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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-17

Definite Integration

SCQ (Single Correct Type) :

1. The value of the definite integral $\int_2^3 \left[\sqrt{2x-\sqrt{5(4x-5)}}+\sqrt{2x+\sqrt{5(4x-5)}}\right]dx$ is equal to

(A) $4\sqrt{3}-\frac{2\sqrt{2}}{3}$ (B) $4\sqrt{2}$ (C) $4\sqrt{3}-\frac{4}{3}$ (D) $\frac{\sqrt{10}}{2}+\frac{7\sqrt{7}-5\sqrt{5}}{3\sqrt{2}}$

2. If $\int_{\ell n2}^{x}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$, then x is equal to

(A) 4 (B) $\ell n\ 8$ (C) $\ell n\ 4$ (D) none of these

3. The tangent to the graph of the function y = f(x) at the point with abscissa x =1 form an angle of and at the point x = 2, an angle of $\pi/3$ and at the point x = 3, an angle of $\pi/4$. The value of $\int_1^3 f'(x)f''(x)dx+\int_2^3 f''(x)dx$ (f''(x) is supposed to be continuous) is :

(A) $\frac{4\sqrt{3}-1}{3\sqrt{3}}$ (B) $\frac{3\sqrt{3}-1}{2}$ (C) $\frac{4-\sqrt{3}}{3}$ (D) None of these

4. $\int_{5/2}^{5}\frac{\sqrt{\left(25-x^2\right)^3}}{x^4}dx$ equals to :

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) none

MCQ (One or more than one correct) :

5. If $\int_{-\alpha}^{\alpha}e^x+\cos x\ln\left(x\sqrt{1+x^2}\right)dx>\frac{3}{2}$, then α can be _____.

(A) 1 (B) 2 (C) 3 (D) 4

6. The value of the definite integral $\int_0^1\frac{dx}{\sqrt{(x+1)^3(3x+1)}}$ equals _____.

(A) $\sqrt{2}-1$ (B) $\tan\frac{\pi}{2}$ (C) $\tan\frac{\pi}{8}$ (D) $\tan\frac{5\pi}{12}$

7. If $f(x) = 2^{\{x\}}$, where $\{x\}$ denotes the fractional part of x. Then which of the following is true ?

(A) f is periodic

(B) $\int_0^1 2^{\{x\}}dx = \frac{1}{\ell n2}$

(C) $\int_0^1 2^{\{x\}}\,dx = \log_2 e$

(D) $\int_0^{100} 2^{\{x\}}\,dx = 100\log_2 e$

Comprehension Type Question:

Comprehension # 1

Consider the integral $I = \int_0^{10\pi} \frac{\cos 6x\cos 7x\cos 8x\cos 9x}{1+e^{2\sin^3 4x}}dx$

8. If $k\int_0^{\pi/2} \cos 6x\cos 7x\cos 8x\cos 9x\,dx$ then k is _____.

(A) 1 (B) 5 (C) 10 (D) 20

9. If $I = \lambda\int_0^{\pi/4} \cos 6x\cos 8x\cos 2x\,dx$ then λ is _____.

(A) 5 (B) 10 (C) 20 (D) $\frac{5}{2}$

10. The value of I is _____.

(A) $\frac{5\pi}{4}$ (B) $\frac{5\pi}{16}$ (C) $\frac{5\pi}{32}$ (D) $\frac{5\pi}{8}$

Numerical based Questions :

11. Given $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$ and $\int_0^1 \frac{\ln x}{1-x^2}dx = -\frac{\pi^2}{\lambda}$ then value of λ is _____.

12. $\int_1^e\left(\frac{1}{\sqrt{x\ln x}} + \sqrt{\frac{\ln x}{x}}\right)dx = L$ then the value of $\frac{3L}{\sqrt{e}}$ is _____.

13. If $\int_0^{\frac{\pi}{2}} \sqrt[3]{\sin 2x}\sin x dx = k\int_0^{\frac{\pi}{4}} \sqrt[3]{\cos 2x}\cos x\,dx$ then the value of k^2 is_____.

14. $\int_2^4 \frac{3x^2+1}{(x^2-1)^3}dx = \frac{\lambda}{n^2}$ where $\lambda, n \in N$ and gcd(λ, n) = 1, then find the value of $\lambda + n$

15. Let f(x) be a function satisfying $f(x) = f\left(\frac{100}{x}\right)\ \forall x > 0$. If $\int_1^{10}\frac{f(x)}{x}dx = 5$ then find the value of $\int_1^{100}\frac{f(x)}{x}dx$

16. Evaluate $\dfrac{2005\displaystyle\int_0^{1002}\dfrac{dx}{\sqrt{1002^2-x^2}+\sqrt{1003^2-x^2}}+\int_{1002}^{1003}\sqrt{1003^2-x^2}\,dx}{\displaystyle\int_0^1\sqrt{1-x^2}\,dx}$ = k, then find the sum of squares of digits of natural number k.

17. Let $I_1=\displaystyle\int_0^{\pi/4}(1+\tan x)^2dx$, $I_2=\displaystyle\int_0^1\frac{dx}{(1+x)^2(1+x^2)}$ then find the value of $\dfrac{I_1}{I_2}$

18. If f, g, h be continuous functions on [0, a] such that f (a – x) = f (x), g (a – x) = – g (x) and 3 h (x) – 4 h (a – x) = 5, then find the value of $\displaystyle\int_0^a$ f (x) g (x) h (x) dx

19. If $f(x)=\dfrac{\sin x}{x}\ \forall\ x\in(0,\pi]$, If $\dfrac{\pi}{k}\displaystyle\int_0^{\pi/2}f(x)\,f\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi}f(x)\,dx$ then find the value of k.

20. Let a be a real number in the interval [0, 314] such that $\displaystyle\int_{-\pi+a}^{3\pi+a}|x-a-\pi|\sin\left(\frac{x}{2}\right)dx=-16$, then determine number of such values of k.

21. If $f(x)=x+\displaystyle\int_0^1 t(x+t)f(t)\,dt$, then the value of the definite integral $\displaystyle\int_0^1 f(x)\,dx$ can be expressed in the form of rational as $\dfrac{p}{q}$ (where p and q are coprime). Find (p + q).

22. If the minimum of the following function f(x) defined at $0<x<\dfrac{\pi}{2}$.

$f(x)=\displaystyle\int_0^x\frac{d\theta}{\cos\theta}+\int_x^{\frac{\pi}{2}}\frac{d\theta}{\sin\theta}$ is equal to $\ell n(a+\sqrt{b})$ where a, b $\in$ N and b is not a perfect square then find the value of (a + b)

Matrix Match Type :

23. Match the following:

[x] $\rightarrow$ greatest integer less than or equal to x

Column-I	Column-II
(A) $25\int_0^{\frac{\pi}{4}}\left(\tan^6(x-[x])+\tan^4(x-[x])\right)dx =$	(p) 4
(B) $I_1=\int_{-2}^{2}\frac{x^6+3x^5+7x^4}{x^4+2}dx$ $I_2=\int_{-3}^{1}\frac{2(x+1)^2+11(x+1)+14}{(x+1)^4+2}dx$ then $I_1+I_2=$	(q) 2
(C) $\frac{2}{\pi}\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx =$	(r) $\frac{100}{3}$
(D) $f(x)=\frac{2-x}{\pi}\cos\pi(x+3)+\frac{1}{x^2}\sin\pi(x+3)$ where $0<x<4$. The number of points of local extrema are	(s) 5
	(t) $\frac{102}{3}$

(A) A $\rightarrow$ s; B $\rightarrow$ r; C $\rightarrow$ q; D $\rightarrow$ q

(B) A $\rightarrow$ s; B $\rightarrow$ s; C $\rightarrow$ q; D $\rightarrow$ r

(C) A $\rightarrow$ s; B $\rightarrow$ r; C $\rightarrow$ s; D $\rightarrow$ q

(D) A $\rightarrow$ s; B $\rightarrow$ r; C $\rightarrow$ r; D $\rightarrow$ q

Subjective Based Questions :

24. Evaluate $\int_0^1\frac{1}{\left(5+2x-2x^2\right)\left(1+e^{(2-4x)}\right)}dx$

25. Evaluate $\int_0^1\left(\{2x\}-1\right)\left(\{3x\}-1\right)dx$, where { } denotes fractional part of x.