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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-23

CIRCLE

SCQ (Single Correct Type) :

1. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line $y = x + 1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is :

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(0, \frac{1}{2\sqrt{2}}\right)$ (C) $\left(0, \frac{2-\sqrt{2}}{4}\right)$ (D) none of these

2. A circle of constant radius 'r' passes through origin O and cuts the axes of coordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is :

(A) $(x^2 + y^2)(x^{-2} + y^{-2}) = 4r^2$ (B) $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = r^2$

(C) $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$ (D) $(x^2 + y^2)(x^{-2} + y^{-2}) = r^2$

3. A pair of tangents are drawn from a point P to the circle $x^2 + y^2 = 1$. If the tangents make an intercept of 2 units on the line $x = 1$, then the locus of P is _____.

(A) a straight line (B) a pair of lines (C) a parabola (D) a hyperbola

4. A circle with centre at the origin and radius equal to a meets the X axis at the points A(–a, 0) and B(a, 0) . P(α) and Q(β) are two points on this circle so that $\alpha - \beta = 2\gamma$, where γ is a constant. The locus of the point of intersection of AP and BQ is _____.

(A) $x^2 - y^2 - 2ay\tan\gamma = a^2$ (B) $x^2 + y^2 - 2ay\tan\gamma = a^2$

(C) $x^2 + y^2 + 2ay\tan\gamma = a^2$ (D) $x^2 - y^2 + 2ay\tan\gamma = a^2$

5. Let the lines $y - 2 = m_1 (x - 5)$ and $(y + 4) = m_2 (x - 3)$ intersect at right angles at a point P, where m_1 and m_2 are parameters. If the locus of P is $x^2 + y^2 + gx + fy + 7 = 0$, then the value of $(f - g)$ equals _____.

(A) 1 (B) 2 (C) 8 (D) 10

6. The circle, which passes through the points of intersection of the circles $x^2 + y^2 - 4x - 6y + 12 = 0$ and $x^2 + y^2 - 8x + 12y + 50 = 0$, and also passes through the origin, is_____.

(A) $19x^2 + 19y^2 - 52x - 222y = 0$ (B) $19 (x^2 + y^2) - 2 (34x + 111y) = 0$

(C) $19 (x^2 + y^2) - 117x + 26y = 0$ (D) such circle does not exist

7. Let P(α, β) be a point in the first quadrant. Circles are drawn through P touching the coordinate axes.

The relation between α and β , for which two circles are orthogonal, is _____.

(A) $\alpha^2 + \beta^2 = 4\alpha\beta$ (B) $\alpha + \beta^{\ 2} = 4\alpha\beta$

(C) $\alpha^2 + \beta^2 = \alpha\beta$ (D) $\alpha^2 + \beta^2 = 2\alpha\beta$

8. The equation of circum-circle of a ΔABC is $x^2 + y^2 + 3x + y - 6 = 0$. If A = (1,−2), B = (−3,2) and the vertex C varies then the locus of ortho-centre of ΔABC is a

(A) Straight line (B) Circle (C) Parabola (D) Ellipse

9. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$which subtends an angle of 90º at the point (2,3) then the locus of the midpoint of AB is a circle whose centre is

(A) (1, 5) (B) $\left(1,\frac{3}{2}\right)$ (C) $\left(1,\frac{5}{2}\right)$ (D) $\left(2,\frac{5}{2}\right)$

10. P and Q are two points on a line passing through (2, 4) and having slope m. If a line segment AB subtends a right angle at P and Q where A ≡ (0, 0) and B ≡ (6, 0), then range of m is

(A) $\left(\frac{2-3\sqrt{2}}{4},\frac{2+3\sqrt{2}}{4}\right)$ (B) $\left(-\infty,\frac{2-3\sqrt{2}}{4}\right)\cup\left(\frac{2+3\sqrt{2}}{4},\infty\right)$

(C) (–4, 4) (D) $-\infty,-4 \cup 4,\infty$

MCQ (One or more than one correct) :

11. If $a\ell^2 - bm^2 + 2d\ell + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$, then the line $\ell x + my + 1 = 0$ touches a fixed circle :

(A) which cuts the x–axis orthogonally

(B) with radius equal to b

(C) on which the length of the tangent from the origin is $\sqrt{d^2 - b}$

(D) none of these.

12. Let A, B, C, D lie on a line such that AB = BC = CD =1. The points A and C are also joined by a semicircle with AC as diameter and P is a variable point on this semicircle such that $\angle PBD=\theta$, $0 \le \theta \le \pi$. Let R is the region bounded by are AP, the straight line PD and line AD.

(A) The maximum possible area of region R is $\frac{2\pi+3\sqrt{3}}{6}$

(B) If 'L' is the perimeter of region 'R', then L is equal to $3+\pi-\theta+\sqrt{5-4\cos\theta}$

(C) The maximum possible area of region R is $\frac{2\pi-3\sqrt{3}}{6}$

(D) If 'L' is the perimeter of region 'R', then L is equal to $3+\pi-\theta+\sqrt{5+4\cos\theta}$

Numerical based Questions :

13. The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms and the new equation $x^2 + y^2 = \frac{\lambda}{4}$, then find the value of λ.

14. A line meets the co–ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively and diameter of the circle is $\lambda_1 d_1 + \lambda_2 d_2$, then find the value of $\lambda_1 + \lambda_2$.

15. Find the number of integral points which lie on or inside the circle $x^2 + y^2 = 4$.

16. Find number of values of 'c' for which the set,

$\{(x, y) \mid x^2 + y^2 + 2x \le 1\} \cap \{(x, y) \mid x - y + c \ge 0\}$ contains only one point is common.

17. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles and the area of the rhombus is $a\sqrt{3}$ sq. units, then find the value of a.

18. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B (1, 7) & D (4, – 2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

19. If a tangent of slope $\frac{1}{2}$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 2 = 0$, then the maximum value of ab is _____.

Subjective Type Questions:

20. Find the equation of the circle passing through the points A(4, 3), B(2, 5) and touching the axis of y. Also find the point P on the y–axis such that the angle APB has largest magnitude.

21. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$. Find the equations of the circles.

22. The centre of the circle S = 0 lies on the line $2x - 2y + 9 = 0$ and S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points and also find their co–ordinates.

23. The lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6 unit. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts of intercepts of length 8 on these lines.

24. Prove that the two circles which pass through the points (0, a), (0, – a) and touch the straight line $y = mx + c$ will cut orthogonaly if $c^2 = a^2 (2 + m^2)$.

25. Show that if one of the circle $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2g_1x + c = 0$ lies within the other, then gg_1 and c are both positive.