competishun — Power of real gurus **MATHEMATICS** **TARGET : JEE- Advanced 2023**	**CAPS-12** **MOD & AOD-1**

SCQ (Single Correct Type) :

1. The curves $C_1 : y = x^2 - 3$; $C_2 : y = kx^2$, $k < 1$ intersect each other at two different points. The tangent drawn to C_2, at one of the points of intersection A= (a, y_1) $(a > 0)$ meets C_1 again at $B(1, y_2)(y_1 \neq y_2)$. Then the value of a = ?

(A) 4 (B) 3 (C) 2 (D) 1

2. Tangent at P_1 other than origin on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve again at P_3 and so on, then $\frac{\text{area of } \Delta P_1P_2P_3}{\text{area of } \Delta P_2P_2P_4}$ equals

(A) 1 : 20 (B) 1 : 16 (C) 1 : 18 (D) 1 : 2

3. The portion of the tangent to the curve $x = \sqrt{a^2 - y^2} + \frac{a}{2}\log\frac{a - \sqrt{a^2 - y^2}}{a + \sqrt{a^2 - y^2}}$ between the point of contact and the x-axis is of length

(A) a^2 (B) 2a (C) a (D) 3a

4. The chord of the parabola $y = -a^2x^2 + 5ax - 4$ touches the curve $y = \frac{1}{1-x}$ at the point x = 2 and is bisected by that point. Then 'a' can be.

(A) 0 (B) 1 (C) 2 (D) 3

5. Angle between the curves $y = [|\sin x| + |\cos x|]$ where [.] denotes the greatest integer function and $x^2 + y^2 = 5$ is $\tan^{-1}(k)$ then k can be -

(A) -2 (B) 3 (C) $\frac{1}{3}$ (D) 2

6. Let f (x) = $\lim\limits_{h \to 0} \frac{(\sin(x+h))^{\ln(x+h)} - (\sin x)^{\ln x}}{h}$ then $f\left(\frac{\pi}{2}\right)$ is

(A) equal to 0 (B) equal to 1 (C) $\ln\frac{\pi}{2}$ (D) non existent

7. If $y = \frac{x}{\sqrt{a^2 - 1}} - \frac{2}{\sqrt{a^2 - 1}}\tan^{-1}\left(\frac{\sin x}{a + \sqrt{a^2 - 1} + \cos x}\right)$ where $a \in (-\infty, -1) \cup (1, \infty)$ then $y'\left(\frac{\pi}{2}\right)$ equals

(A) $\frac{1}{a}$ (B) $\frac{2}{a}$ (C) $\frac{1}{2a}$ (D) a

8. Let $f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then $f'\left(\frac{\pi}{2}\right) =$

(A) 0 (B) – 12 (C) 6 (D) 12

9. If a curve is represented parametrically by the equations x = f (t) and y = g(t) then $\left(\frac{d^2y}{dx^2}\right) \Big/ \left(\frac{d^2x}{dy^2}\right)$ is equal to (where f ' (t) ≠ 0, g ' (t) ≠ 0)

(A) 1 (B) $\frac{g'(t)}{f'(t)}$ (C) $-\left(\frac{g'(t)}{f'(t)}\right)^2$ (D) $-\left(\frac{g'(t)}{f'(t)}\right)^3$

10. If $f(x) = \sin^{-1}\left\{[3x+2] - \left\{3x + \left(x - \{2x\}\right)\right\}\right\}$, $x \in \left(0, \frac{\pi}{12}\right)$ and gof(x) = x, $x \in \left(0, \frac{\pi}{12}\right)$

then $g'\left(\frac{\pi}{6}\right)$ is equal to :

Note : {y} and [y] denote fractional part function and greatest integer function respectively.

(A) $\frac{\sqrt{3}}{8}$ (B) $\frac{-1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{-\sqrt{3}}{4}$

MCQ (One or more than one correct) :

11. The equation of a straight line which is tangent to one point and normal to the other point on the curve $x = 2t^2 + 1$ & $y = 4t^3$ is/are

(A) $\sqrt{2}x - y = \frac{31\sqrt{2}}{27}$ (B) $\sqrt{2}x - y = \frac{39\sqrt{2}}{27}$

(C) $\sqrt{2}x + y = \frac{31\sqrt{2}}{27}$ (D) $\sqrt{2}x + y = \frac{39\sqrt{2}}{27}$

12. Suppose ' f ' and ' g ' are functions having second derivatives f " and g" everywhere, if f (x).g(x) =1 for all ' x ' and f ’ and g’ are never zero, then $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$ equals

(A) $\frac{-2f'(x)}{f(x)}$ (B) $\frac{-2g'(x)}{g(x)}$ (C) $\frac{-f'(x)}{f(x)}$ (D) $\frac{2f'(x)}{f(x)}$

13. Let ' f ' be a real valued function defined on the interval (0, ∞) by $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt$.

Then which of the following statement(s) is/are true?

(A) f '(x) exists for all $x \in (0,\infty)$ and f ' is continuous on $(0,\infty)$, but not differentiable on $(0, \infty)$

(B) f "(x) exists for all $x \in (0,\infty)$

(C) There exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha,\infty)$

(D) There exists $\beta > 0$ such that $|f(x)| + |f'(x)| < \beta$ for all $x \in (0,\infty)$

14. Let a curve be given parametrically $x(t)=3t^2,\ y(t)=2t^3,\ t\in R$

Suppose a line L is tangent at one point and normal at another point of the curve. Then which of the following statements is/are correct?

(A) There are two possible situation for the line L

(B) A possible equation of line L is $\sqrt{2}x-y-2\sqrt{2}=0$

(C) A possible equation of line L is $\sqrt{2}x+y-2\sqrt{2}=0$

(D) A possible equation of line L is $x+\sqrt{2}y-2\sqrt{2}=0$

15. Two functions f & g have first & second derivatives at x = 0 & satisfy the relations,

$f(0)=\dfrac{2}{g(0)}$, $f'(0) = 2g'(0) = 4g(0)$, $g''(0) = 5f''(0) = 6f(0) = 3$ then

(A) if $h(x)=\dfrac{f(x)}{g(x)}$ then $h'(0)=\dfrac{15}{4}$ (B) if $k(x) = f(x) \cdot g(x)\sin x$ then $k'(0) = 2$

(C) $\displaystyle\operatorname{Limit}_{x\to 0}\frac{g'(x)}{f'(x)}=\frac{1}{2}$ (D) none

16. Which of the following statements are true?

(A) If $xe^{xy} = y + \sin^2 x$, then at x = 0, (dy/dx) = 1.

(B) If $f(x) \equiv a_0 x^{2m+1} + a_1 x^{2m} + a_2 x^{2m-1} + \ldots\ldots + a_{2m+1} = 0$ $(a_0 \neq 0)$ is a polynomial equation with rational co-efficients then the equation $f'(x) = 0$ must have a real root.($m\in N$).

(C) If $(x - r)$ is a factor of the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots.. + a_0$ repeated m times where $1 \le m \le n$ then r is a root of the equation $f'(x) = 0$ repeated (m – 1) times.

(D) If $y = \sin^{-1}(\cos\sin^{-1}x) + \cos^{-1}(\sin\cos^{-1}x)$ then $\dfrac{dy}{dx}$ is dependent on x.

Comprehension Type Question:

Comprehension # 1

To find the point of contact P = (x_1, y_1) of a tangent to the graph of y = f (x) passing through origin O, we equate the slope of tangent to y = f (x) at 'P' to the slope of OP. Hence, we solve the equation $f'(x_1)=\dfrac{f(x_1)}{x_1}$ to get x_1 and y_1 .

17. The equation $|\ln(mx)|$ = p x, where 'm' is a positive constant has a single root for

(A) $0<p<\dfrac{m}{e}$ (B) $p<\dfrac{e}{m}$ (C) $0<p<\dfrac{e}{m}$ (D) $p>\dfrac{m}{e}$

18. The equation $|\ln(mx)|$ = px, where 'm' is a positive constant has exactly two root for

(A) $p=\dfrac{m}{e}$ (B) $p=\dfrac{e}{m}$ (C) $0<p\le\dfrac{e}{m}$ (D) $0<p\le\dfrac{m}{e}$

19. The equation $|\ln(mx)| = px$, where 'm' is a positive constant has exactly three root for

(A) $p < \frac{m}{e}$ (B) $0 < p < \frac{m}{e}$ (C) $0 < p < \frac{e}{m}$ (D) $p < \frac{e}{m}$

Comprehension # 2

f(x) is a polynomial function $f : \rightarrow \mathbb{R} \rightarrow \mathbb{R}$ such that $f(2x) = f'(x)\, f''(x)$.

20. The value of f (3) is _____.

(A) 4 (B) 12 (C) 15 (D) 18

21. f(x) is ______.

(A) one-one and onto (B) one-one and into

(C) many-one and onto (D) many-one and into

22. Equation f(x) = x has ______.

(A) Three real and distinct roots (B) one real root

(C) Four real and distinct roots (D) Two real and distinct roots

Numerical based Questions :

23. Suppose a room, containing $12000 m^3$ of air is originally free of carbon monoxide. Beginning at t = 0, smoke containing 4% carbon monoxide is introduced into the room at a rate of $1.0 m^3$ / min and the well circulated mixture is allowed to leave the room at the same rate. Extended exposure to carbon monoxide concentration as much as 0.00012 is harmful to the human body. If T (in min) is the approximate time at which this concentration is reached, then $\left[\frac{T}{4}\right]$ (in minutes) (where [.] denotes the greatest integer function) is _______.

$(\ln(0.997) = -3.005 \times 10^{-3}$

24. The line y = mx +1 touches the curves $y = -x^4 + 2x^2 + x$ at two distinct points P (x_1, y_1) and Q (x_1, y_1) . The value of $x_1^2 + x_2^2 + y_1^2 + y_2^2$ is ____

25. If a curve is represented parametrically by the equations

$x = \sin\left(t + \frac{7\pi}{12}\right) + \sin\left(t - \frac{\pi}{12}\right) + \sin\left(t + \frac{3\pi}{12}\right)$, $y = \cos\left(t + \frac{7\pi}{12}\right) + \cos\left(t - \frac{\pi}{12}\right) + \cos\left(t + \frac{3\pi}{12}\right)$

then find the value of $\frac{d}{dt}\left(\frac{x}{y} - \frac{y}{x}\right)$ at $t = \frac{\pi}{8}$.

26. Find the derivative with respect to x of the function:

Let $f(x) = \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)$, find the value of $\frac{d(\tan f(x))}{d(\tan^{-1} x)}$ when $x = \frac{1}{\sqrt{2}}$.

27. If y = y(x) and it follows the relation $e^{xy} + y \cos x = 2$, then find y '' (0).

28. If $\lim\limits_{x\to 0} \dfrac{1-\cos 3x\cdot\cos 9x\cdot\cos 27x.........\cos 3^n x}{1-\cos\frac{1}{3}x\cdot\cos\frac{1}{9}x\cdot\cos\frac{1}{27}x.........\cos\frac{1}{3^n}x} = 3^{10}$, find the value of n.

Matrix Match Type :

29. Match the follwoing

Column-I	**Column-II**
(A) The number of non-differentiable points on the curve $y=\left\|e^{\|x\|}-4\right\|$ is	(p) 1
(B) Length of the latus rectum of the parabola defined by $x = \cos t - \sin t$ and $y = \sin 2t$ is	(q) 0
(C) The number of real solution of the equation $x^{2\log_3(x+3)}=16$ is	(r) 3
(D) If in a ΔABC, $2R = r_1 - r$, then $2\cos A$ is equal to	(s) 2

(A) A → (r); B → (p); C → (p); D →(q) (B) A → (q); B → (p); C → (q); D →(q)

(C) A → (r); B → (r); C → (q); D →(q) (D) A → (p); B → (p); C → (q); D →(q)

30. Column-I contains function defined on R and Column-II contains their properties. Match them.

Column-I	**Column-II**
(A) $\lim\limits_{n\to\infty}\left(\dfrac{1+\tan\frac{\pi}{2n}}{1+\sin\frac{\pi}{3n}}\right)^n$ equal	(P) e
(B) $\lim\limits_{x\to 0^+}\dfrac{1}{(1+\text{cosec}\,x)^{\frac{1}{\ln(\sin x)}}}$ equals	(Q) e^2
(C) $\lim\limits_{x\to 0}\left(\dfrac{2}{\pi}\cos^{-1}x\right)^{1/x}$ equals	(R) $e^{-2/\pi}$
	(S) $e^{\pi/6}$

(A) A → (S); B → (Q); C → (R) (B) A → (S); B → (P); C → (R)

(C) A → (Q); B → (P); C → (R) (D) A → (P); B → (Q); C → (S)