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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-21

3D

SCQ (Single Correct Type) :

1. For the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$, which one of the following is incorrect?

(A) it lies in the plane x – 2y + z = 0

(B) it is same as line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(C) it passes through (2, 3, 5)

(D) it is parallel to the plane x – 2y + z – 6 = 0

2. Given planes

$P_1 : cy + bz = x$

$P_2 : az + cx = y$

$P_3 : bx + ay = z$

P_1, P_2 and P_3 pass through one line, if

(A) $a^2 + b^2 + c^2 = ab + bc + ca$

(B) $a^2 + b^2 + c^2 + 2abc = 1$

(C) $a^2 + b^2 + c^2 = 1$

(D) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + 2abc = 1$

3. The equation of the line passing through A(1, 0, 3), intersecting the line $\frac{x}{2}=\frac{x-1}{3}=\frac{z-2}{1}$ and which is parallel to the plane x + y + z = 2 is _____.

(A) $\frac{3x-1}{2}=\frac{2y-3}{3}=\frac{2z-5}{-1}$

(B) $\frac{x-1}{2}=\frac{y-0}{3}=\frac{z-3}{-1}$

(C) $\frac{x-1}{2/3}=\frac{y-1}{3/2}=\frac{z-3}{-1/2}$

(D) $\frac{3x-1}{2}=\frac{2y-3}{-3}=\frac{6z-13}{5}$

4. L_1 and L_2 are two lines whose vector equations are given below.

$$L_1 : \bar{r} = \lambda\left(\left(\cos\theta+\sqrt{3}\right)\hat{i}+\left(\sqrt{2}\sin\theta\right)\hat{j}+\left(\cos\theta-\sqrt{3}\right)\hat{k}\right)$$

$$L_1 : \bar{r} = \mu\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$$

Here, λ and μ are scalars. If the angle α is the acute angle between the two lines and is independent of θ, then a possible value of α is _____.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

5. Let $P_1 \equiv \bar{r}\cdot\bar{r}_1 = d$, $P_2 \equiv \bar{r}\cdot\bar{r}_2 = d_2$, $P_3 \equiv \bar{r}\cdot\bar{r}_3 = d_3$ be three planes where $\bar{r}_1, \bar{r}_2, \bar{r}_3$ are three non-coplanar vectors. Then the lines $P_1 = 0 = P_2$, $P_2 = 0 = P_3$ and $P_3 = 0 = P_1$ are ______.

(A) parallel lines (b) coplanar lines (c) co-incident lines (d) concurrent lines

6. Let P(x,y,1) and Q(x,y,z) be points on the curves $\frac{x^2}{9}+\frac{y^2}{4}=4$ and $\frac{x+2}{1}+\frac{\sqrt{3}-y}{\sqrt{3}}=\frac{z-1}{2}$ respectively. Then, the minimum distance between P and Q is ______.

(A) $\sqrt{2}$ (B) $\sqrt{\frac{7}{2}}$ (C) 2 (D) none of these

7. A line L_1 with direction ratios –3, 2, 4 passes through the point A(7, 6, 2) and a line L_2 with direction ratios 2, 1, 3 passes through the point B(5, 3, 4). A line L_3 with direction ratios 2, –2, –1 intersects L_1 and L_3 at C and D.

The equation of the plane parallel to the line L_1 and containing the line L_2 is equal to _____.

(A) $x + 3y + 4z = 30$ (B) $x + 2y + z = 15$

(C) $2x - y + z = 11$ (D) $2x + 17y - 7z = 33$

MCQ (One or more than one correct) :

8. Let $\bar{a}=2\hat{i}-\hat{j}+\hat{k},\ \bar{b}=\hat{i}+2\hat{j}-\hat{k},\ \bar{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\bar{b}$ and $\bar{c}$ whose projection on $\bar{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is _____.

(A) $2\hat{i}+3\hat{j}-3\hat{k}$ (B) $2\hat{i}+3\hat{j}+3\hat{k}$ (C) $-2\hat{i}-\hat{j}+5\hat{k}$ (D) $2\hat{i}+\hat{j}+5\hat{k}$

9. Let DABC be a tetrahedron such that AD is perpendicular to the base ABC and $\angle ABC = 30^\circ$. The volume of the tetrahedron is 18 cubic units. If the value of AB + BC + AD is minimum, then the length of AC is _____.

(A) $6\sqrt{2-\sqrt{3}}$ (B) $3\left(\sqrt{6}-\sqrt{2}\right)$ (C) $6\sqrt{2+\sqrt{3}}$ (D) $3\left(\sqrt{6}+\sqrt{2}\right)$

10. If the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ intersects line $3\beta^2x+3(1-2\alpha)y+z=3=-\frac{1}{2}\left(6\alpha^2x+3(1-2\beta)y+2z\right)$, then the point $(\alpha,\beta,1)$ lies on the plane ______.

(A) $2x - y + z = 4$ (b) $x + y - z = 2$ (c) $x - 2y = 0$ (d) $2x - y = 0$

11. Consider the planes $P_1 : 2x + y + z + 4 = 0$, $P_2 : y - z + 4 = 0$ and $P_3 : 3x + 2y + z + 8 = 0$. Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2 respectively. Then,

(A) Atleast two of the lines L_1, L_2 and L_3 are non–parallel

(B) Atleast two of the lines L_1, L_2 and L_3 are parallel

(C) The three planes intersect in a line

(D) The three planes form a triangular prism

Comprehension Type Question:

Comprehension # 1

Consider a plane

$x + y - z = 1$ and the point A(1, 2, –3)

A line L has the equation

$x = 1 + 3r$

$y = 2 - r$

$z = 3 + 4r$

12. The co-ordinate of a point B of line L, such that AB is parallel to the plane, is

(A) 10, –1, 15 (B) –5, 4, –5 (C) 4, 1, 7 (D) –8, 5, –9

13. Equation of the plane containing the line L and the point A has the equation

(A) $x - 3y + 5 = 0$ (B) $x + 3y - 7 = 0$ (C) $3x - y - 1 = 0$ (D) $3x + y - 5 = 0$

Comprehension # 2

A ray of light emanating from the point source $P\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and travelling parallel to the line $\frac{x-2}{1} = \frac{y}{3} = \frac{z+1}{2}$ is incident on the plane $x + 3y - 3z = 0$ at the point Q. After reflection from the plane the ray travels along the line QR. It is also known that the incident ray, reflected ray and the normal to the plane at the point of incident are in the same plane.

14. The position vector of Q is _____.

(A) $3\hat{i} + 15\hat{j} + 6\hat{k}$ (B) $3\hat{i} + 6\hat{j} + 3\hat{k}$ (C) $-3\hat{i} - 6\hat{j} - 3\hat{k}$ (D) $-3\hat{i} - 15\hat{j} - 6\hat{k}$

15. The vector equation of line containing QR is _____.

(A) $\bar{r} = \left(12\hat{i} + 22\hat{j} + 4\hat{k}\right) + \lambda\left(15\hat{i} + 37\hat{j} + 10\hat{k}\right)$

(B) $\bar{r} = \left(3\hat{i} + 15\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$

(C) $\bar{r} = \left(3\hat{i} + 6\hat{j} + 3\hat{k}\right) + \lambda\left(15\hat{i} + 37\hat{j} + 10\hat{k}\right)$

(D) $\bar{r} = \left(\hat{i} + \hat{j} + \hat{k}\right) + \lambda\left(-\hat{i} - \hat{j} - \hat{k}\right)$

16. The equation of the plane in Cartesian form is _____.

(A) $5x + 2y - z + 3 = 0$ (B) $11x - 5y + 2z = 30$

(C) $5x - y - z = 6$ (D) $x - y + z = 6$

Numerical based Questions :

17. The lengths of two opposite edges of a tetrahedron are 3 and 4 units, the shortest distance between them is equal to 6 unit and angle between them is 30 . Then the volume of tetrahedron in cubic units is _____.

18. A line L_1 with direction ratios –3, 2, 4 passes through the point A(7, 6, 2) and a line L_2 with direction ratios 2, 1, 3 passes through the point B(5, 3, 4). A line L_3 with direction ratios 2, –2, –1 intersects L_1 and L_3 at C and D.

The length CD is equal to _____.

19. A line L_1 with direction ratios –3, 2, 4 passes through the point A(7, 6, 2) and a line L_2 with direction ratios 2, 1, 3 passes through the point B(5, 3, 4). A line L_3 with direction ratios 2, –2, –1 intersects L_1 and L_3 at C and D.

The volume of the parallelepiped formed by $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$ is equal to _____.

20. If the length of shortest distance between the two lines $\frac{1}{2}(x-1)=\frac{1}{4}(y-3)=z+2$ and $3x-y-2z+4=0=2x+y+z+1$ is $s\sqrt{14}$, then the value of s is ______.

21. The perpendicular distance of the point (1,−2,3) to plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

22. Let the equations of two straight lines L_1, L_2 be respectively be respectively be $x-5=\frac{y-3}{5}=\frac{z-15}{2}$ and $\frac{x}{2}=\frac{y+1}{5}=\frac{z+6}{3}$. A,B are two distinct points on the x – axis such that two straight lines l_1, l_2 both perpendicular to the x – axis (l_1 through A, l_2 through B) are drawn so as to intersect both L_1, L_2 .

If θ is the acute angle between the lines l_1 , l_2 and $\cos\theta=\frac{\lambda}{5\sqrt{794}}$ = then λ =

23. Let the equations of two straight lines L_1, L_2 be respectively be respectively be $x-5=\frac{y-3}{5}=\frac{z-15}{2}$ and $\frac{x}{2}=\frac{y+1}{5}=\frac{z+6}{3}$. A,B are two distinct points on the x – axis such that two straight lines l_1, l_2 both perpendicular to the x – axis (l_1 through A, l_2 through B) are drawn so as to intersect both L_1, L_2 .

The shortest distance between the lines l_1, l_2 is

24. If the perpendicular distance of a corner of a unit cube from a diagonal not passing through it is d, then the value of $3d^2$ is

Matrix Match Type :

25. Consider the following four pairs of lines in **column-I** and match them with one or more entries in **column-II**.

	Column-I		Column-II
(A)	$L_1 : x = 1 + t,\ y = t,\ z = 2 - 5t$ $L_2 : \vec{r} = (2, 1, -3) + \lambda(2, 2, -10)$	(P)	non coplanar lines
(B)	$L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$ $L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$	(Q)	lines lie in a unique plane
(C)	$L_1 : x = -6t,\ y = 1 + 9t,\ z = -3t$ $L_2 : x = 1 + 2s,\ y = 4 - 3s,\ z = s$	(R)	infinite planes containing both the lines
(D)	$L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ $L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$	(S)	lines do not intersect