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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-24

PARABOLA

SCQ (Single Correct Type) :

1. PQ is a chord of parabola $x^2 = 4y$ which subtends right angle at vertex. Then locus of centroid of triangle PSQ, where S is the focus of given parabola, is

(A) $x^2 = 4(y + 3)$ (B) $x^2 = \frac{4}{3}(y - 3)$

(C) $x^2 = \frac{-4}{3}(y + 3)$ (D) $x^2 = \frac{-4}{3}(y + 3)$

2. PN is an ordinate of the parabola $y^2 = 4ax$ (P on $y^2 = 4ax$ and N on x-axis). A straight line is drawn parallel to the axis to bisect NP and meets the curve in Q. NQ meets the tangent at the vertex A in apoint T such that AT = kNP, then the value of k is (where A is the vertex)

(A) 3/2 (B) 2/3 (C) 1 (D) 1/3

3. Locus of the feet of the perpendiculars drawn from vertex of the parabola $y^2 = 4ax$ upon all such chords of the parabola which subtend a right angle at the vertex is

(A) $x^2 + y^2 - 4ax = 0$ (B) $x^2 + y^2 - 2ax = 0$

(C) $x^2 + y^2 + 2ax = 0$ (D) $x^2 + y^2 + 4ax = 0$

4. If the normal to a parabola $y^2 = 4ax$ at P meets the curve again in Q and if PQ and the normal at Q makes angles α and β respectively with the x-axis then $\tan\alpha(\tan\alpha + \tan\beta)$ has the value equal to

(A) 0 (B) – 2 (C) $-\frac{1}{2}$ (D) – 1

5. From a variable point P on the tangent at the vertex of a parabola $y^2 = 2x$, a line is drawn perpendicular at chord of contact. These variable lines always passes through a fixed point, where co-ordinates are

(A) $\left(\frac{1}{2}, 0\right)$ (B) (1, 0) (C) $\left(\frac{3}{2}, 0\right)$ (D) (2, 0)

6. If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet passes through a fixed point whose co-ordinates are :

(A) (– 2a, 0) (B) (a, 0) (C) (2a, 0) (D) none

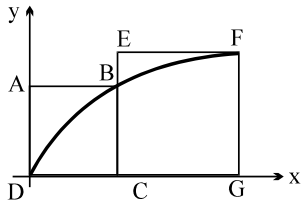
7. ABCD and EFGC are squares and the curve $y = k\sqrt{x}$ passes through the origin D and the points B and F. The ratio $\frac{FG}{BC}$ is

(A) $\frac{\sqrt{5}+1}{2}$

(B) $\frac{\sqrt{3}+1}{2}$

(C) $\frac{\sqrt{5}+1}{4}$

(D) $\frac{\sqrt{3}+1}{4}$



MCQ (One or more than one correct) :

8. P is a point on the parabola $y^2 = 4ax$ $(a > 0)$ whose vertex is A. PA is produced to meet the directrix in D and M is the foot of the perpendicular from P on the directrix. If a circle is described on MD as a diameter then it intersects the x–axis at a point whose co–ordinates are :

(A) $(-3a, 0)$ (B) $(-a, 0)$ (C) $(-2a, 0)$ (D) $(a, 0)$

9. The focus of the parabola is (1, 1) and the tangent at the vertex has the equation $x + y = 1$. Then

(A) equation of the parabola is $(x - y)^2 = 2(x + y - 1)$

(B) equation of the parabola is $(x - y)^2 = 4(x + y - 1)$

(C) the co-ordinates of the vertex are $\left(\frac{1}{2}, \frac{1}{2}\right)$

(D) length of the latus rectum is $2\sqrt{2}$

10. Consider the parabola whose equation is $y = x^2 - 4x$ and the line $y = 2x - b$. Then which of the following is/are correct?

(A) For $b = 9$ the line is a tangent to the parabola.

(B) For $b = 7$ the line cuts the parabola in A and B such that the $\angle AOB$ is a right angle when 'O' is the origin.

(C) For some $b \in R$ the line cuts the parabola in C and D such that x-axis bisects the $\angle COD$.

(D) For $b > 9$ the line passes outside the parabola.

11. The straight line $y + x = 1$ touches the parabola

(A) $x^2 + 4y = 0$ (B) $x^2 - x + y = 0$ (C) $4x^2 - 3x + y = 0$ (D) $x^2 - 2x + 2y = 0$

12. If the two parabolas $y^2 = 4x$ and $y^2 = (x - k)$ have a common normal other than the x-axis then k can be equal to

(A) 1 (B) 2 (C) 3 (D) 4

13. PQ is a double ordinate of the parabola $y^2 = 4ax$. If the normal at P intersect the line passing through Q and parallel to axis of x at G, then locus of G is a parabola with

(A) length of latus rectum equal to 4a. (B) vertex at (4a, 0).

(C) directrix as the line $x - 3a = 0$ (D) focus as (5a, 0)

Comprehension Type Question:

Comprehension

A variable circle passes through the point A (2, 1) and touches the x-axis. Locus of the other end of the diameter through A is a parabola.

14. Find the length of the latus rectum of the parabola.

(A) 2 (B) 3 (C) 4 (D) 5

15. Find the coordinates of the foot of the directrix of the parabola.

(A) (2, – 1) (B) (1, – 2) (C) (–2, 1) (D) (–2, – 1)

16. The two tangents and two normals at the extremities of the latus rectum of the parabola constitutes a quadrilateral. Find area of quadrilateral.

(A) 3 sq. unit (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units

Numerical based Questions :

17. Let P (a, b) and Q (c, d) are the two points on the parabola $y^2 = 8x$ such that the normals at them meet in (18, 12). Find the product (abcd).

18. Normals are drawn from the point P with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1m_2 = \alpha$ is a part of the parabola itself, then find α.

19. Three nromals are drawn from the point (14, 7) to the curve $y^2 - 16x - 8y = 0$. Find the coordinates of the feet of the normals.

20. The normal chord at a point t on the parabola $y^2 = 4ax$ subtends a right angle at its vertex. Find the value of t^2.

21. From the origin, tangents OA and OB are drawn to the curve $(x-2)^2 + (y-2)^2 = 1$. If the line PQ, where P and Q are respectively the midpoints of OA and OB, touches the curve $(y+3)^2 = 4\alpha(x+4)$ and the length of latus rectum of the parabola is ℓ, then $\frac{\ell}{7}$ is _____.

22. If two distinct chords of a parabola $y^2 = 4ax$ passing through (a,2a) are bisected by the line $x + y = 1$, and 4a is a natural number, then the maximum length of the latus–rectum is _____.

Matrix Match Type :

23. Identify the conic whose equations are given in column-I.

	Column-I (Equation of a conic)		**Column-II (Nature of conic)**
(A)	$xy + a^2 = a(x + y)$	(P)	Ellipse
(B)	$2x^2 – 72xy + 23y^2 – 4x – 28y – 48 = 0$	(Q)	Hyperbola
(C)	$6x^2 – 5xy – 6y^2 + 14x + 5y + 4 = 0$	(R)	Parabola.
(D)	$14x^2 – 4xy + 11y^2 – 44x – 58y + 71 = 0$	(S)	line pair
(E)	$4x^2 – 4xy + y^2 – 12x + 6y + 8 = 0$		

24. Consider the parabola $y^2 = 12x$

Column-I	**Column-II**
(A) Tangent and normal at the extremities of the latus rectum intersect the x axis at T and G respectively. The coordinates of the middle point of T and G are	(P) (3, 0)
(B) Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the squares of the reciprocals of the two parts of the chords through K, is a constant. The coordinate of the point K are	(Q) (6, 0) (R) (12, 0)
(C) All variable chords of the parabola subtending a right angle at the origin are concurrent at the point	