competishun
Power of real gurus

MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-15

Probability

SCQ (Single Correct Type) :

1. Of all the function that can be defined from the set A : {1, 2, 3, 4} $\rightarrow$ B(5, 6, 7, 8, 9}, a mapping is randomly selected. The chance that the selected mapping is strictly monotonic, is

(A) $\frac{1}{125}$ (B) $\frac{2}{125}$ (C) $\frac{5}{4096}$ (D) $\frac{5}{2048}$

2. On a Saturday night 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is

(A) 3/7 (B) 4/7 (C*) 5/7 (D) 6/7

3. There are six families each consisting of a husband, a wife and a child. A group of three consisting of a man, a woman and a child is said to form a trio. If 6 persons are selected, the probability that there will be two trios in which exactly one trio is of the same family is _____.

(A) $\frac{125}{1547}$ (B) $\frac{25}{221}$ (C) $\frac{30}{1547}$ (D) $\frac{60}{1547}$

4. There are four seats numbered 1, 2, 3, 4 in a room and four persons having tickets corresponding to these seats (one person having one ticket). Now the person having the ticket number 1, enters into the room and sits on any of the seat at random. Then the person having the ticket number 2, enters in room. If his seat is empty then he sits on his seat otherwise he sits on any of the empty seat at random. Similarly the other persons sit. Probability that the person having ticket numbered 4 gets the seat number 4 is

(A) 1/2 (B) 1/4 (C) 1/8 (D) 1/16

5. 'A' tosses a fair coin. If it shows a tail ' A' is asked to roll a fair die and A' s score is the number that die shows. If the coin shows a head, 'A' is asked to toss five more coins and A' s score is total number of heads shown (including the first coin). If ' A' tells you that his score is only 3 then the probability that 'A' rolls a die is

(A) $\frac{23}{96}$ (B) $\frac{1}{6}$ (C) $\frac{8}{23}$ (D) $\frac{7}{44}$

MCQ (One or more than one correct) :

6. A fair die is rolled four times. Find the probability that each number is no smaller than the preceding number.

(A) $\frac{7}{72}$ (B) $< \frac{1}{3}$ (C) $> \frac{1}{3}$ (D) $\frac{11}{72}$

7. If three numbers are chosen randomly from the set $\{1,3,3^2,....,3^n\}$ without replacement, then the probability that they form an increasing geometric progression is

(A) $\frac{3}{2n}$ if n is odd (B) $\frac{3}{2n}$ if n is even

(C) $\frac{3n}{2(n^2-1)}$ if n is even (D) $\frac{3n}{2(n^2-1)}$ if n is odd

8. Let 'A' and 'B' be independent events such that $P(A) > \frac{1}{2}, P(A \cap \bar{B}) = \frac{3}{25}$ and $P(\bar{A} \cap B) = \frac{8}{25}$. Then which of the following statements is/are correct ?

(A) $(P(A))^2 + (P(B))^2 = 1$ (B) P(A) satisfies $25x^2 - 20x + 3 = 0$

(C) P(A) satisfies $5x^2 - 20x + 3 = 0$ (D) $(P(A))^2 + (P(B))^2 = 2$

Comprehension Type Question:

Comprehension # 1

A bag contains 6 different balls of three colours white, green and red (at least one ball of each colour.

9. The probability that the bag contains 2 balls of each colour is

(A) $\frac{1}{3}$ (B) $\frac{1}{7}$ (C) $\frac{1}{9}$ (D) $\frac{1}{10}$

10. Three balls are picked up at random and found to be one of each colour. The probability that the bag contained 4 red balls

(A) $\frac{1}{10}$ (B) $\frac{1}{14}$ (C) $\frac{1}{7}$ (D) $\frac{7}{25}$

11. Three balls are picked up at random and found to be one of each colour. The probability that the bag contained equal number of white and green balls is

(A) $\frac{3}{14}$ (B) $\frac{3}{10}$ (C) $\frac{2}{7}$ (D) $\frac{7}{25}$

Comprehension # 2

A player tosses a coin and scores one point for every head and two points for every tail that turns up. He plays on until his score reaches 'n ' or passes n. P_n denotes the probability of getting a score of exactly n.

12. The value of P_n is equal to

(A) $\frac{1}{2}[P_{n-1} + P_{n-2}]$ (B) $\frac{1}{2}[2P_{n-1} + P_{n-2}]$ (C) $\frac{1}{2}[P_{n-1} + 2P_{n-2}]$ (D) None of these

13. The value of $P_n + \frac{1}{2}P_{n-1}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{1}{4}$

14. Which of the following is not true?

(A) $P_{100} > \frac{2}{3}$ (B) $P_{101} < \frac{2}{3}$ (C) $P_{100}, P_{101} < \frac{2}{3}$ (D) None of these

Comprehension # 3

2^n ($n \in N$, $n \geq 2$) players of equal strength are playing a knock out tournament. They are paired randomly in all the rounds, and the winner reaches the next round.

15. The probability P_n that exactly one of the two specified players P_1 and P_2 reaches the semifinals is given by

(A) $\frac{8\left(2^n-4\right)}{2^n\left(2^n-1\right)}$ (B) $\frac{6\left(2^n-4\right)}{2^n\left(2^n-1\right)}$ (C) $\frac{4\left(2^n-4\right)}{\left(2^n-2\right)\left(2^n-1\right)}$ (D) $\frac{8\left(2^n-4\right)}{\left(2^n-2\right)\left(2^n-1\right)}$

16. If there are 16 players including P_1 and P_2 then the chance that exactly one of either P_1 or P_2 reaches the semifinals, is

(A) $\frac{8}{35}$ (B) $\frac{3}{10}$ (C*) $\frac{2}{5}$ (D) $\frac{16}{25}$

17. As n tends to infinity then the value of P_n

(A) approaches $\frac{1}{2}$.

(B) decreases and tends to approches zero.

(C) equals zero.

(D) first increases upto 0.5 and then decreases to zero value.

Numerical based Questions :

18. During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the polygraph says he is guilty is a/b where *a* and *b* are relatively prime, find the value of (a + b).

19. Six faces of a cube are numbered randomly 1, 2, 3, 4, 5, 6. The probability that faces 1 and 6, 6 and 3, 3 and 1 will share an edge is $\frac{m}{n}$ (in its lowest form). m+ n = _____.

20. There are two dice A and B both having six faces. Die A has three faces marked with 1, two faces marked with 2 and one face marked with 3. Die B has one face marked with 1, two faces marked with 2 and three faces marked with 3. Both the dice are thrown randomly once. If E be the event of getting sum of the numbers appearing on top faces equal to x and if P(E) be the probability of event E, then find the value of x when P(E) is maximum.

21. 3 coins are thrown at one time and we remove those coins which show tails. Then we throw the remaining coins at one time and we remove those coins which show tails. This is done repeatedly until all of coins are removed. If the probability that the trials are ended in the 2^{nd} round is P, then the value of [10P] is _____ (where [.] denotes the greatest integer function).

22. Suppose n(> 4) people are seated at a round table. If three people are selected at random, and p_n is the probability that atleast two of them are sitting next to each other, then $p_7 = \frac{a}{b}$ (when $\frac{a}{b}$ is expressed in lowest terms) where a + b =

23. Five cards are drawn randomly one by one with replacement from a well shuffled pack of 52 playing cards. The probability that these cards will contain cards of each suit is

(A) $\frac{15}{64}$ (B) $\frac{17}{64}$ (C) $\frac{21}{64}$ (D) $\frac{19}{64}$

Matrix Match Type :

24. Match the following :

Column-I	**Column-**II
(A) A and B play a game with a pair of dice each. A has a "good throw" if the sum on A's dice is 7, while B has a "good throw" if the sum on B's dice is 4. In each round they throw simultaneously. If one of them has a "good throw" and the other does not, the player having a "good throw" is declared a winner, otherwise they throw again. The probability that A wins is	(p) $\frac{3}{8}$
(B) Suppose A nad B shoot independently until each hits his target. They have probabilities $\frac{3}{5}$ and $\frac{5}{7}$ respectively of hitting the targets at each shot. The probability that B will requre more tries than A to hit the target is	(r) $\frac{11}{16}$
(C) The probability of a missile being destroyed before hitting the target is $\frac{1}{3}$. If the missile is not destroyed, the probability of hittingthe target is $\frac{3}{4}$. Three missiles are fired. The probability of exactly 2 hitting the target is	(r) $\frac{11}{16}$
(D) The probability of a bomb hitting a bridge is $\frac{1}{2}$. At least two direct hits are needed to destroy it. If n is the least number of bombs required so that the probability of the bridge getting destroyed is greater than 0.9, then the value of $\frac{2}{n}$ is	(s) $\frac{6}{31}$
	(t) $\frac{3}{16}$

(A) A→s; B→s; C→p; D→q (B) A→p; B→s; C→p; D→q

(C) A→t; B→s; C→p; D→q (D) A→r; B→s; C→p; D→q

25. n whole numbers are randomly chosen and multiplied.

Column-I	Column-II
(A) The probability that the last digit is 1,3,7 or 9 is	(p) $\frac{8^n - 4^n}{10^n}$
(B) The probability that the last digit 2,4,6 or 8 is	(q) $\frac{5^n - 4^n}{10^n}$
(C) The probability that last digit is 5 is	(r) $\frac{4^n}{10^n}$
(D) The probability that the last digit is zero is	(s) $\frac{10^n - 8^n - 5^n + 4^n}{10^n}$
	(t) none of these

(A) A→r; B→p; C→q; D→s

(B) A→s; B→p; C→q; D→r

(C) A→r; B→q; C→p; D→s

(D) A→q; B→p; C→r; D→s