [b, \infty)$, where 'a' is the largest real number for which f (x) is bijective.

22. The value of (a + b) is equal to

(A) – 2 (B) – 1 (C) 0 (D) 1

23. Let f : $R \to R$, then range of values of k for which equation f (| x |) = k has 4 distinct real roots is

(A) (– 2, – 1) (B) (– 2, 0) (C) (– 1, 0) (D) (0, 1)

Paragraph for question nos. 24 to 26

An even periodic function f : $R \to R$ with period 4 is such that

$$f(x) = \begin{cases} \max.(|x|, x^2) & ; 0 \le x < 1 \\ x & ; 1 \le x \le 2 \end{cases}$$

24. The value of {f(x)} at x = 5.12 (where { } represents fractional part), is

(A) {f (7.88) } (B) {f (3.26) } (C) { f (2.12) } (D) { f (5.88) }

25. The equation of circle with centre lies on the curve f(x) at x = 9 and touches x-axis, is

(A) $x^2 + y^2 - 14x - 2y + 49 = 0$ (B) $x^2 + y^2 - 18x - 4y + 84 = 0$

(C) $x^2 + y^2 - 18x - 2y + 81 = 0$ (D) $x^2 + y^2 - 18x + 2y + 81 = 0$

26. If g(x) = |3sin x|, then the number of solutions of f(x) = g(x) for $x \in (-6, 6)$, are

(A) 5 (B) 7 (C) 3 (D) 9

Matrix Match Type :

27. Match the following :

Column-I		Column-II	
A	Let f : $[-1, \infty) \to (0, \infty)$ defined by f (x) = $e^{x^2+\|x\|}$ then f (x), is	P	one-one
B	Let f : $(1, \infty) \to [3, \infty)$ defined by f (x) = $\sqrt{10-2x+x^2}$, then f (x) is	Q	into
C	Let f : $R \to I$ defined by f (x) = $\tan^5\pi[x^2 + 2x + 3]$ where [ ] denotes greatest integer function, then f (x) is	R	many one
D	Let f : $[3, 4] \to [4, 6]$ defined by f (x) = $\|x-1\|+\|x-2\|+\|x-3\|+\|x-4\|$ then f (x)	S	onto
		T	periodic

28. Match the following :

Column-I		Column-II	
A	Let f : $R^+ \to \{-1,0, 1\}$ defined by f (x) = sgn $(x - x^4 + x^7 - x^8 - 1)$ where sgn denotes signum function, then f(x) is	P	Into
B	Let f : $R \to R$ and satisfies f (x) + x f (–x) = x + 1, then f (x) is	Q	One-one
C	Let f : $[0, 4] \to [0, 9]$ defined by f (x) = $\sqrt{6x-x^2}$, then f (x) is	R	Many-one
D	Let f : $[0, 3] \to [2, 8]$ defined by f (x) = $2^{\|x-1\|+\|x-2\|}$, then f (x) is	S	Even
		T	Onto