competishun — Power of real gurus **MATHEMATICS** TARGET : JEE- Advanced 2023	**CAPS-11** **CONTINUITY & DIFFERENEIABILITY**

SCQ (Single Correct Type) :

1. The value of f(0), so that the function $f(x) = \dfrac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$ (a > 0) becomes continuous for all x, is given by -

(A) $a\sqrt{a}$ (B) $\sqrt{a}$ (C) $-\sqrt{a}$ (D) $-a\sqrt{a}$

2. Let f(x) = [cos x + sinx] , 0 < x < 2π, where [.] denotes G.I.F. The number of points of discontinuity of f(x) is-

(A) 6 (B) 5 (C) 4 (D) 3

3. The function $f(x) = \begin{cases} x^2\left[\dfrac{1}{x^2}\right] & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, is (where [.] denotes G.I.F.)

(A) Continuous at x = 1 (B) Continuous at x = – 1

(C) Discontinuous at x = 0 (D) Continuous at x = 2

4. If $f(x) = [x^2] + \sqrt{\{x\}^2}$, where [.] and {.} denote the greatest integer and fractional part functions respectively, then-

(A) f(x) is continuous at all integral points except 0

(B) f(x) is continuous and differentiable at x = 0

(C) f(x) is discontinuous for all $x \in I - \{1\}$

(D) f(x) is not differentiable for all $x \in I$.

5. If f(x) is a continuous function for all real values of x satisfying $x^2 + (f(x) - 2)x + 2\sqrt{3} - 3 - \sqrt{3}\,f(x) = 0$, then the value of $f(\sqrt{3})$ is -

(A) $\sqrt{3}$ (B) $1 - \sqrt{3}$ (C) $2(1 - \sqrt{3})$ (D) $2(\sqrt{3} - 1)$

6. If g(x) is a polynomial satisfying g(x)g(y) = g(x) + g(y) + g(xy) − 2 for all real x and y and g(2) = 5, then $\lim_{x\to 3} g(x)$ is ______.

(A) 9 (B) 25 (C) 10 (D) none of these

7. A differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfies f (xy) = f (x) + f(y) $\forall$ x, $y \in \mathbb{R}^+$. If f(16) = 3, then the value of f(2) is _____.

(A) $\dfrac{3}{8}$ (B) $\dfrac{3}{4}$ (C) $\sqrt{3}$ (D) $\dfrac{3}{2}$

8. Let f(x), $f:\mathbb{R}\to\mathbb{R}$ be a non-constant continuous function such that f(2x) = (e^x+1) f(x). the value of f'(0) is ______ .

(A) $\lim\limits_{h\to 0}\frac{f(h)}{e^h+1}$ (B) $\lim\limits_{h\to 0}\frac{f(h)}{e^h-1}$ (C) $\lim\limits_{h\to 0}\frac{f(h)}{e^h-h-1}$ (D) $\lim\limits_{h\to 0}\frac{f(h)}{e^{-h}-1}$

9. If f(x) be positive, continuous and differentiable on the interval (a, b). If $\lim\limits_{x\to a^+} f(x)=1$ and $\lim\limits_{x\to b^-} f(x)=3^{\frac{1}{4}}$ also $f'(x)>(f(x))^3+\frac{1}{f(x)}$ then _____.

(A) $b-a>\frac{\pi}{24}$ (B) $b-a<\frac{\pi}{24}$ (C) $b-a=\frac{\pi}{12}$ (D) $b-a=\frac{\pi}{24}$

10. If f(x) = sgn ($\sin^2 x$ − sin x −1) has exactly four points of discontinuity for $x\in(0, n\pi)$ $n\in\mathbb{N}$ then n can be

(A) only 4 (B) 4 or 5 (C) only 5 (D) 5 or 6

MCQ (One or more than one correct) :

11. Suppose that f(x) is a differentiable invertible function with $f'(x)\neq 0$ and $h(x)=\int_1^x f(t)\,dx$. Given that $f(1)=f'(1)=1$ and g(x) is inverse of f(x). Let G(x) = x^2g(x)–xh(g(x)) $\forall x\in\mathbb{R}$. Which of the following are correct?

(A) G'(1) = 2 (B) G'(1) = 3 (C) G"(1) = 2 (D) G"(1) = 3

12. consider the function $f(x)=\begin{cases}\int_0^x (4+|t-2|)dt, & x>3\\ ax^2+bx & x\le 3\end{cases}$. If f(x) is differentiable at x = 3, then ____.

(A) $a+b=\frac{83}{18}$ (B) $a+b=\frac{85}{18}$ (C) $ab=\frac{7}{27}$ (D) $\frac{b}{a}=84$

13. If $f(x)=\left|x^2-4|x|+3\right|$ then _____.

(A) f(x) is non-differentiable at 5 points (B) f(x) is non-differentiable at 4 points

(C) f(x) has local maxima at x = 0 (D) f(x) has local minima at x = –1

14. If $f(x)=\begin{cases} a+\frac{\sin^3[x]}{x} & x>0\\ 3, & x=0\\ 2b+\left[\frac{\sin x-x}{x^3}\right] & x<0\end{cases}$ and f(x) is continuous at x = 0, then

(A) a = 2 (B) a = 3 (C) b = 2 (D) b = 3

15. If $f(x)=-1+|x-2|, 0\le x\le 4$ and $g(x)=2-|x|, -1\le x\le 3$ then (fog) (x) is ______.

(A) Discontinuous at x = 0 (B) Continuous at x = 0

(C) Not differentiable at x = 0 (D) Differentiable at x = 0

16. Suppose f is a function that satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, Then _____.

(A) $f(x) > 0$ for $x > 0$ and $f(x) < 0$ for $x < 0$

(B) $f'(0) = 1$

(C) $f''(0) = 1$

(D) $f'''(0) = 6$

17. Let [x] denote the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then f(x) is :

(A) continuous at $x = 0$

(B) continuous in $(-1, 0)$

(C) differentiable at $x = 1$

(D) differentiable in $(-1, 1)$

Numerical based Questions :

18. A function $f : \mathbb{R} \to \mathbb{R}$ satisfies $\sin x \cos y\,(f(2x + 2y) - f(2x-2y)) = \cos x \sin y\,(f(2x+2y) + f(2x-2y))$. If $f'(0) = \frac{1}{2}$, then the value of $4f''(x) + f(x)$ is ______

19. Let f(x) be a real valued function not identically zero such that $f(x + y^3) = f(x) + [f(y)]^3 \;\forall\; x, y \in R$ and $f'(0) \geq 0$, then find f(10)

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23. If g(x) = $\begin{cases} \dfrac{1-a^x+xa^x.\ \ell na}{x^2a^x} &, \quad x<0 \\ \dfrac{(2a)^x - x\,\ell n 2a - 1}{x^2} &, \quad x>0 \end{cases}$

(where a > 0) , then find ‘a’ and g(0) so that g(x) is continuous at x = 0.

24. If f : R $\to$ ($-\pi$, π) be a derivable function such that f(x) + f(y) = $f\left(\dfrac{x+y}{1-xy}\right)$, xy < 1.

If f(1) = $\dfrac{\pi}{2}$ and $\lim\limits_{x\to 0} \dfrac{f(x)}{x}$ = 2 , find f(x).

25. Given f(x) = $\cos^{-1}\left(\text{sgn}\left(\dfrac{2[x]}{3x-[x]}\right)\right)$, where sgn ( ) denotes the signum function and [.] denotes the greatest integer function. Discuss the continuity and differentiability of f (x) at x = ± 1.