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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-16

Indefinite Integration

SCQ (Single Correct Type) :

1. The value of $\int X \cdot \frac{\ell n\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$ equals:

(A) $\sqrt{1+x^2}\ \ell n\left(x+\sqrt{1+x^2}\right)-x+C$ (B) $\frac{x}{2}\cdot \ell n^2\left(x+\sqrt{1+x^2}\right)-\frac{x}{\sqrt{1+x^2}}+C$

(C) $\frac{x}{2}\cdot \ell n^2\left(x+\sqrt{1+x^2}\right)+\frac{x}{\sqrt{1+x^2}}+C$ (D) $\sqrt{1+x^2}\,\ell n\left(x+\sqrt{1+x^2}\right)+x+C$

2. The value of $2\int \sin x\cdot \operatorname{cosec} 4x\,dx$ is equal to

(A) $\frac{1}{2\sqrt{2}}\ \ell n\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|-\frac{1}{4}\ \ell n\left|\frac{1+\sin x}{1-\sin x}\right|+C$

(B) $\frac{1}{2\sqrt{2}}\ \ell n\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|+\frac{1}{4}\ \ell n\left|\frac{1+\sin x}{1-\sin x}\right|+C$

(C) $\frac{1}{2\sqrt{2}}\ \ell n\left|\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right|-\frac{1}{4}\ \ell n\left|\frac{1+\sin x}{1-\sin x}\right|+C$

(D) none of these

3. The values of a nad b which satisfy $\int \frac{2\sin 2x-\cos x}{4-\cos^2 x-4\sin x}dx = a\log|\sin x-1|+b\log|\sin x-3|+c$, are _____.

(A) $-\frac{3}{2},\frac{11}{2}$ (B) $\frac{3}{2},\frac{11}{2}$ (C) $\frac{3}{2},-\frac{11}{2}$ (D) $-\frac{3}{2},-\frac{11}{2}$

4. Let $\int \frac{g(x)((1+x)\ln x-5)}{x(\ln x)^6}dx=\frac{g(x)}{(\ln x)^5}+c, g(1)=e$ and $g:(0,e)\to(0,\infty)$. If $g^n(x)$ denotes the n^{th} derivative of g(x), then the value of $g^{2012}(1)$ is _____.

(A) $2011g(1)$ (B) $2012g(1)$ (C) $2013g(1)$ (D) $2014g(1)$

5. $\int e^{\sin x}\left(\frac{x\cos^3 x-\sin x}{\cos^2 x}\right)dx=$

(A) $e^{\sin x}(x+\sec x)+c$ (B) $e^{\sin x}(1+\sec x)+c$

(C) $e^{\sin x}(1-\sec x)+c$ (D) $e^{\sin x}(x-\sec x)+c$

6. If $\int\frac{xdx}{\sqrt[2012]{(1+x^2)^{1012}(2+x^2)^{3012}}}=\frac{\alpha}{\beta}(1-f(x))^{\frac{\beta}{2\alpha}}+k$ then which is true

(A) $\alpha=503;\beta=500,f(\sqrt{2})=\frac{1}{\beta-\alpha}$ (B) $\alpha=503;\beta=250,f(\sqrt{2})=\frac{1}{\alpha-\beta}$

(C) $\alpha=503;\beta=500,f(1)=\frac{1}{\alpha-\beta}$ (D) $\alpha=503;\beta=225,f(\sqrt{3})=\frac{1}{\alpha-\beta}$

7. If $\int\frac{\sec^2 x-2010}{\sin^{2010}x}dx=\frac{P(x)}{\sin^{2010}x}+C$, then value of $P\left(\frac{\pi}{3}\right)$ is _____.

(A) 0 (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) None of these

8. If $g(x)=\frac{f(x)}{(x-a)(x-b)(x-c)}$ where f(x) is a polynomial of degree < 3, then ____.

(A) $\int g(x)dx=\begin{vmatrix}1 & a & f(a)\log|x-a|\\ 1 & b & f(b)\log|x-b|\\ 1 & c & f(c)\log|x-c|\end{vmatrix}\div\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}+k$

(B) $\frac{d}{dx}g(x)=\begin{vmatrix}1 & a & f(a)\log(x-a)^{-2}\\ 1 & b & f(b)\log(x-b)^{-2}\\ 1 & c & f(c)\log(x-c)^{-2}\end{vmatrix}\div\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$

(C) $\int g(x)dx=\begin{vmatrix}1 & a & f(a)\log|x-a|\\ 1 & b & f(b)\log|x-b|\\ 1 & c & f(c)\log|x-c|\end{vmatrix}\div\begin{vmatrix}a^2 & a & 1\\ b^2 & b & 1\\ c^2 & c & 1\end{vmatrix}+k$

(D) None of these

9. If $y=2\cot^{-1}\left(\frac{x}{2}\right)+x$, then $\int\frac{x^2dx}{(x^2-4)\sin x+4x\cos x}$

(A) $\log|\operatorname{cosec}2y+\cot y|+c$ (B) $\log|\operatorname{cosec}2y-\cot 2y|+c$

(C) $\log|\operatorname{cosec}y+\cot y|+c$ (D) $\log|\operatorname{cosec}y-\cot y|+c$

Comprehension Type Question:

Comprehension # 1

In calculating a number of integrals we had to use the method of integration by parts several times in succession. The result could be obtained more rapidly and in a more concise form by using the so-called generalized formula for integration by parts

$\int u(x)\,v(x)\,dx = u(x)\,v_1(x) - u'(x)\,v_2(x) + u''(x)\,v_3(x) + \ldots\ldots + (-1)^{n-1}\,u^{n-1}(x)\,v_n(x) - (-1)^{n-1}\int u^n(x)\,v_n(x)\,dx$

where $v_1(x) = \int v(x)\,dx$, $v_2(x) = \int v_1(x)\,dx$ ...., $v_n(x) = \int v_{n-1}(x)\,dx$

Of course, we assume that all derivatives and integrals appearing in this formula exist. The use of the generalized formula for integration by parts is especially useful when calculating $\int P_n(x)Q(x)\,dx$, where $P_n(x)$ is polynomial of degree n and the factor Q(x) is such that it can be integrated successively n + 1 times.

10. If $\int (x^3 - 2x^2 + 3x - 1)\cos 2x\,dx = \frac{\sin 2x}{4}u(x) + \frac{\cos 2x}{8}v(x) + C$ then

(A) $u(x) = x^3 - 4x^2 + 3x$ (B) $u(x) = 2x^3 - 4x^2 + 3x$

(C) $v(x) = 3x^2 - 4x + 3$ (D) $v(x) = 6x^2 - 8x$

11. If $\int (3x^2 + x - 2)\sin^2(3x+1)\,dx = -\frac{u(x)}{72}\sin(6x+2) + \frac{v(x)}{72}\cos(6x+2) + \frac{1}{2}x^3 + \frac{1}{4}x^2 - x + C$

(A) $u(x) = 3x^2 + 6x - 13$ (B) $u(x) = 18x^2 + 2x - 13$

(C) $v(x) = 3x + 1$ (D) $v(x) = -(6x + 1)$

Numerical based Questions :

12. If $\int \frac{\sqrt{4+x^2}}{x^6}dx = \frac{\left(a+x^2\right)^{3/2}\cdot\left(x^2-b\right)}{120\,x^5} + C$ then a + b equals to :

13. Let $g(x) = \int \frac{1+2\cos x}{(\cos x+2)^2}dx$ and g(0) = 0 then value of $32\,g\left(\frac{\pi}{2}\right)$ is.

14. If $f(x) = \sqrt{x-1}$; $g(x) = e^x$ and $\int fog(x)dx = Afog(x) + B\tan^{-1}(fog(x)) + C$ then $A^3 + B^2$ equals

15. If $\int \frac{1+x\cos x}{x\left(1-x^2e^{2\sin x}\right)}dx = k\,\ell n\sqrt{\frac{x^2e^{2\sin x}}{1-x^2e^{2\sin x}}} + C$ then k is equal to :

16. Let $\int \frac{f'(x)g(x) - g'(x)f(x)}{\left(f(x)+g(x)\right)\sqrt{f(x)g(x)-g^2(x)}}dx = \sqrt{m}\tan^{-1}\left(\sqrt{\frac{f(x)-g(x)}{ng(x)}}\right) + C$,

where m, n $\in$ N and 'C' is constant of integration $\left(g(x) > 0\right)$. Find the value of $(m^2 + n^2)$.

Matrix Match Type :

17. Let $I = \int \frac{e^x}{e^{4x}+1}dx$ and $J = \int \frac{e^{-x}}{e^{-4x}+1}dx$

Then for any arbitrary constant C, match the following

Column-I		**Column-II**	
(A)	I	(P)	$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{e^{2x}-1}{\sqrt{2}e^x}\right)+C$
(B)	J + I	(Q)	$\frac{1}{2\sqrt{2}}\ln\left(\frac{e^{2x}-\sqrt{2}e^x+1}{e^{2x}+\sqrt{2}e^x+1}\right)+C$
(C)	J – I	(R)	$\frac{1}{2\sqrt{2}}\left(\tan^{-1}\left(\frac{e^{2x}-1}{\sqrt{2}e^x}\right)-\frac{1}{2}\ln\left(\frac{e^{2x}-\sqrt{2}e^x+1}{e^{2x}+\sqrt{2}e^x+1}\right)\right)+C$
		(S)	$\frac{1}{2\sqrt{2}}\left(\tan^{-1}\left(\frac{e^{2x}-1}{\sqrt{2}e^x}\right)+\frac{1}{2}\ln\left(\frac{e^{2x}-\sqrt{2}e^x+1}{e^{2x}+\sqrt{2}e^x+1}\right)\right)+C$

18.

Column-I	**Column-II**
(A) $\int \frac{x^2(x^6+x^5-1)dx}{(2x^6+3x^5+2)^2}$	(P) $\frac{1}{6}\left(\frac{1}{x^{-3}}+\frac{3}{x^{-2}}\right)^{\frac{1}{2}}+C$
(B) $\int \frac{(x^5+x^4+x^2)dx}{\sqrt{4x^7+5x^6+10x^4}}$	(Q) $\frac{1}{2}(1+x^{-2}+x^{-5})^{-2}+C$
(C) $\int \frac{(2x^{12}+5x^9)dx}{(x^5+x^3+1)^3}$	(R) $\frac{-1}{6}(2x^3+3x^2+2x^{-3})^{-1}+C$
(where C is the constant of integration.)	(S) $x\left(\frac{x^3}{25}+\frac{x^2}{20}+\frac{1}{10}\right)^{\frac{1}{2}}+C$

Subjective Type Questions :

19. Evaluate : $\int \cos x \cdot e^x \cdot x^2 dx$

20. Evaluate : $\tan^{-1} x \cdot \ell n\,(1+x^2)\,dx$.

21. Evaluate : $\int e^x \frac{1+nx^{n-1}-x^{2n}}{\left(1-x^n\right)\sqrt{1-x^{2n}}}dx$

22. If $\int \frac{x\cos\alpha+1}{\left(x^2+2x\cos\alpha+1\right)^{3/2}}dx = \frac{f(x)}{\sqrt{g(x)}}+C$ then find f(x) and g(x).

23. $\int \frac{dx}{(\sin^2 x+2\cos^2 x)^2}dx$

24. Evaluate $\int \frac{(x-\sin x)^{3/2}}{\sqrt{x}}\left\{\frac{6x^2\sin^2\frac{x}{2}}{x-\sin x}+3x\right\}dx$