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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-25

Conic Sections

SCQ (Single Correct Type) :

1. If α, β are eccentric angles of the extremities of a focal chord of an ellipse, then eccentricity of the ellipse is

(A) $\frac{\cos\alpha+\cos\beta}{\cos(\alpha+\beta)}$ (B) $\frac{\sin\alpha-\sin\beta}{\sin(\alpha-\beta)}$ (C) sec α + sec β (D) $\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$

2. A series of concentric ellipses E_1, E_2, ..., E_n are drawn such that E_n touches the extremities of the major axis of E_{n-1} and the foci of E_n coincide with the extremities of minor axis of E_{n-1}. If the eccentricity of the ellipses is independent of n, then the value of the eccentricity, is

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) $\frac{1}{\sqrt{5}}$

3. The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2 : 1, the equation of the hyperbola is :

(A) $4x^2 - 5y^2 = 4a^2$ (B) $4x^2 - 5y^2 = 5a^2$ (C) $5x^2 - 4y^2 = 4a^2$ (D) $5x^2 - 4y^2 = 5a^2$

4. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB (O is the origin) is an equilateral triangle, then the eccentricity 'e' of the hyperbola satisfies

(A) e > $\sqrt{3}$ (B) $1 < e < 2\frac{2}{\sqrt{3}}$ (C) e = $\frac{2}{\sqrt{3}}$ (D) e > $\frac{2}{\sqrt{3}}$

5. An ellipse whose major axis is parallel to X –axis such that the segments of the focal chords are 1 and 3 units. The lines ax + by + c = 0 are the chords of the ellipse such that a,b,c are in A.P. and bisected by the point at which they intersect. The equation of its auxiliary circle is $x^2 + y^2 + 2\alpha x + 2\beta y - 2\alpha - 1 = 0$ then _____.

Equation of the auxiliary circle is

(A) $x^2+y^2-2x+4y+1=0$ (B) $x^2+y^2+2x+2y-3=0$

(C) $x^2+y^2+2x+4y+1=0$ (D) $x^2+y^2-4x+2y-3=0$

MCQ (One or more than one correct) :

6. A point moves such that the sum of the squares of its distances from the two sides of length 'a' of a rectangle is twice the sum of the squares of its distances from the other two sides of length 'b'. The locus of the point can be :

(A) a circle (B) an ellipse (C) a hyperbola (D) a pair of lines

7. Which of the following equations in parametric form can represent a hyperbolic profile, where 't' is a parameter.

(A) $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ & $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$ (B) $\frac{tx}{a} - \frac{y}{b} + t = 0$ & $\frac{x}{a} + \frac{ty}{b} - 1 = 0$

(C) $x = e^t + e^{-t}$ & $y = e^t - e^{-t}$ (D) $x^2 - 6 = 2\cos t$ & $y^2 + 2 = 4\cos^2\frac{t}{2}$

8. Let C_1 : $9x^2 - 16y^2 - 18x + 32y - 23 = 0$ and C_2 : $25x^2 + 9y^2 - 50x - 18y + 33 = 0$ are two conics then

(A) eccentricity of C_1 is $\frac{5}{4}$. (B) eccentricity of C_2 is $\frac{5}{3}$.

(C) area of the quadrilateral with vertices at the foci of the conics is $\frac{8}{9}$.

(D) latus rectum of C_1 is greater than latus rectum of C_2.

9. A hyperbola centred at C has one focus at P(6, 8) . If its directrices are $3x + 4y + 10 = 0$ and $3x + 4y - 10 = 0$, then _____.

(A) CP =10 (B) eccentricity = $\sqrt{5}$

(C) CP = 8 (D) eccentricity = $\frac{\sqrt{5}}{2}$

10. Two ellipses $\frac{x^2}{\cos^2\alpha} + \frac{y^2}{\sin^2\alpha} = 1$ and $\frac{x^2}{\sin^2\alpha} + \frac{y^2}{\cos^2\alpha} = 1\left(0 < \alpha < \frac{\pi}{4}\right)$ intersect at four points P,Q, R, S then which of the following statement(s) is /are true?

(A) PQRS is a square with length of the side $\sin 2\alpha$

(B) PQRS lie on a circle whose centre is origin and with radius $\frac{\sin 2\alpha}{\sqrt{2}}$

(C) eccentricity of the two given ellipses are same

(D) there are two points on $\frac{x^2}{\sin^2\alpha} + \frac{y^2}{\cos^2\alpha} = 1$ whose reflection in y = x lie on the same ellipse

Comprehension Type Question:

Comprehension # 1

Paragraph for question nos. 11 to 13

The graph of the conic $x^2 - (y - 1)^2 = 1$ has one tangent line with positive slope that passes through the origin. the point of tangency being (a, b). Then

11. The value of $\sin^{-1}\left(\frac{a}{b}\right)$ is

(A) $\frac{5\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

12. Length of the latus rectum of the conic is

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) none

13. Eccentricity of the conic is

(A) $\frac{4}{3}$ (B) $\sqrt{3}$ (C) 2 (D) none

Numerical based Questions :

14. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D, then the area of the quadrilateral ABCD is $\lambda\sqrt{2}$ the λ is equal to

15. If two points P & Q on the hyperbola $x^2/a^2 - y^2/b^2 = 1$ whose centre is C be such that CP is perpendicular to CQ & a < b, then $\frac{1}{CP^2} + \frac{1}{CQ^2} = \lambda\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$ where λ is :

16. If the distance between the centres of the hyperbolas :

$x^2 - 16xy - 11y^2 - 12x + 6y + 21 = 0$ ..... (i)

$9x^2 - 16y^2 - 18x - 32y - 151 = 0$ ..... (ii)

is d then $125\ d^2$ = ...

17. If $(a^2, a - 2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points (2, 2) and (8, – 4), then the number of all possible integral values of a is :

18. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 1 = 0$, then maximum value of ab is _____.

19. If the length of the latus rectum of a standard hyperbola of eccentricity 2 is equal to the limit of the series $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + ...$ and r is the radius of the director circle of its conjugate hyperbola, then r^2 = _____.

20. Let $C_1 : x^2 - y^2 = 5$ and $C_2 : x^2 + y^2 - 8y + 3 = 0$ be the equations of a hyperbola and a circle respectively. The curves C_1 and C_2 touch each other at $(\pm 3, 2)$. P$(0, 4 - \sqrt{13})$ is a point on the curve C_2 . Let a line through P meet C_1 at m number of points and C_2 at n number of points. If (m+ n) = 3, then the number of such straight lines is_____.

21. Coordinates of the vertices B and C are (2, 0) and (8, 0) respectively. The vertex A is varying in such a way that $4\tan\frac{B}{2}\tan\frac{C}{2} = 1$. If the locus of A is an ellipse then the length of its semi major axis is _____.

Matrix Match Type :

22. Match the following:

For the ellipse $3x-6^{2}+3y-9^{2}=\frac{4}{169}5x+12y+6^{2}$

Column-I	Column-II
(a) Length major axis	(p) $\frac{24}{5}$
(b) Length minor axi	(q) $\frac{16}{5}$
(c) Length of Latus Rectum	(r) $\frac{16}{3}$
(d) Distance between directrices	(s) $\frac{72}{5}$
	(t) $\frac{48}{5}$

(A) a → r; b → q; c → r; d → s (B) a → t; b → q; c → r; d → s

(C) a → t; b → s; c → r; d → s (D) a → t; b → q; c → q; d → s

Subjective Type Questions :

23. If S and H be the focii of an ellipse and any point A be taken on the curve and the chords ASB, BHC, CSD and DHE be drawn and eccentric angles of A, B, C, D, E, ... be θ_1, θ_2, q_3, q_4, ... Prove that $\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}=\cot\frac{\theta_2}{2}\cot\frac{\theta_3}{2}=\tan\frac{\theta_3}{2}\tan\frac{\theta_4}{2}=.....$

24. A variable point P on an ellipse of eccentricity e is joined to joined to its foci S, S'. Prove that the locus of the incentre of $\Delta PSS'$ is an ellipse of eccentricity $\sqrt{\frac{2e}{1+e}}$.

25. Let P and Q be two points on the ellipse $x^2+4y^2=16$ whose eccentric angles are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ respectively. The tangent at P and the normal at Q cut each other at R and the normal at Q cuts the ellipse again at M. Find the area of the triangle PRM.

26. A rectangular hyperbola, with centre C, is intersected by a circle of radius r in four points p, Q, R and S. Prove that $CP^2+CQ^2+CR^2+CS^2=4r^2$