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MATHEMATICS

TARGET : JEE- Advanced 2023

CAPS-10

Limit

SCQ (Single Correct Type) :

1. If $[x]$ is the greatest integer less than or equal to x, then $\lim\limits_{x\to 0}\left[\dfrac{x^2}{\sin x \tan x}\right] =$

(A) –1 (B) 2 (C) 0 (D) 1

2. If $\lim\limits_{t\to x}\dfrac{e^t f(x) - e^x f(t)}{(t-x)(f(x))^2} = 2$ and $f(0) = \dfrac{1}{2}$, then find the value of f '(0).

(A) 4 (B) 2 (C) 0 (D) 1

3. The value of $\lim\limits_{x\to 0}\dfrac{\left(1+3x+2x^2\right)^{\frac{1}{x}} - \left(1+3x-2x^2\right)^{\frac{1}{x}}}{x}$ is _____.

(A) $-e^3$ (B) $3e^2$ (C) $\dfrac{1}{3}e^{-2}$ (D) $4e^3$

4. If $f(x) = \dfrac{\tan x}{x}$ then $\lim\limits_{x\to 0}\left(\left[f(x)\right] + x^2\right)^{\frac{1}{\{f(x)\}}} =$ (where [.] denotes the greatest integer function and {.} denotes fractional part)

(A) 3 (B) log3 (C) e^3 (D) Does not exist

5. Let $f(x) = \left|\cos^{-1}(\sin x)\right| - \left|\sin^{-1}(\cos x)\right|$ and $\dfrac{3\pi}{2} \le x \le 2\pi$ $\lim\limits_{x\to 2\pi}\dfrac{\sin x - \tan x}{[f(x)]^3} =$ _____.

(A) $\dfrac{-1}{16}$ (B) $\dfrac{1}{32}$ (C) $\dfrac{-1}{8}$ (D) $\dfrac{1}{16}$

6. The value of $\lim\limits_{x\to 1}\left(\dfrac{p}{1-x^p} - \dfrac{q}{1-x^q}\right)$; p, q $\in$ N equals

(A) $\dfrac{p+q}{2}$ (B) $\dfrac{pq}{2}$ (C) $\dfrac{p-q}{2}$ (D) $\sqrt{\dfrac{p}{q}}$

7. $\lim\limits_{n\to\infty}\dfrac{1^2\, n + 2^2\,(n-1) + 3^2\,(n-2) + \ldots\ldots + n^2 \cdot 1}{1^3 + 2^3 + 3^3 + \ldots\ldots + n^3}$ is equal to:

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{6}$

8. The value of $\lim\limits_{x\to\frac{1}{\sqrt{2}}}\dfrac{\cos^{-1}(2x\sqrt{1-x^2})}{x - \dfrac{1}{\sqrt{2}}}$ equals

(A) $2\sqrt{2}$ (B) $-2\sqrt{2}$ (C) 0 (D) does not exist

9. The value of $\lim\limits_{x\to\pi}\dfrac{1}{(x-\pi)}\left(\sqrt{\dfrac{4\cos^2 x}{2+\cos x}}-2\right)$ is

(A) 0 (B) 2 (C) – 2 (D) does not exist

10. For $n \in N$, let x_n be defined as $\left(1+\dfrac{1}{n}\right)^{n+x_n} = e$, then $\lim\limits_{n\to\infty} x_n$ equals

(A) 1 (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{e}$ (D) 0

MCQ (One or more than one correct) :

11. If $\lim\limits_{x\to1}\left(2-x+a[x-1]+b[1+x]\right)$ exists, then a and b can take the values _____, (where [ ] denotes the greatest integer function).

(A) $a=\dfrac{1}{3},\ b=1$ (B) $a=1,\ b=-1$ (C) $a=9,\ b=-9$ (D) $a=9,\ b=\dfrac{2}{3}$

12. If $A=\lim\limits_{x\to0}\dfrac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B=\lim\limits_{x\to0}\dfrac{[|x|]}{x}$, where [ ] represents GIF, then _____.

(A) A = 1 (B) A does not exist (C) B = 0 (D) B = 1

13. $\lim\limits_{x\to\infty}\left[mx\sin\dfrac{1}{x}\right]$ is equal to (where [.] denotes the greatest integer function and $m \in I$

(A) m if m < 0 (B) m – 1 if m > 0 (C) m – 1 if m < 0 (D) m if m > 0

14. If f : R $\to$ R be a differentiable function and f(0) = 0 and f'(0) = 1, then

$\lim\limits_{x\to0}\dfrac{\left[f(x)+f\left(\dfrac{x}{2}\right)+f\left(\dfrac{x}{3}\right)+...+f\left(\dfrac{x}{n}\right)\right]}{x}$, where $n \in N$, equals

(A) 0 (B) $1+\dfrac{1}{2}+\dfrac{1}{3}+....+\dfrac{1}{n}$

(C) ${}^nC_1-\dfrac{{}^nC_2}{2}+\dfrac{{}^nC_3}{3}-.....+(-1)^{n-1}\dfrac{{}^nC_n}{n}$ (D) does not exist

Comprehension Type Question:

Comprehension # 1

Let $f(x)=\dfrac{\sin^{-1}(1-\{x\})\times\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\times(1-\{x\})}$ where $\{x\}$ denotes the fractional part of x.

15. $L=\lim\limits_{x\to0^+} f(x)$ is equal to

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{2\sqrt{2}}$ (C) $\dfrac{\pi}{\sqrt{2}}$ (D) $\sqrt{2}\pi$

16. $R=\lim\limits_{x\to0^-} f(x)$ is equal to

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{2\sqrt{2}}$ (C) $\dfrac{\pi}{\sqrt{2}}$ (D) $\sqrt{2}\pi$

17. Which of the following is not true ?

(A) cos L < cos R (B) tan (2L) > tan R (C) sin L > sin R (D) None of these

Comprehension # 2

Let f(x) = $\lim_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}\right)^n$, g(x) = $\lim_{n\to\infty}\left(1-x+x\sqrt[n]{e}\right)^n$. Now, consider the function y = h(x), where

h(x) = $\tan^{-1}\left(g^{-1}f^{-1}(x)\right)$

18. $\lim_{n\to0^+}\frac{\ln(f(x))}{\ln(g(x))}$ is equal to _____.

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) 1

19. Domain of the function y = h(x) is _____.

(A) $(0,\infty)$ (B) R (C) (0, 1) (D) [0, 1]

20. Range of the function y = h(x) is _____.

(A) $\left(0,\frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2},0\right)$ (C) R (D) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Comprehension # 3

Column-I	Column-II
(I) If $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=0$, then	(i) a = 2 (P) $b\in R$
(II) $\lim_{x\to0}\left(1+ax+bx^2\right)^{\frac{2}{x}}=e^3$, then	(ii) $a=\frac{3}{2}$ (Q) $b=2$
(III) $\lim_{x\to0}\left(\frac{ae^x-b}{x}\right)=2$	(iii) a = 1 (R) b = –1
(IV) $\lim_{x\to-\infty}\left\{\sqrt{\left(x^2-x+1\right)}-ax-b\right\}=0$	(iv) a = –1 (S) $b=\frac{1}{2}$

21. Which of the following is correct?

(A) (III) – (ii) – (R) (B) (III) – (iv) – (R) (C) (III) – (i) – (Q) (D) (III) – (ii) – (S)

22. Which of the following is correct?

(A) (II) – (ii) – (P) (B) (II) – (ii) – (R) (C) (II) – (iii) – (Q) (D) (II) – (iv) – (S)

23. Which of the following is correct?

(A) (I) – (ii) – (R) (B) (I) – (iii) – (R) (C) (I) – (iii) – (S) (D) (I) – (iv) – (P)

Comprehension # 4

Given $g(x)=\sin\frac{1}{x}, h(x)=x^2+2x+(\lambda+1)$ and $u(x)=\frac{1}{x}+\cos\frac{1}{x^2}$ Let

$f(x)=\lim_{n\to\infty}\frac{x^{2n+1}g(x)+h(x)}{x^{2n}+3x.u(x)}, n\in I$

On the basis of above information, answer the following questions:

24. $\lim_{x\to\infty}f(x)$ is equal to :

(A) 0 (B) 2 (C) 5 (D) 1

25. If $\lim_{x\to0}f(x)=2$, then the value of λ is ;

(A) 10 (B) 5 (C) 2 (D) does not exist

Numerical based Questions :

26. If $\lim\limits_{x\to0}\dfrac{\sin x+ae^{x}+be^{-x}+c\ln(1+x)}{x^{3}}$ exists, then find the value of a + b + c

27. Let K > 0 and $\lambda=\lim\limits_{n\to0}\dfrac{K\left(1-4\sqrt{K^{2}-x^{2}}\right)}{x^{2}\sqrt{K^{2}-x^{2}}}$ is finite then the value of λK is

28. If $f(x)=\begin{cases}x-1, & x\ge1\\ 2x^{2}-2, & x<1\end{cases}$, $g(x)=\begin{cases}x+1, & x>0\\ -x^{2}+1, & x\le0\end{cases}$ and $h(x)=|x|$,

then $\lim\limits_{x\to0}$ f(g(h(x))) is equal to :

29. Let $\underset{x\to\infty}{\text{Lim}}\ x\ \ell n\left(e\left(1+\dfrac{1}{x}\right)^{1-x}\right)$ equal $\dfrac{m}{n}$ where m and n are relatively prime positive integer.

Find (m + n).

30. Let $f(x)=ax^{3}+bx^{2}+cx+d$ and $g(x)=x^{2}+x-2$.

If $\underset{x\to1}{\text{Lim}}\dfrac{f(x)}{g(x)}=1$ and $\underset{x\to-2}{\text{Lim}}\dfrac{f(x)}{g(x)}=4$, then find the value of $\dfrac{c^{2}+d^{2}}{a^{2}+b^{2}}$

31. If $\underset{x\to3}{\text{Lim}}\left(\dfrac{\sqrt{2x+3}-x}{\sqrt{x+1}-x+1}\right)^{\frac{x-1-\sqrt{x^{2}-5}}{x^{2}-5x+6}}$ can be expressed in the form $\dfrac{a\sqrt{b}}{c}$ where a, b, c $\in$ N,

then find the least value of $(a^{2}+b^{2}+c^{2})$.

32. Let $\underset{x\to1}{\text{Lim}}\dfrac{x^{6}+1+\frac{5}{3}\left(x^{4}+x^{2}\right)-\frac{8}{3}(x^{5}+x)}{x^{5}+x+6x^{3}-4x^{4}-4x^{2}}=\dfrac{p}{q}$ where p, q $\in$ N.

Find the smallest value of (p + q).

Matrix Match Type :

33. Match the following

Column – I	**Column - II**
(A) $\lim\limits_{x\to0}\dfrac{\cos^{2}x-1}{x+\sin x\cos x}$	(P) 8
(B) $\lim\limits_{x\to0}\dfrac{\tan[-\pi^{2}]x^{2}-[-\pi^{2}]x^{2}}{\sin^{2}x}$ [.] denotes greatest integer function	(Q) 0
(C) $\lim\limits_{x\to0}\left[\dfrac{x^{2}}{\sin x\tan x}\right]$ where [] denotes G.I. function	(R) 4
(D) $\lim\limits_{x\to0}\dfrac{2\sin^{2}x}{2-\sqrt{2(1+\cos x)}}$	(S) –1

34. **Column-I** **Column-II**

(A) If $L = \lim\limits_{x \to -1} \frac{\sqrt[3]{(7-x)} - 2}{(x+1)}$, then 12 L = (P) – 2

(B) If $L = \lim\limits_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$, then $\frac{-L}{4}$ = (Q) 2

(C) If $L = \lim\limits_{x \to 1} \frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2 + x - 3}$, then 20L = (R) 1

(D) If $L = \lim\limits_{x \to \infty} \frac{\log x^n - [x]}{[x]}$, where $n \in N$, then – 2L = (S) –1

(Note : [x] denotes greatest integer less than or equal to x)

35. **Column-I** **Column-II**

(A) If $\lim\limits_{x \to \infty} \left(\sqrt{x^2 - x - 1} - ax - b\right) = 0$, where a > 0, then there exists at least one a and b for which point (a, 2b) lies on the line. (P) y = –3

(B) If $\lim\limits_{x \to 0} \frac{(1+a^3) + 8e^{1/x}}{1 + (1-b^3)\, e^{1/x}} = 2$, then there exists at least one a and b for which point (a, b^3) lies on the line. (Q) 3x – 2y – 5 = 0

(C) If $\lim\limits_{x \to \infty} \left(\sqrt{\left(x^4 - x^2 + 1\right)} - ax^2 - b\right) = 0$, then there exists at least one a and b for which point (a, –4b) lies on the line. (R) 15x – 2y – 11 = 0

(D) If $\lim\limits_{x \to -a} \frac{x^7 + a^7}{x + a} = 7$, where a < 0, ten there exists at least one a for which point (a, 2) lies on the line. (S) y = 2