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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-20

VECTOR

SCQ (Single Correct Type) :

1. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle of $\cos^{-1}\frac{11}{14}$ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The value of 'x' is:

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 2

2. If the acute angle that the vector, $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ makes with the plane of the two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $\hat{i}-\hat{j}+2\hat{k}$ is $\cot^{-1}\sqrt{2}$ then :

(A) $\alpha(\beta+\gamma)=\beta\gamma$ (B) $\beta(\gamma+\alpha)=\gamma\alpha$

(C) $\gamma(\alpha+\beta)=\alpha\beta$ (D) $\alpha\beta+\beta\gamma+\gamma\alpha=0$

3. If $\bar{p},\bar{q},\bar{r}$ are three mutually perpendicular vectors of the same magnitude and if a vector $\bar{x}$ satisfies the equation $\bar{p}\times\left[(\bar{x}-\bar{q})\times\bar{p}\right]+\bar{q}\times\left[(\bar{x}-\bar{r})\times\bar{q}\right]+\bar{x}\times\left[(\bar{x}-\bar{p})\times\bar{r}\right]=\bar{0}$, then vector $\bar{x}$ is equal to _____.

(A) $\frac{1}{2}(\bar{p}+\bar{q}-2\bar{r})$ (B) $\frac{1}{2}(\bar{p}+\bar{q}+\bar{r})$ (C) $\frac{1}{3}(\bar{p}+\bar{q}+\bar{r})$ (D) $\frac{1}{3}(2\bar{p}+\bar{q}-\bar{r})$

4. Let $\bar{a}$ and $\bar{b}$ be two vectors of equal magnitude of 5 units each. Let p and $\bar{p}$ be $\bar{q}$ be vectors such that $\bar{p}=\bar{a}+\bar{b}$ and $\bar{q}=\bar{a}-\bar{b}$. If $|\bar{p}\times\bar{q}|=2\left\{\lambda-\left(\bar{a}-\bar{b}\right)^2\right\}^{\frac{1}{2}}$, then value of λ is _____.

(A) 25 (B) 125 (C) 625 (D) None of these

5. $|\bar{a}|=|\bar{b}|=|\bar{c}|=|\bar{a}+\bar{b}|=1, \bar{a}\cdot\bar{c}=0$ and $\bar{a}=\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$, where $\bar{a},\hat{k}$ and $\bar{b}$ are linearly dependent vectors. If for some $\bar{c}, \left|\bar{b}\times\bar{c}\right|=\left|\bar{a}\times\bar{c}\right|$ and $\bar{c}=p\hat{i}+q\hat{j}+r\hat{k}$, then $16\left(p^4+q^4+r^4\right)$ is _____.

(A) 8 (B) 21 (C) $\frac{21}{3}$ (D) 4

6. Let $\bar{a},\bar{b}$ and $\bar{c}$ be three unit vectors such that $\left|\bar{a}+\bar{b}+\bar{c}\right|=\sqrt{3}$. If $\left(\bar{a}\times\bar{b}\right)\cdot\left(\bar{b}\times\bar{c}\right)+\left(\bar{b}\times\bar{c}\right)\cdot\left(\bar{c}\times\bar{a}\right)+\left(\bar{c}\times\bar{a}\right)\cdot\left(\bar{a}\times\bar{b}\right)=\lambda$, then the maximum value of λ is

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) 2

7. Let $\bar{r}$ be the position vector of a variable point in the Cartesian OXY plane such that $\bar{r}\cdot\left(10\hat{j}-8\hat{i}-\bar{r}\right)=40$ and $P_1=\max\left\{\left|\bar{r}+2\hat{i}-3\hat{j}\right|^2\right\}$, $P_2=\min\left\{\left|\bar{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. P_1^2 is equal to _____.

(A) 9 (B) $2\sqrt{2}-1$ (C) $6\sqrt{2}+1$ (D) $9-4\sqrt{2}$

8. Let $\bar{r}$ be the position vector of a variable point in the Cartesian OXY plane such that $\bar{r}\cdot\left(10\hat{j}-8\hat{i}-\bar{r}\right)=40$ and $P_1=\max\left\{\left|\bar{r}+2\hat{i}-3\hat{j}\right|^2\right\}$, $P_2=\min\left\{\left|\bar{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. P_1 + P_2 is equal to ____.

(A) 2 (B) 10 (C) 18 (D) 5

9. Given $|\vec{a}|=|\vec{b}|=1$ and $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}$ -is a vector such that $\vec{c}-\vec{a}-2\vec{b}=3\left(\vec{a}\times\vec{b}\right)$, then $\left(\vec{c}\cdot\vec{b}\right)$ is equal to

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

MCQ (One or more than one correct) :

10. If $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$, $\vec{b}=y\hat{i}+z\hat{j}+x\hat{k}$ and $\vec{c}=z\hat{i}+x\hat{j}+y\hat{k}$, then $\vec{a}\times(\vec{b}\times\vec{c})$ is

(A) parallel to $(y-z)\ \hat{i}+(z-x)\ \hat{j}+(x-y)\ \hat{k}$ (B) orthogonal to $\hat{i}+\hat{j}+\hat{k}$

(C) orthogonal to $(y+z)\ \hat{i}+(z+x)\ \hat{j}+(x+y)\ \hat{k}$ (D) orthogonal to $x\hat{i}+y\ \hat{j}+z\hat{k}$

11. The value(s) of $\alpha\in[0, 2\pi]$ for which vector $\vec{a}=\hat{i}+3\hat{j}+(\sin2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors $\vec{b}=(\tan\alpha)\hat{i}-\hat{j}+2\sqrt{\sin\frac{\alpha}{2}}\hat{k}$ and $\vec{c}=(\tan\alpha)\ \hat{i}+(\tan\alpha)\ \hat{j}-3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\hat{k}$ are orthogonal, is/are:

(A) $\tan^{-1}3$ (B) $\pi-\tan^{-1}2$ (C) $\pi+\tan^{-1}3$ (D) $2\pi-\tan^{-1}2$

12. Let $\bar{a}$ and $\bar{c}$ be unit vectors such that $\bar{a}\times\bar{b}=2\bar{a}\times\bar{c}$, where $|\bar{b}|=4$. The angle between $\bar{a}$ and $\bar{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If $\bar{b}-2\bar{c}=k\bar{a}$, then k is equal to _____.

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) –4 (D) 3

13. If $\left(\vec{a}\times b\right)\times\left(\vec{c}\times\vec{d}\right)\cdot\left(\vec{a}\times\vec{d}\right)=0$, then which of the following may be true?

(A) $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are necessarily coplanar (B) $\vec{a}$ lies in the plane of $\vec{c}$ and $\vec{d}$

(C) $\vec{b}$ lies in the plane of $\vec{c}$ and $\vec{d}$ (D) $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{d}$

Numerical based Questions :

14. If $\vec{d}$ is a unit vector and $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, then the value of $\left|\frac{\left(\bar{a}\cdot\bar{d}\right)\left(\bar{b}\times\bar{c}\right)+\left(\bar{b}\cdot\bar{d}\right)\left(\bar{c}\times\bar{a}\right)+\left(\bar{c}\cdot\bar{d}\right)\left(\bar{a}\times\bar{b}\right)}{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}\right|$ is equal to ______.

15. Let $\left(\hat{p}\times\bar{q}\right)\times\hat{p}+\left(\hat{p}\cdot\bar{q}\right)\bar{q}=\left(x^2+y^2\right)\bar{q}+(14-4x-6y)\hat{p}$, where $\hat{p}$ and $\bar{q}$ are two non-collinear vectors (and $\hat{p}$ is a unit vector) and x, y are scalars. Find the value of (x+y).

16. Let $\bar{r} = (\bar{a}\times\bar{b})\sin x + (\bar{b}\times\bar{c})\cos y + (\bar{c}\times\bar{a})$, where $\bar{a}, \bar{b}$ and $\bar{c}$ are non-zero non-coplanar vectors. If $\bar{r}$ is orthogonal to $3\bar{a}+5\bar{b}+2\bar{c}$, then the value of $\sec^2 y + \operatorname{cosec}^2 x + \sec y \operatorname{cosec} x$ is _____.

17. Given four non zero vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$. The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and $(\vec{a}\,\hat{}\,\vec{b}) = (\vec{b}\,\hat{}\,\vec{c}) = \frac{\pi}{3}$, $(\vec{d}\,\hat{}\,\vec{a}) = \alpha$ and $(\vec{d}\,\hat{}\,\vec{b}) = \beta$, if $(\vec{d}\,\hat{}\,\vec{c}) = \cos^{-1}(m\cos\beta + n\cos\alpha)$ then m – n is :

18. Given $f^2(x) + g^2(x) + h^2(x) \le 9$ and U(x) = 3f(x) + 4g(x) + 10h(x), where f(x), g(x) and h(x) are continuous $\forall x\in R$. If maximum value of U(x) is $\sqrt{N}$. Then the value of cube root of (N–1000) is:

19. In an equilateral ΔABC find the value of $\frac{|\overrightarrow{PA}|^2 + |\overrightarrow{PB}|^2 + |\overrightarrow{PC}|^2}{R^2}$ where P is any arbitrary point lying on its circumcircle, is

20. If represents the position vector of point R in which the line AB cuts the plane CDE, where position vectors of points A, B, C, D, E are respectively $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}+\hat{j}+2\hat{k}$, $\vec{c} = -4\hat{j}+4\hat{k}$, $\vec{d} = 2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{e} = 4\hat{i}+\hat{j}+2\hat{k}$, then $\vec{r}^2$ is :

21. Line L_1 is parallel to vector $\vec{\alpha} = -3\hat{i}+2\hat{j}+4\hat{k}$ and passes through a point A(7, 6, 2) and line L_2 is parallel to a vector $\vec{\beta} = 2\hat{i}+\hat{j}+3\hat{k}$ and passes through a point B(5, 3, 4). Now a line L_3 parallel to a vector $\vec{r} = 2\hat{i}-2\hat{j}-\hat{k}$ intersects the lines L_1 and L_2 at points C and D respectively, then $|4\overrightarrow{CD}|$ is equal to :

Matrix Match Type :

22. Match the following.

Column – I	Column – II
(A) If D, E and F are the mid points of the sides BC, CA and AB respectively of a triangle ABC and λ is a scalar such that $\overline{AD} + \frac{2}{3}\overline{BE} + \frac{1}{3}\overline{CF} - \lambda\overline{AC}$, then λ is equal to	(p) 0
(B) If $\bar{A}, \bar{B}$ and $\bar{C}$ are vectors such that $\lvert\bar{B}\rvert = \lvert\bar{C}\rvert$, then the value of $\left(\left(\left(\bar{A}+\bar{B}\right)\times\left(\bar{A}+\bar{C}\right)\right)\times\left(\bar{B}\times\bar{C}\right)\right)\cdot\left(\bar{B}+\bar{C}\right)$ is	(q) $\frac{1}{3}$
(C) In a ΔABC, points D, E and F are taken on the sides BC, CA and AB respectively such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = 2$. If area($\Delta DEF$) $=\lambda$ area(ΔABC) , then λ is equal to	(r) $\frac{1}{2}$
(D) The corner P of the square OPQR is folded up so that the plane OPQ is perpendicular to the plane OQR. If θ is angle between OP and QR, then $\lvert\cos\theta\rvert$ is equal to	(s) $\frac{3}{5}$
	(t) $\frac{3}{4}$

(A) A–p; B–r; C–q; D–s
(B) A–r; B–p; C–s; D–r
(C) A–s; B–q; C–p; D–r
(D) A–r; B–p; C–q; D–r

Subjective based Questions :

23. Let ABC be a triangle.Points M, N and P are taken on the sides AB, BC and CA respectively such that $\frac{AM}{AB} = \frac{BN}{BC} = \frac{CP}{CA} = \lambda$. Prove that the vectors $\overrightarrow{AN}$, $\overrightarrow{BP}$ and $\overrightarrow{CM}$ form a triangle. Also find λ for which the area of the triangle formed by these vectors is the least.

24. Let P be an interior point of a triangle ABC and AP, BP, CP meet the sides BC, CA, AB is D, E, F respectively. Show that

$$\frac{AP}{PD} = \frac{AF}{FB} + \frac{AE}{EC}$$

25. Let PM be the perpendicular from the point P(1, 2, 3) to the x-y plane. If OP makes an angle θ with the positive direction of the z-axis and OM makes an angle ϕ with the positive direction of the x-axis, where O is the origin, then find θ and ϕ