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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-2

Quadratic Equation

SCQ (Single Correct Type) :

1. If α, β be the roots of $x^2 + x + 2 = 0$ and γ, δ be the roots of $x^2 + 3x + 4 = 0$, then $(\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$ is equal to

(A) – 18 (B) 18 (C) 24 (D) 44

2. The number of value of k for which $(x^2 - (k - 2)x - 2k)(x^2 + kx + 2k - 4)$ is a perfect square is

(A) 1 (B) 2 (C) 2 (D) 0

3. The value of 'a' for which the quadratic expression $ax^2 + |2a - 3|x - 6$ is positive for exactly three integral values of x is

(A) $\left(-\frac{3}{5}, -\frac{1}{2}\right]$ (B) $\left[-\frac{3}{5}, -\frac{1}{2}\right]$ (C) $\left[-\frac{3}{5}, -\frac{1}{6}\right]$ (D) None of these

4. Consider the quadratic equation $ax^2 - bx + c = 0$, a, b, c$\in$N. If the given equation has two distinct real roots belonging to (1,2) then

(A) $1 < a < 5$ (B) $a \ge 5$ (C) $a = 4$ (D) $a = 3$

5. The set of values of 'a ' for which in-equation $(a - 1)x^2 - (a + 1)x + (a - 1) \ge 0$ is true for all $x \ge 2$ is

(A) $\left(\frac{7}{3}, \infty\right)$ (B) $\left(1, \frac{7}{3}\right]$ (C) $(-\infty, 1)$ (D) $(-3, -2)$

6. Let a_m (m = 1, 2, 3, ...p) be the possible integral values of 'a' for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$ meets at some point for all real values of b.

Let $T_r = \prod_{m=1}^{p}(r - a_m)$ and $S_n = \sum_{r=1}^{n} T_r (n \in N)$

sum of all the possible values of 'n' for which T_n vanishes, is ____

(A) 10 (B) 15 (C) 21 (D) 20

7. If p, q, r, s $\in$ R, then equaton $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx - 2q) = 0$ has

(A) 6 real roots (B) at least two real roots

(C) 2 real and 4 imaginary roots (D) 4 real and 2 imaginary roots

MCQ (One or more than one correct) :

8. If $|ax^2 + bx + 1| \le 1$ for all x in [0, 1] then

(A) $|a| \le 8$ (B) $|b| > 8$ (C) $b > 0$ (D) $|a| + |b| \le 16$

9. The set 'S' of all real 'x' for which $(x^2 - x + 1)^{x-1} < 1$ contains

(A) (–5, –1) (B) (–1, 1) (C) (–1, 0) (D) (–3, 1)

10. Let Δ^2 be the discriminant and α, β be the roots of the equation $ax^2 + bx + c = 0$. Then, $2a\alpha + \Delta$ and $2a\beta - \Delta$ can be the roots of the equation

(A) $x^2 + 2bx + b^2 = 0$ (B) $x^2 - 2bx + b^2 = 0$

(C) $x^2 + 2bx - 3b^2 + 16ac = 0$ (D) $x^2 - 2bx - 3b^2 + 16ac = 0$

Comprehension Type Question:

Comprehension # 1

If α, β, γ be the roots of the equation $ax^3 + bx^2 + cx + d = 0$. To obtain the equation whose roots are $f(\alpha)$, $f(\beta)$, $f(\gamma)$, where f is a function, we put $y = f(\alpha)$ and simplify it to obtain $\alpha = g(y)$ (some function of y). Now α is a root of the equation $ax^3 + cx + d = 0$, then we obtain the desired equation which is $a\{g(y)\}^3 + b\{g(y)\}^2 + c\{g(y)\} + d = 0$

For example, if α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$. To find equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ we put $y = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{\gamma}$

As α is a root of $ax^3 + bx^2 + cx + d = 0$

we get $\frac{a}{y^3} + \frac{b}{y^2} + \frac{c}{y} + d = 0 \quad \Rightarrow dy^3 + cy^2 + by + a = 0$

This is desired equation.

On the basis of above information, answer the following questions :

11. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $a(2x + 1)^2 + b(2x + 1)(x - 1) + c(x - 1)^2 = 0$ are-

(A) $\frac{2\alpha+1}{\alpha-1}, \frac{2\beta+1}{\beta-1}$ (B) $\frac{2\alpha-1}{\alpha+1}, \frac{2\beta-1}{\beta+1}$ (C) $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$ (D) $\frac{2\alpha+3}{\alpha-1}, \frac{2\beta+3}{\beta-1}$

12. If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is -

(A) $x^2 + 10x - 11 = 0$ (B) $11x^2 + 10x + 1 = 0$

(C) $x^2 + 10x + 11 = 0$ (D) $11x^2 - 10x + 1 = 0$

Numerical based Questions :

13. If $X^2 - 2P_SX + s = 0$, s = 1, 2, 3 are three equations of which each pair has exactly one root common and no root is common to all three equations, then the number of solutions of the triplet (p_1, p_2, p_3) is

14. If α and β are the roots of the equation $x^2 - a(x + 1) - b = 0$, where $a,b \in R-\{0\}$ and $(a + b) \neq 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} - \frac{2}{a+b}$ is equal to

15. Find product of all real values of x satisfying $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$

16. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Then find the value of $(a - c)(b - c)(a + d)(b + d)/(q^2 - p^2)$.

17. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - Kx^3 + Kx^2 + Lx + M = 0$, where K, L & M are real numbers, then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is – n. Find the value of n.

18. Consider $y = \frac{2x}{1+x^2}$, where x is real , then the range of expression $y^2 + y - 2$ is [a, b].

Find b – 4a.

19. Find the values of a, for which the quadratic expression $ax^2 + (a - 2)x - 2$ is negative for exactly two integral values of x.

20. If α, β are roots of the equation $x^2 - 34x + 1 = 0$, evaluate $\sqrt[4]{\alpha} - \sqrt[4]{\beta}$

21. Find the number of real roots of $\left(x + \frac{1}{x}\right)^3 + \left(x + \frac{1}{x}\right) = 0$

Matrix Match Type :

22. Consider the equation $x^2 + 2(a - 1)x + a + 5 = 0$, where 'a' is parameter. Match of the real values of 'a' so that the given equation has

Column-I		**Column-II**	
A	imaginary roots	p	$\left(-\infty, -\frac{8}{7}\right)$
B	one root smaller than 3 and other root greater than 3	q	(–1, 4)
C	exactly one root in the interval (1, 3) & 1 and 3 are not be root of the equation	r	$\left(-\frac{4}{3}, -\frac{8}{7}\right)$
D	one root smaller than 1 and other root greater than 3	s	$\left(-\infty, -\frac{4}{3}\right)$

Subjective Type Questions :

23. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has non-trivial solution, prove that

$$\frac{b^2}{q^2} = \frac{ac}{pr}$$

24. The equation $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ have a common root and the difference of their other roots is one. Then show that $|ac| = |a^2 - c^2|$. Hence or otherwise find the maximum and minimum value of $\left|\frac{a}{c}\right|$.

25. If the roots of the equation $ax^2 - 2bx + c = 0$ are imaginary, find the number of real roots of $4e^x + (a + c)^2 (x^3 + x) = 4b^2x$.

26. Prove that roots of $a^2x^2 + (b^2 + a^2 - c^2)x + b^2 = 0$ are not real, if $a + b > c$ and $|a - b| < c$. (where a, b, c are positive real numbers)