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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-22

Straight Line

SCQ (Single Correct Type) :

1. Let A(4, −1), B and C be the vertices of a triangle. Let the internal angular bisectors of angles B and C be x – 1 = 0 and x – y –1= 0 respectively. Let D, E and F be the points of contact of the sides BC, CA and AB respectively with the incircle of triangle ABC. The slope of BC is _____.

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) 12

2. Let A(4, −1) , B and C be the vertices of a triangle. Let the internal angular bisectors of angles B and C be x – 1 = 0 and x – y –1= 0 respectively. Let D, E and F be the points of contact of the sides BC, CA and AB respectively with the incircle of triangle ABC. If D′, E′, F′ are the images of D, E, F in the internal angular bisectors of angles A, B, C respectively, then the equation of the circumcircle of $\Delta D'E'F'$ is _____.

(A) $(x-1)^2+y^2=5$ (B) $x^2+(y-1)^2=25$

(C) $(x-1)^2+(y-1)^2=5$ (D) $x^2+y^2=25$

3. ABC is a triangle right angled at A with vertices A,B,C in the anti-clockwise sense in that order. A= (1,2), B = (−3,1) and vertex C lies on the X – axis. BCEF is a square with vertices B,C,E,F in the clockwise sense in that order. ACD is an equilateral triangle with vertices A,C,D in the anti-clockwise sense in that order. The abscissa of centroid of ΔBCE is

(A) –1 (B) $\frac{-1}{2}$ (C) $\frac{-1}{3}$ (D) $\frac{-2}{3}$

4. Statement 1: Consider the point A(0,1) and B(2,0) and 'P' be a point on the line 4x+3y+9 = 0, then coordinates of 'P' such that |PA− PB| is maximum is $\left(\frac{-12}{5},\frac{17}{5}\right)$

Statement 2: $|PA-PB| \le |AB|$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

5. The vertices of a triangle are $\left(1,\sqrt{3}\right)$, $\left(2\cos\theta,2\sin\theta\right)$ and $\left(2\sin\theta,-2\cos\theta\right)$ where $\theta\in R$. The locus of orthocentre of the triangle is :

(A) $\left(x-1\right)^2+\left(y-\sqrt{3}\right)^2=4$ (B) $\left(x-2\right)^2+\left(y-\sqrt{3}\right)^2=4$

(C) $\left(x-1\right)^2+\left(y-\sqrt{3}\right)^2=8$ (D) $\left(x-2\right)^2+\left(y-\sqrt{3}\right)^2=8$

6. The true set of real values of a such that the point M(a, sina) lies inside the triangle formed by the lines $x-2y+2=0,\ x+y=0$ and $x-y-\pi=0$, is

(A) $\left(0,\pi\right)$ (B) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (C) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{2},\pi\right)\cup\left(\frac{2\pi}{3},2\pi\right)$-

7. The equations $3x+2y+1=0,\ 2x+4y-1=0$ and $3x^2+4xy+4y^2+2x-2y+1+\alpha=0$ will have a unique solution if α equals

(A) $\frac{2}{3}$ (B) $\frac{4}{5}$ (C) $\frac{3}{8}$ (D) $\frac{3}{4}$

8. If the points (– 2, 0), $\left(-1,\frac{1}{\sqrt{3}}\right)$ and (cos θ, sin θ) are collinear, then the number of values of θ when $0\le\theta\le\frac{\pi}{2}$.

(A) 0 (B) 1 (C) 2 (D) Infinite

MCQ (One or more than one correct) :

9. Consider the three linear equations, ax + by + c = 0, bx + cy + a = 0, cx + ay + b = 0, where a, b, $c\in\mathbb{R}$. Which of the following is (are) correct?

(A) If a + b + c = 0 and $a^2+b^2+c^2=ab+bc+ca$, then the lines represent the entire XY plane.

(B) If a + b + c = 0 and $a^2+b^2+c^2\neq ab+bc+ca$, then the lines are concurrent.

(C) If $a+b+c\neq0$ and $a^2+b^2+c^2=ab+bc+ca$, then the lines are coincident.

(D) If $a+b+c\neq0$ and $a^2+b^2+c^2\neq ab+bc+ca$, then the lines are neither coincident nor concurrent.

10. The triangle formed by the lines x + y = 0, 3x + y −4 = 0 and x +3 y −4 = 0 is

(A) isosceles (B) scalene (C) acute angled (D) obtuse angled

11. Consider the straight lines L_1 : x + y = 2, L_2 : 2 x − y + 3 = 0 and a variable point $P(a,a^2)$ where $a\in R$. 'P' lies in the acute angle not containing the origin if 'a ' lies in the interval

(A) $\left(-4,-3\right)$ (B) $\left(\frac{3}{2},\frac{5}{2}\right)$ (C) $\left(-2,\frac{-3}{2}\right)$ (D) (5, 7)

12. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then :

(A) the lines will pass through a fixed point (B) there will be a set of parallel lines

(C) all the lines intersect the line $x = x_1$ (D) all the lines will be parallel to the line $y = x_1$.

13. If $a^2 + 9b^2 - 4c^2 = 6ab$ then the family of lines $ax + by + c = 0$ are concurrent at :

(A) (1/2, 3/2) (B) (– 1/2, – 3/2) (C) (– 1/2, 3/2) (D) (1/2, – 3/2)

Comprehension Type Question:

Comprehension # 1

Paragraph for question nos. 14 to 16

Let ABCD is a square with sides of unit length. Points E and F are taken on sides AB and AD respectively so that AE = AF. Let P be a point inside the square ABCD.

14. The maximum possible area of quadrilateral CDFE is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

15. The value of $(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2$ is equal to

(A) 3 (B) 2 (C) 1 (D) 0

16. Let a line passing through point A divides the square ABCD into two parts so that area of one portion is double the other, then the length of portion of line inside the square is

(A) $\frac{\sqrt{10}}{3}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{\sqrt{11}}{3}$ (D) $\frac{2}{\sqrt{3}}$

Numerical based Questions :

17. The equation $9x^3 + 9x^2y - 45x^2 = 4y^3 + 4xy^2 - 20y^2$ represents 3 straight lines, two of which pass through the origin. Find the area of the triangle formed by these lines (in sq. units).

18. If the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a, b, c and $a \neq 1$, $b \neq 1$ and $c \neq 1$, then find the value of abc– (ab + bc + ac) + 3 (a + b + c).

19. If the straight lines joining the origin and the points of intersection of the curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis, then find the value of $|k|$.

20. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so find λ.

Matrix Match Type :

21. Let ABC be a triangle such that the coordinates of A are (– 3, 1). Equation of the median through B is 2x + y – 3 = 0 and equation of the angular bisector of C is 7x – 4y – 1 = 0. Then match the entries of column-I with their corresponding correct entries of column-II.

	Column-I		**Column-II**
(A)	Equation of the line AB is	(P)	2x + y – 3 = 0
(B)	Equation of the line BC is	(Q)	2x – 3y + 9 = 0
(C)	Equation of CA is	(R)	4x + 7y + 5 = 0
		(S)	18x – y – 49 = 0

SUBJECTIVE:

22. Two vertices of a triangle are (4, –3) and (–2, 5). If the orthocentre of the triangle is at (1, 2), then find the third vertex.

23. A pair of perpendicular straight lines is drawn through the origin and forming with the line 2x + 3y = 6 an isosceles Δ right angled at the origin. Find the equation of the pair of straight lines and area of the Δ.

24. A straight line passing through O (0, 0) cuts the lines $x = \alpha$, $y = \beta$ and x + y = 8 at A, B and C respectively such that $OA \cdot OB \cdot OC = 48\sqrt{2}$ and $f(\alpha, \beta) \le 0$

where $f(x, y) = \left|\frac{y}{x} - \frac{3}{2}\right| + (3x - 2y)^6 + \sqrt{ex + 2y - 2e - 6}$.

(i) Find the point of intersection of lines $x = \alpha$ and $y = \beta$.

(ii) Find the value of (OA + OB + OC).

(iii) Find the equation of line OA.

25. The vertices of a triangle OBC are O(0,0) B(–3,–1) and C(–1,–3). Find the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$.