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# MATHEMATICS

**TARGET : JEE- Advanced 2023**

# CAPS-13

## AOD-2

**SCQ (Single Correct Type) :**

**1.** f : [0, 4] $\to$ R is a differentiable function. Then for some a, b $\in$ (0, 4) , $f^2(4) - f^2(0)$ =

(A) 8f′(a) . f(b) (B) 4f′(b) f(a) (C) 2f′ (b) f(a) (D) f′(b) f(a)

**2.** The values of the parameter 'k' for which the equation $x^4 + 4x^3 - 8x^2 + k = 0$ has all roots real is given by

(A) k $\in$ (0,3) (B) k $\in$ (0, 128) (C) k $\in$ (3, 128) (D) k $\in$ (128, $\infty$)

**3.** A composite function $(f_1 o\ f_2 o\ f_3 o.....of_{21})(X)$ is an increasing function. If number of increasing functions in the set $\{f_1, f_2, ......f_{21}\}$ is r and remaining are decreasing functions, then maximum value of r(21−r) is

(A) 110 (B) $\frac{441}{4}$ (c) 105 (d) None of these

**4.** A sector subtends an angle 2$\alpha$ at the centre then the greatest area of the rectangle inscribed in the sector is (R is radius of the circle)

(A) $R^2 \tan\frac{\alpha}{2}$ (B) $\frac{R^2}{2}\tan\frac{\alpha}{2}$ (C) $R^2 \tan\alpha$ (D) $\frac{R^2}{2}\tan\alpha$

**5.** If the graphs of the functions y = ln x & y = ax intersect at exactly two points, then

(A) a$\in$(0,e) (B) $a \in \left(0, \frac{1}{e}\right)$ (C) a$\in$(−e,1) (d) a$\in$(1,e)

**6.** Let f (x) be a polynomial of degree 3 satisfying f(3) = 5, f (−1) = 9, f (x) has minimum at x = 0 and f′( x) has maximum at x =1. The distance between local maximum and local minimum of f (x) is

(A) $3\sqrt{2}$ (B) $\sqrt{15}$ (C) $2\sqrt{5}$ (D) $4\sqrt{3}$

**7.** Statement 1: In $\Delta$ABC, sinA + sinB sin C $\le \frac{3\sqrt{3}}{2}$

Statement 2: Let y = f (x) be a twice differentiable function such that f ''(x) < 0 in [a,b] then $\frac{f(a_1)+f(a_2)+f(a_3)}{3} \ge f\left(\frac{a_1+a_2+a_3}{3}\right)$ for $a_1, a_2, a_3 \in [a,b]$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

**8.** Let P(x) be a fourth degree polynomial with derivative P'( x) . Such that P(1) = P(2) = P(3) = P'(7) = 0 . Let k is the real number k $\neq$1,2,3 such that P(k) = 0, then k is equal to

(A) $\frac{317}{37}$ (B) $\frac{319}{37}$ (C) $\frac{321}{37}$ (D) $\frac{15}{37}$

**9.** Let f (x) = 2x(2− x), 0 ≤ x ≤ 2. The number of solution of $f\left(f\left(f(x)\right)\right)=\frac{x}{2}$ is

(A) 2 (B) 4 (C) 8 (D) 12

**10.** Set of values of a for which one negative and two positive real roots of the equation $x^3 - 3x + a = 0$ are possible, is _____.

(A) (0, 2) (B) (0, 4) (C) (2, 4) (D) (0, 10)

**11.** $f(x)=\begin{cases} e^x - 2 - e^{-2}, x < -2 \\ x^2 - x + \lambda, -2 \le x \le 2 \\ -\mu \ell n x, x > 2 \end{cases}$

If y = f(x) has local maxima at x = –2, then range of $\lambda$ is

(A) $(-\infty, 8]$ (B) $[-8, \infty)$ (C) [–8, 8] (D) $(-\infty, -8] \cup [8, \infty)$

**MCQ (One or more than one correct) :**

**12.** Consider the function f : R $\rightarrow$ R defined as f (x) = x + sin x. Which of the following is/are the correct statement(s)?

(A) The function is strictly increasing at every point on R except at 'x' equal to an odd integral multiple of $\pi$ where the derivative of f (x) is zero and where the function f is not strictly increasing.

(B) The function is bounded in every bounded interval but unbounded on whole real line.

(C) The graph of the function y = f (x) lies in the first and third quadrants only.

(D) The graph of the function y = f (x) cuts the line y = x at infinitely many points.

**13.** Let f (x) be a non constant twice derivable function defined on R such that f (2 + x) = f (2 – x) and $f'\left(\frac{1}{2}\right)$ = 0 = f ' (1). Then which of the following alternative(s) is/are correct?

(A) f (– 4) = f (8).

(B) Minimum number of roots of the equation f '' (x) = 0 in (0, 4) are 4.

(C) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(2+x)\sin x\,dx = 0$.

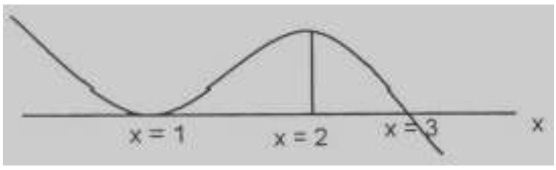
(D) $\int_0^2 f(t)5^{\cos \pi t}\,dt = \int_2^4 f(4-t)5^{\cos \pi t}\,dt$ .

**14.** If $\lim_{x\to a} f(x) = \lim_{x\to a}[f(x)]$ (a $\in$ R), where [·] denotes greatest integer function and f (x) is a non constant continuous function, then

(A) $\lim_{x\to a} f(x)$ is an integer. (B) $\lim_{x\to a} f(x)$ is non integer.

(C) f(x) has a local minimum at x = a. (D) f(x) has a local maximum at x = a.

**15.** If graph of $y = f'(x)$ is




then which of the following can be true for $y = f(x)$

(A) point of inflection at $x = 1$ and $x = 2$

(B) concave down in $(-\infty,1) \cup (2,\infty)$

(C) point of local maxima at $x = 3$

(D) decreasing in interval $(3,\infty)$

**16.** Let $g(x) = f(\tan x) + f(\cot x) \forall x \in \left(\frac{\pi}{2}, \pi\right)$. If $f''(x) < 0 \forall x \in \left(\frac{\pi}{2}, \pi\right)$, then

(A) g(x) is increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

(B) g(x) is increasing in $\left(\frac{3\pi}{4}, \pi\right)$

(C) g(x) is decreasing in $\left(\frac{3\pi}{4}, \pi\right)$

(D) g(x) has local maximum at $x = \frac{3\pi}{4}$

**Comprehension Type Question:**

**Comprehension # 1**

Consider f, g and h be three real valued differentiable functions defined on R. Let $g(x) = x^3 + g''(1) x^2 + (3g'(1) - g''(1) - 1)x + 3g'(1)$, $f(x) = x\,g(x) - 12x + 1$ and $f(x) = (h(x))^2$ where $h(0) = 1$.

**17.** The function $y = f(x)$ has

(A) Exactly one local minima and no local maxima

(B) Exactly one local maxima and no local minima

(C) Exactly one local maxima and two local minima

(D) Exactly two local maxima and one local minima

**18.** Which of the following is/are true for the function $y = g(x)$?

(A) g(x) monotonically decreases in $\left(-\infty, 2-\frac{1}{\sqrt{3}}\right) \cup \left(2+\frac{1}{\sqrt{3}}, \infty\right)$

(B) g(x) monotonically increases in $\left(2-\frac{1}{\sqrt{3}}, 2+\frac{1}{\sqrt{3}}\right)$

(C) There exists exactly one tangent to $y = g(x)$ which is parallel to the chord joining the points (1, g(1)) and (3, g(3))

(D) There exists exactly two distinct Lagrange's mean value in (0, 4) for the function $y = g(x)$.

**19.** Which one of the following does not hold good for $y = h(x)$?

(A) Exactly one critical point

(B) No point of inflection

(C) Exactly one real zero in (0, 3)

(D) Exactly one tangent parallel to x-axis

**Comprehension # 2**

Consider f(x) = $\begin{cases} |-x^2 - x| + \lambda; & x \le 0 \\ \lim\limits_{n\to\infty} \dfrac{x^n - x^{-n}}{x^n + x^{-n}} + k; & 0 < x < 1 \quad n \in N, k \in R \\ b \quad ; & x = 1 \\ \text{sgn}(\ell n(e^x + e^{-x} + 1)); & x > 1 \end{cases}$

**20.** The value of k + b + $\lambda$ so that f(x) is continuous in R is

(A) 3 (B) 2 (C) 4 (D) 1

**21.** Number of point(s) where continuous function f(x) is non differentiable, is

(A) 0 (B) 1 (C) 2 (D) 3

**22.** If f(x) is continuous then set of values of x for which f ′(x) is decreasing, is

(A) (– ∞, – 1) (B) (– 1, 0) (C) (0, 1) (D) (– 1, 1)

**Comprehension # 3**

A function f(x) having the following properties;

(i) f(x) is continuous except at x = 3

(ii) f(x) is differentiable except at x = – 2 and x = 3

(iii) f(0) = 0, $\lim\limits_{x\to 3} f(x) \to -\infty$, $\lim\limits_{x\to -\infty} f(x) = 3$, $\lim\limits_{x\to \infty} f(x) = 0$

(iv) $f'(x) > 0 \ \forall\ x \in (-\infty, -2) \cup (3, \infty)$ and $f'(x) \le 0 \ \forall\ x \in (-2, 3)$

(v) $f''(x) > 0 \ \forall\ x \in (-\infty, -2) \cup (-2, 0)$ and $f''(x) < 0 \ \forall\ x \in (0, 3) \cup (3, \infty)$

then answer the following questions

**23.** Maximum possible number of solutions of f(x) = | x | is

(A) 2 (B) 1 (C) 3 (D) 4

**24.** Graph of function y = f (– | x |) is

(A) differentiable for all x, if f ′(0) = 0

(B) continuous but not differentiable at two points, if f ′(0) = 0

(C) continuous but not differentiable at one points, if f ′(0) = 0

(D) discontinuous at two points, if f ′(0) = 0

**25.** f(x) + 3x = 0 has five solutions if

(A) f(–2) > 6 (B) f ′(0) < – 3 and f(– 2) > 6

(C) f ′ (0) > – 3 (D) f ′(0) > – 3 and f(– 2) > 6

**Numerical based Questions :**

**26.** If $\ln 2\pi < \log_2\left(2+\sqrt{3}\right) < \ell n 3\pi$, then number of roots of the equation $4\cos(e^x) = 2^x + 2^{-x}$, is

**27.** Let f(x) = Max. $\{x^2, (1-x)^2, 2x(1-x)\}$ where $x \in [0, 1]$ If Rolle's theorem is applicable for f(x) on largest possible interval [a, b] then the value of 2(a + b + c) when $c \in (a, b)$ such that f'(c) = 0, is

**28.** $f(x)=\begin{cases}\left(\sqrt{2}+\sin\frac{1}{x}\right)e^{\frac{-1}{|x|}} & \text{if} \quad x\neq 0\\ 0 & \text{if} \quad x=0\end{cases}$

Number of points where f (x) has local extrema when $x \neq 0$ be $n_1$ . $n_2$ be the value of global minimum of f (x) then $n_1 + n_2$ =

**29.** f (x) is a polynomial of 6th degree and f (x) = f (2− x) $\forall$ $x \in R$. If f (x) = 0 has 4 distinct real roots and two real and equal roots then sum of roots of f (x) = 0

**30.** ABCD and PQRS are two variable rectangles, such that A,B,C and D lie on PQ,QR,RS and SP respectively and perimeter ' x ' of ABCD is constant. If the maximum area of PQRS is 32, then $\frac{x}{4}=$

**31.** Find number of distinct read roots of $x^4-4x^3+12x^2+x-1=0$

**Matrix Match Type :**

**32. Match the following:**

| **Column-I** | **Column-II** |
|---|---|
| (A) If $x^2 + y^2 = 1$, then minimum value of x + y is | (p) |
| (B) If maximum value of y = $a\cos x-\frac{1}{3}\cos 3x$ occurs at $x=\frac{\pi}{6}$, then value of ‘a’ is | (q) $-\sqrt{2}$ |
| (C) If f(x) = x –2 sin x, $0 \le x \le 2\pi$ is increasing in the interval $(a\pi, b\pi)$ then a + b is | (r) 3 |
| (D) If equation of tangent to the curve $y=-e^{-x/2}$ where it crosses the y-axsi is $\frac{x}{p}+\frac{y}{q}=1$, then p–q is | (s) 2 |
| | (t) –2 |

(A) A–p; B–r; C–q; D–s
(B) A–q; B–s; C–s; D–r
(C) A–s; B–q; C–p; D–r
(D) A–r; B–p; C–q; D–r

**Subjective based Questions :**

**33.** Find the possible values of a such that the inequality $3 - x^2 > |x - a|$ has atleast one negative solution

**34.** A cone is made from a circular sheet of radius $\sqrt{3}$ by cutting out a sector and keeping the cut edges of the remaining piece together. Then find the maximum volume attainable for the cone

**35.** f (x) and g (x) are differentiable functions for $0 \le x \le 2$ such that f (0) = 5, g (0) = 0, f (2) = 8, g (2) = 1. Show that there exists a number c satisfying 0 < c < 2 and f ' (c) = 3 g' (c).

**36.** Find maximum value of function $g(x)=\frac{\log(\pi+x)}{\log(e+x)}$ $(0 \le x \le \pi e)$.