competishun — Power of real gurus **MATHEMATICS** **TARGET : JEE- Advanced 2023**	**CAPS-18** **Area Under the Curves**

SCQ (Single Correct Type) :

1. Let f (x) = $2\sqrt{x}$ and g(x) = $2\sqrt{1-x}$ be two functions then the area bounded by y = f(x), y = g(x) and x axis is

(A) $\frac{1}{3\sqrt{2}}$ sq unit (B) $\frac{2}{3\sqrt{2}}$ sq unit (C) $\frac{1}{\sqrt{2}}$ sq unit (D) $\frac{4}{3\sqrt{2}}$ sq unit

2. The area bounded by the lines y = 2, x = 1, x = a and the curve y = f(x), which cuts the last two lines above the first line for all $a \geq 1$, is equal to $\frac{2}{3}((2a))^{3/2} - 3a + 3 - 2\sqrt{2}$ then f(x) =

(A) $2\sqrt{2x}; x \geq 1$ (B) $\sqrt{2x}; x \geq 1$ (C) $2\sqrt{x}; x \geq 1$ (D) None of these

3. A variable circle C touches y = 0 and passes through the point (0,1). Let the locus of the centre of C is the curve P. The area enclosed by the curve P and the line x + y = 2 is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{15}{3}$ (D) $\frac{11}{3}$

4. Let n S_n be the area of the figure enclosed by a curve $y = x^3(1-x^2)^n, 0 \leq x \leq 1$ and the x-axis. The value of $\lim_{n\to\infty}\sum_{k=1}^{n} S_k$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

5. Find the area of the region enclosed between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of the points where the curve has local minima

(A) $\frac{7}{60}$ (B) $\frac{7}{80}$ (C) $\frac{7}{120}$ (D) $\frac{7}{160}$

6. Let $f(x) = \begin{cases} Ln(x-[x]); & x \notin I \\ 0 \; ; & x \in I \end{cases}$ Area enclosed by curves $y = f(x), y = f(|x|), -5 \leq x \leq 5$ and y = 0 is

(A) 5 (1 – 1n2) (B) 0 (C) 51n2 (D) none of these

7. The area bounded by the curves y = 2 − |x −1| , y = sin x; x = 0 and x = 2 is _____.

(A) 1+ $2\cos^2 1$ (B) 2 + $\sin^2 1$ (C) $\frac{\pi}{2}$ (D) 1+ log2

8. The area bound by the curve y = f (x) , the x–axis and the line y =1, where $f(x) = 1 + \frac{1}{x}\int_1^x f(t)\,dt, (x \neq 0)$ is _____.

(A) $2\left(1-\frac{1}{e}\right)$ (B) $2\left(1+\frac{1}{e}\right)$ (C) $\left(1+\frac{1}{e}\right)$ (D) $\left(1-\frac{1}{e}\right)$

9. Given f (x) = $x^3 + x - 2$, f :R $\to$ R, area of region bounded by $y = f(x), y = f^{-1}(x)$ and xy = 0 in 1^{st} quadrant is A, where $A = 3t - \frac{5}{2}$, where $t = a^{1/b}$ and $3(a+b) = n; a,b,n \in N$ and a is prime. Difference of last two digits of $a^{a^{11}-36}$ is

(A) 0 (B) 1 (C) 2 (D) 3

MCQ (One or more than one correct) :

10. The area bounded by the curve $y = f^{-1}(x)$, x-axis, x = 0 and x = π, where f(x) is given by f(x) = x + sin x is $\frac{\pi^2 - a}{b}$ then the correct statement are :

(A) a = 2b (B) a + 2b = 8 (C) 3a =b (d) $\left(\frac{2\pi}{b}, \frac{a\pi}{4}\right)$ lies on f(x)

11. If A_i is the area bounded by $|x - a_i| + |y| = b_i, i \in \mathbb{N}$ where 1 $a_{i+1} = a_i + \frac{3}{2}b_i$ and $b_{i+1} = \frac{b_i}{2}, a_1 = 0, b_1 = 32$, then _____.

(A) A_3 =128 (B) A_3 = 256 (C) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$ (D) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{4}{3}(16)^2$

12. Let T be the triangle with vertices (0, 0), (0, c^2) and (c, c^2) and let R be the region between y = cx and y = x^2 where c > 0 then

(A) Area (R) = $\frac{c^3}{6}$ (B) Area of R = $\frac{c^3}{3}$

(C) $\lim_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = 3$ (D) $\lim_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = \frac{3}{2}$

Comprehension Type Question:

Comprehension # 1

Let $f : \mathbb{R} \to \mathbb{R}$ be continuous function and bijective, defined such that f(α) = 0($\alpha \neq 0$). The area bounded by y = f (x), x =α, x =α−t is equal to area bounded by y = f (x), x =α, x = α + t , $\forall t \in \mathbb{R}$ then

13. y = f (x) is symmetrical about the point

(A) (0,0) (B) (0, α) (C) (α,0) (D) (α,α)

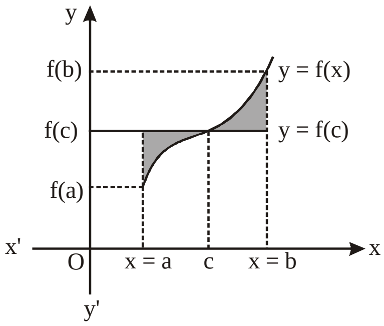
14. f(2α) equal to _____.

(A) f (α) (B) −f(α) (c) f(0) (d) − f(0)

Comprehension # 2

If $y = f(x)$ is a monotonic function in (a, b), then the area bounded by the ordinates at $x = a$, $x = b$, $y = f(x)$ and $y = f(c)$ (where $c \in (a, b)$) is minimum when $c = \frac{a+b}{2}$.

Proof : $A = \int_a^c \left(f(c) - f(x)\right) dx + \int_c^b \left(f(x) - f(c)\right) dx$



$= f(c)(c-a) - \int_a^c \left(f(x)\right) dx + \int_c^b \left(f(x)\right) dx - f(c)(b-c)$

$\Rightarrow$ $A = [2c - (a + b)\ f(c) + \int_c^b \left(f(x)\right) dx - \int_a^c \left(f(x)\right) dx$

Differetiating w.r.t. c,

$\frac{dA}{dc} = \left[2c - (a+b)\right]$ f '(x) + 2 f(c) + 0 – f(c) – (f(c))

for maxima and minima $\frac{dA}{dc} = 0$

$\Rightarrow$ f '(x)[2c – (a + b)] = 0 (as $f'(c) \neq 0$)

hence $c = \frac{a+b}{2}$

Also $c < \frac{a+b}{2}$, $\frac{dA}{dc} < 0$ and $c > \frac{a+b}{2}, \frac{dA}{dc} > 0$.

Hence A is minimum when $c = \frac{a+b}{2}$.

15. If the area bounded by f(x) = and the straight lines x = 0, x = 2 and the x-axis is minimum, then the value of a is

(A) $\frac{1}{3}$ (B) 2 (C) 1 (D) $\frac{2}{3}$

16. The valueof the parameter a for which the area of the figure bounded by the abscissa axis, the graph of the function $y = x^3 + 3x^2 + x + a$ and the straight lines, which are parallel to the axis of ordinates and cut the abscissa axis at the point of extremum of the function, is the least, is

(A) 2 (B) 0 (C) – 1 (D) 1

17. If the area enclosed by $f(x) = \sin x + \cos x$, $y = a$ between the two consecutive points of extremum is minimum, then the value of a is

(A) 0 (B) – 1 (C) 1 (D) 2

Comprehension # 3

Consider the areas S_0, S_1, S_2 ..... bounded by the x-axis and half-waves of the curve $y = e^{-x} \sin x$, where $x \geq 0$.

18. The value of S_0 is

(A) $\frac{1}{2}\left(1+e^{\pi}\right)$ sq. units (B) $\frac{1}{2}\left(1+e^{-\pi}\right)$ sq. units

(C) $\frac{1}{2}\left(1-e^{-\pi}\right)$ sq. units (D) $\frac{1}{2}\left(e^{\pi}-1\right)$ sq. units.

19. The sequence S_0, S_1, S_2, ..., forms a G.P. with common ratio

(A) $\frac{e^{\pi}}{2}$ (B) $e^{-\pi}$ (C) e^{π} (D) $\frac{e^{-\pi}}{2}$

20. $\sum_{n=0}^{\infty} S_n$ is equal to

(A) $\frac{1+e^{\pi}}{1-e^{-\pi}}$ (B) $\frac{\frac{1}{2}(1+e^{\pi})}{1-e^{-\pi}}$ (C) $\frac{1}{2(1-e^{-\pi})}$ (D) None of these

Numerical based Questions :

21. If A is the area of the figure bounded by the straight line x = 0 and x = 2 and the curves $y = 2^x$ and $y = 2x - x^2$, then the value of $6\left(\frac{3}{\ell n2} - A\right)$ is

22. The graph of $y^2 + 2xy + 40|x| = 400$ divides the plane into regions. The area of the bounded region is

23. Area enclosed by the curve $4 \le x^2 + y^2 \le 2(|x| + |y|)$ is (in sq. units)

24. The area bounded by the curves $y = -\sqrt{4-x^2}$, $x^2 = -y\sqrt{2}$ and x = y can be expressed in the form of $\pi + \frac{\lambda}{\mu}$, ($\lambda$, μ being relatively prime positive integers) then $\lambda + \mu$ equals to _____.

25. Find the area enclosed between the curves : $y = \log_e(x + e)$, $x = \log_e(1/y)$ & the x–axis.

Matrix Match Type :

26. Match the following

Column-I	Column-II
(a) Let $f(K) = \frac{K}{2009}$ and $g(K) = \frac{(f(K))^4}{(1-f(K))^4 + (f(K))^4}$ and $S = \sum_{K=0}^{2009} g(K)$ then sum of the diagonal in S =	(p) 0
(b) $f : D \to \mathbb{R}$ such that $f(k) = \sqrt{\sin(\cos x)} + \log_e\left(-2\cos^2 x + 3\cos x + 1\right)$ then $\int_{x_1}^{x_2}\left[\cos x - \frac{1}{2}\right]dx$ is equal to, $x_1, x_2 \in D$ and [.] denotes greatest integer function.	(q) 1
(c) The area of region, represented by $\|x\| + \|y\| \le 2$ and $x^2 \le y$ is S square units then 3S = _____.	(r) 7
(d) $\lim_{y\to\infty} \frac{\int_1^y \left[\tan^{-1} x\right]dx}{\int_1^y \left[1+\frac{1}{x}\right]dx}$ = _____. ([.]) denotes greatest integer function)	(s) 6

(A) A→ s; B → p; C → s; D → q (B) A → p; B → s; C → r; D → q

(C) A → s; B → p; C → r; D → q (D) A → s; B → p; C → r; D → s

27. **Column - I** | **Column-II**

Column - I	Column-II
(A) The area bounded y the curve $y = x\|x\|$, x-axis and the ordinates $x = 1$, $x = -1$	(P) $\frac{10}{3}$ sq. units
(B) The area of the region lying between the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$	(Q) $\frac{64}{3}$ sq. units
(C) The area enclosed between the curves $y^2 = x$ and $y = \|x\|$	(R) $\frac{2}{3}$ sq. units
(D) The area bounded by parabola $y^2 = x$, straight line $y = 4$ and y-axis	(S) $\frac{1}{6}$ sq. units

(A) A→Q; B→S; C→P; D→R

(B) A→S; B→R; C→Q; D→P

(C) A→R; B→P; C→S; D→Q

(D) A→Q; B→R; C→P; D→S