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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-19

Differential Equation

SCQ (Single Correct Type) :

1. If $\left(1+x^2\right)\frac{dy}{dx}=x(1-y), y(0)=\frac{4}{3}$, then the value of $y\left(\sqrt{8}\right)+\frac{8}{9}$ is :

(A) 4 (B) 2 (C) 3 (D) 5

2. Solution of the differential $y'=\frac{3yx^2}{x^3+2y^4}$ is

(A) $x^3y^{-1}=\frac{2}{3}y^3+c$ (B) $x^2y^{-1}=\frac{2}{3}y^3+c$ (C) $xy^{-1}=\frac{2}{3}y^3+c$ (D) None of these

3. If $ydx-dy=e^{-x}y^4dy$ and y = 1 at x = –ln3, then at y = 3, x = lnk, k is

(A) 21 (B) 24 (C) 27 (D) 81

4. The solution of the differential equation $\left(x\cot y+\ln(\cos x)\right)dy+\left(\ln(\sin y)-y\tan x\right)dx=0$ is _____.

(A) $(\sin x)^y(\cos y)^x=c$ (B) $(\sin y)^x(\cos x)^y=c$

(C) $(\sin x)^y(\cos y)^x=c$ (D) $(\cot x)^y(\cot y)^x=c$

5. If x and y are related, Also $\frac{d^2x}{dy^2}\cdot\left(\frac{dy}{dx}\right)^3+k\frac{d^2y}{dx^2}=0$. Then value of k is _____.

(A) –1 (B) 1 (C) –2 (D) 2

6. Solution of the differential equation $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$ is given by

(A) $2\log|x|-\log|y|-\frac{1}{xy}=C$ (B) $2\log|y|-\log|x|-\frac{1}{xy}=C$

(C) $2\log|x|+\log|y|+\frac{1}{xy}=C$ (D) $2\log|y|+\log|x|+\frac{1}{xy}=C$

7. The solution of the differential equation $\frac{dy}{dx}=\frac{1}{xy\left(x^2\sin y^2+1\right)}$ is, ('c' being an arbitrary constant).

(A) $x^2\left(\cos y^2-\sin y^2-2ce^{-y^2}\right)=2$ (B) $x^2\left(\cos y^2-\sin y^2-2ce^{-y^2}\right)=4c$

(C) $y^2\left(\cos x^2-\sin x^2-2ce^{-y^2}\right)=2$ (D) $y^2\left(\cos x^2-\sin x^2-ce^{-y^2}\right)=4c$

MCQ (One or more than one correct) :

8. The solution of $\left(\frac{dy}{dx}\right)^2 - 2\left(x + \frac{1}{4x}\right)\frac{dy}{dx} + 1 = 0$

(A) $y = x^2 + c$ (B) $y = \frac{1}{2}\ln(x) + c, x > 0$ (C) $y = \frac{x}{2} + c$ (D) $y = \frac{x^2}{2} + c$

9. A function y = f(x) satisfying the differential equation $\frac{dy}{dx}$. sin x – y cos x + $\frac{\sin^2 x}{x^2}$ = 0 is such that, $y \to 0$ as $x \to \infty$ then the statement which is correct is

(A) $\underset{x \to 0}{\text{Limit}}$ f(x) = 1

(B) $\int_0^{\pi/2}$ f(x) dx is less than $\frac{\pi}{2}$

(C) $\int_0^{\pi/2}$ f(x) dx is greater than unity

(D) f(x) is an odd function

10. Let T be the triangle with vertices (0, 0), (0, c^2) and (c, c^2) and let R be the region between y = cx and y = x^2 where c > 0 then

(A) Area (R) = $\frac{c^3}{6}$

(B) Area of R = $\frac{c^3}{3}$

(C) $\lim_{c \to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = 3$

(D) $\lim_{c \to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = \frac{3}{2}$

Comprehension Type Question:

Comprehension # 1

Suppose f(x) and g(x) are differentiable functions such that x g(f(x)). f'(g(x)).g'(x)=f(g(x)).g'(f(x)) f'(x) $\forall$ x $\in$ R and f(x) & g(x) are positive for all x $\in$ R. Also $\int_0^x f(g(t))\,dt = \frac{1}{2}(1 - e^{-2x}) \forall x \in R$, g(f(0)) = 1 and h(x) = $\frac{g(f(x))}{f(g(x))} \forall x \in R$.

11. The graph of y = h(x) is symmetric with respect to the line:

(A) x = –1 (B) x = 0 (C) x = 1 (D) x = 2

12. The value of f(g(0)) + g(f(0)) is equal to:

(A) 1 (B) 2 (C) 3 (D) 4

13. The largest possible value of h(x) $\forall$ x $\in$ R is

(A) 1 (B) $e^{1/3}$ (C) e (D) e^2

Comprehension # 2

Let y = f(x) satisfy the equation $f(x) = (e^{-x} + e^{x})\cos x - 2x - \int_0^x (x-t)f'(t)\,dt$. Then,

14. f(x) satisfies the differential equation

(A) $\frac{dy}{dx} + y = e^x(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$

(B) $\frac{dy}{dx} = y + e^x(\cos x + \sin x) + e^{-x}(\cos x - \sin x)$

(C) $\frac{dy}{dx} + y = e^x(\cos x + \sin x) - e^{-x}(\cos x + \sin x)$

(D) None of these

15. $f'(0) + f''(0) =$

(A) –1 (B) 2 (C) 0 (D) 1

16. f(x) as a function of x equals

(A) $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x + \sin x) + \frac{2}{5}e^{-x}$

(B) $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

(C) $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x - \sin x) + \frac{2}{5}e^{-x}$

(D) None of these

Numerical based Questions :

17. If $ye^y dx = \left(y^3 + 2xe^y\right)dy, y(0) = 1$, then the value of x when y = 1 is

18. If particular solution of $p + \cos px \sin y = \sin px \cos y$ where $\left(p = \frac{dy}{dx}\right)$ is $y = a\sqrt{x^2 - b} - \sin^{-1}\frac{\sqrt{x^2 - c}}{x}$ then evaluate $\left|\frac{b+c}{a}\right|$

19. The solution of $x^2dy - y^2dx + xy^2(x-y)dy = 0$ is $\ln\left|\frac{1}{x} - \frac{1}{y}\right| = \frac{y^2}{k} + C$, then the value of k is ______.

20. Suppose a solution of the differential equation $\left(xy^3 + x^2y^7\right)\frac{dy}{dx} = 1$ satisfies the initial condition $y\left(\frac{1}{4}\right) = 1$. Then the value of $-\frac{5}{4} \times \frac{dy}{dx}$ when y = −1, is _____.

21. Let y = f (x) be a curve C_1 passing through (2,2) and $\left(8, \frac{1}{2}\right)$ and satisfying a differential equation $y\left(\frac{d^2y}{dx^2}\right) = 2\left(\frac{dy}{dx}\right)^2$. Curve C_2 is the director circle of the circle $x^2 + y^2 = 2$. If the shortest distance between the curves C_1 and C_2 is $\left(\sqrt{p} - q\right)$ where p, q $\in$ N, then find the value of $(p^2 - q)$.

Matrix Match Type :

22. Match the following

Column-I	Column-II
(A) Solution of the differential equation $y''+e^{2y}(y')^3=0$ is	(p) $x=\frac{1}{4}e^{2y}+c_1y+c_2$
(B) Solution of the differential equation $2x(x+y)y'=3y^2+4xy$ at y(1) = 1 is	(q) $y^2+2xy=3x^3$
(C) Equation of orthogonal trajectories of the curve y = tanx + k is	(r) $2x+4y+\sin 2x=c$
(D) Solution of the differential equation $y'+4xy+xy^3=0$ is	(s) $y^2=\left(ce^{4x^2}-\frac{1}{4}\right)^{-1}$

(A) A-p; B-q; C-r; D-s
(B) A-s; B-p; C-r; D-q
(C) A-p; B-q; C-s; D-r
(D) A-p; B-s; C-q; D-r

23. **Column I** **Column II**

(A) If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3}-13\frac{dy}{dx}}{y}$ = K then the value of K/3 is — (P) 3

(B) Number of straight lines which satisfy the differential equation $\frac{dy}{dx}+x\left(\frac{dy}{dx}\right)^2-y=0$ is — (Q) 4

(C) If real value of m for which the subtitution, $y = u^m$ will transform the differential equation, $2x^4y\frac{dy}{dx}+y^4=4x^6$ into a homogenous equation then the value of 2m is — (R) 2

(D) If the solution of differential equation $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=12y$ is $y = Ax^m + Bx^{-n}$ then | m + n | can be — (S) 7

(A) A(R); B(Q); C(S); D(P)
(B) A(P); B(S); C(Q); D(R)
(C) A(Q); B(R); C(P); D(S)
(D) A(S); B(P); C(Q); D(R)

Subjective based Questions :

24. Let the curve y = f(x) passes through (4, –2) satisfy the differential equation,

$y(x + y^3)\,dx = x(y^3 - x)\,dy$ & let $y = g(x) = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$,

$0 \le x \le \frac{\pi}{2}$. Then find the area of the region bounded by curves y = f(x), y = g(x) and x = 0.

25. Solve the equation $x\int_0^x y(t)\,dt = (x+1)\int_0^x t\,y(t)\,dt$, x > 0