

Que Find number of solution of the system of equation

$$x + 2y + 3z = 5, \quad (\lambda + 1)x + (3\lambda + 1)y + (\lambda + 5)z = 2\lambda + 1,$$

$$(\lambda + 2)x + (\lambda + 5)y + (2\lambda + 7)z = \lambda + 14.$$

Sol

$$x + 2y + 3z = 5$$

$$(\lambda + 1)x + (3\lambda + 1)y + (\lambda + 5)z = 2\lambda + 1$$

$$(\lambda + 2)x + (\lambda + 5)y + (2\lambda + 7)z = \lambda + 14$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ \lambda + 1 & 3\lambda + 1 & \lambda + 5 \\ \lambda + 2 & \lambda + 5 & 2\lambda + 7 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ \lambda + 1 & \lambda - 1 & -2\lambda + 2 \\ \lambda + 2 & -\lambda + 1 & -\lambda + 1 \end{vmatrix}$$

$$= 0 \Rightarrow (\lambda - 1)(1 - \lambda) - 2(\lambda - 1)(\lambda - 1)$$

$$3(\lambda - 1)^2 = 0 \Rightarrow \lambda = 1$$

C-1 If $\lambda \neq 1$ then

No. of solution = 1

C-2 If $\lambda = 1$ then System of equation will become

$$\begin{cases} x + 2y + 3z = 5 \\ 2x + 4y + 6z = 3 \\ 3x + 6y + 9z = 15 \end{cases} \Rightarrow x + 2y + 3z = 3/2$$

No. of soln = 0

Ques

Let M be a 3×3 matrix satisfying

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{ and } M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}. \text{ Then the sum of the diagonal entries of } M \text{ is}$$

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$M \cdot \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -1 & 3 & 12 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -1 & 3 & 12 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$[A : I]$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 5 & 1 & 0 & 1 & 0 \\ 7 & 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{tr}(M) = 0 + 2 + (-6 + 12) = 9$$

Ques $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1 , U_2 , and U_3 are columns matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and

$AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. If U is 3×3 matrix whose columns are U_1 , U_2 , U_3 then answer the following questions

The value of $|U|$ is

(A) 3

(B) -3

(C) $3/2$

(D) 2

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

(A)

$$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$AU = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow |A| \cdot |U| = |R.H.S.]$$

$$1. |U| = 3 \Rightarrow |U| = 3$$

Ques

Let p be an odd prime number and T_p be the following set of 2×2 matrices:

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \dots, p-1\} \right\}$$

The number of A in T_p such that A is either symmetric or skew-symmetric or both, and $\det(A)$ divisible by p is

(A) $(p-1)^2$

(B) $2(p-1)$

(C) $(p-1)^2 + 1$

(D) $2p-1$

$p \rightarrow$ odd prime

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \dots, p-1\} \right\}$$

$A \rightarrow$ symt., skew sym both.

$|A|$ is div. by ' p '

for symt. matrix

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$|A| = a^2 - b^2 = (a-b)(a+b)$$

C-1 If $a=b$ then $p \mid |A| \Rightarrow$ No. of matrices $= p$.

C-2 If $p \mid a+b$ then $a+b=p$ (leave 0 as it is covered in C-1)
 $\hookrightarrow$ No. of soln $= p-1$

Total no. of matrices $= p + p-1 = 2p-1$



Ques

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \dots, p-1\} \right\}$$

The number of A in T_p such that the trace of A is not divisible by p but $\det(A)$ is divisible by p is

(A) $(p-1)(p^2 - p + 1)$

(B) $p^3 - (p-1)^2$

(C) $(p-1)^2$

(D) $(p-1)(p^2 - 2)$

Sol

$$A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$$

$$p \nmid \text{tr}(A) \Rightarrow p \nmid 2a$$

$$\Rightarrow a \not\equiv 0.$$

$$p \mid |A| \Rightarrow p \mid a^2 - bc$$

$$\downarrow$$
$$k_1 p + r_1$$

$$\downarrow$$
$$k_2 p + r_2$$

$$1, 2, 3, \dots, (p-1)$$

$$a^2 = kp + \boxed{r_1}$$

$$(1, 2, \dots, p-1)$$

$$1 \cdot b, 2 \cdot b, 3 \cdot b, \dots, (p-1) \cdot b$$
$$2 \quad 3 \quad 1 \quad 4 \quad p-1$$
$$ib, jb$$

$$ib - jb = \boxed{(i-j)b}$$

$$\begin{matrix} a & b & c \\ \downarrow & \downarrow & \downarrow \\ (p-1) & (p-1) & 1 \end{matrix}$$

$$(p-1)^2 \cdot \text{C}$$

Que

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \dots, p-1\} \right\}$$

The number of A in T_p such that $\det(A)$ is not divisible by p is

(A) $2p^2$

(B) $p^3 - 5p$

(C) $p^3 - 3p$

(D) $p^3 - p^2$

$|A| = a^2 - bc$ is not div. by p .

$a=0$
 $b, c \neq 0$

$1 \cdot (p-1) \cdot (p-1)$
 $\leq (p-1)^2$

$a \neq 0$

$(p-1) \cdot (p \cdot p) - (p-1)^2$

$\downarrow +$

$R.Ans = p^2(p-1) = \underline{p^3 - p^2}$

D

Ques

Let

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

, Then find the minimum

value of Δ if ab is constant.

Sol

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$C_1 \rightarrow C_1 - bC_3$
 $C_2 \rightarrow C_2 + aC_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(2-1+a^2+b^2) & a(-2+1-a^2-b^2) & 1-a^2-b^2 \end{vmatrix}$$

$\begin{matrix} \underbrace{2-1+a^2+b^2}_{=1+a^2+b^2} & \underbrace{-2+1-a^2-b^2}_{=-(1+a^2+b^2)} \end{matrix}$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

$R_3 \rightarrow R_3 - bR_1$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 (1-a^2+b^2+2a^2)$$

$$= (1+a^2+b^2)^3 \geq \boxed{27a^2b^2}$$

$$\frac{1+a^2+b^2}{3} \geq \sqrt[3]{1 \cdot a^2 \cdot b^2}$$

$$1+a^2+b^2 \geq 3 \cdot (|ab|)^{2/3}$$

$$(1+a^2+b^2)^3 \geq 27 |ab|^2$$

Que

Let $m, p \in \mathbb{N}$ such that $m \geq p+2$, Now let

$$\Delta(m, p) = \begin{vmatrix} {}^m C_p & {}^m C_{p+1} & {}^m C_{p+2} \\ {}^{m+1} C_p & {}^{m+1} C_{p+1} & {}^{m+1} C_{p+2} \\ {}^{m+2} C_p & {}^{m+2} C_{p+1} & {}^{m+2} C_{p+2} \end{vmatrix}$$

find λ if $\frac{\Delta(m, p)}{\Delta(m-1, p-1)} = \lambda$

$$r \cdot {}^n C_r = n \cdot {}^{n-1} C_{r-1}$$

and hence find $\Delta(m, p)$.

$$\begin{matrix} C_1 & \rightarrow & p \cdot C_1 & \textcircled{Q} & C_3 & \rightarrow & (p+2) C_3 \\ C_2 & \rightarrow & (p+1) C_2 & & & & \end{matrix}$$

Sol

$$\Delta(m, p) = \frac{1}{p \cdot (p+1) \cdot (p+2)}$$

$$= \frac{m(m+1)(m+2)}{p(p+1)(p+2)} \cdot \Delta(m-1, p-1) = \frac{{}^{m+2} C_3}{{}^{p+2} C_3} \cdot \Delta(m-1, p-1)$$

$$\begin{vmatrix} {}^{m-1} C_{p-1} & {}^{m-1} C_p & {}^{m-1} C_{p+1} \\ (m+1) {}^m C_{p-1} & (m+1) {}^m C_p & (m+1) {}^m C_{p+1} \\ (m+2) {}^{m+1} C_{p-1} & (m+2) {}^{m+1} C_p & (m+2) {}^{m+1} C_{p+1} \end{vmatrix}$$

$$\Delta(m, p) = \frac{m+2}{p+2} C_3 \Delta(m-1, p-1).$$

$$= \frac{m+2}{p+2} C_3 \cdot \frac{m+1}{p+1} C_3 \cdot \Delta(m-2, p-2).$$

$$= \frac{m+2}{p+2} C_3 \cdot \frac{m+1}{p+1} C_3 \cdot \frac{m}{p} C_3 \cdot \frac{m-1}{p-1} C_3 \cdots \frac{m-p+3}{3} C_3 \cdot \Delta(m-p, 0)$$

$$\Delta(m-p, 0) = \begin{vmatrix} m-p & m-p & m-p & \cdots & m-p \\ C_0 & C_1 & C_2 & \cdots & C_3 \\ m-p+1 & m-p+1 & m-p+1 & \cdots & m-p+1 \\ C_0 & C_1 & C_2 & \cdots & C_3 \\ m-p+2 & m-p+2 & m-p+2 & \cdots & m-p+2 \\ C_0 & C_1 & C_2 & \cdots & C_3 \end{vmatrix}$$

AB

Que

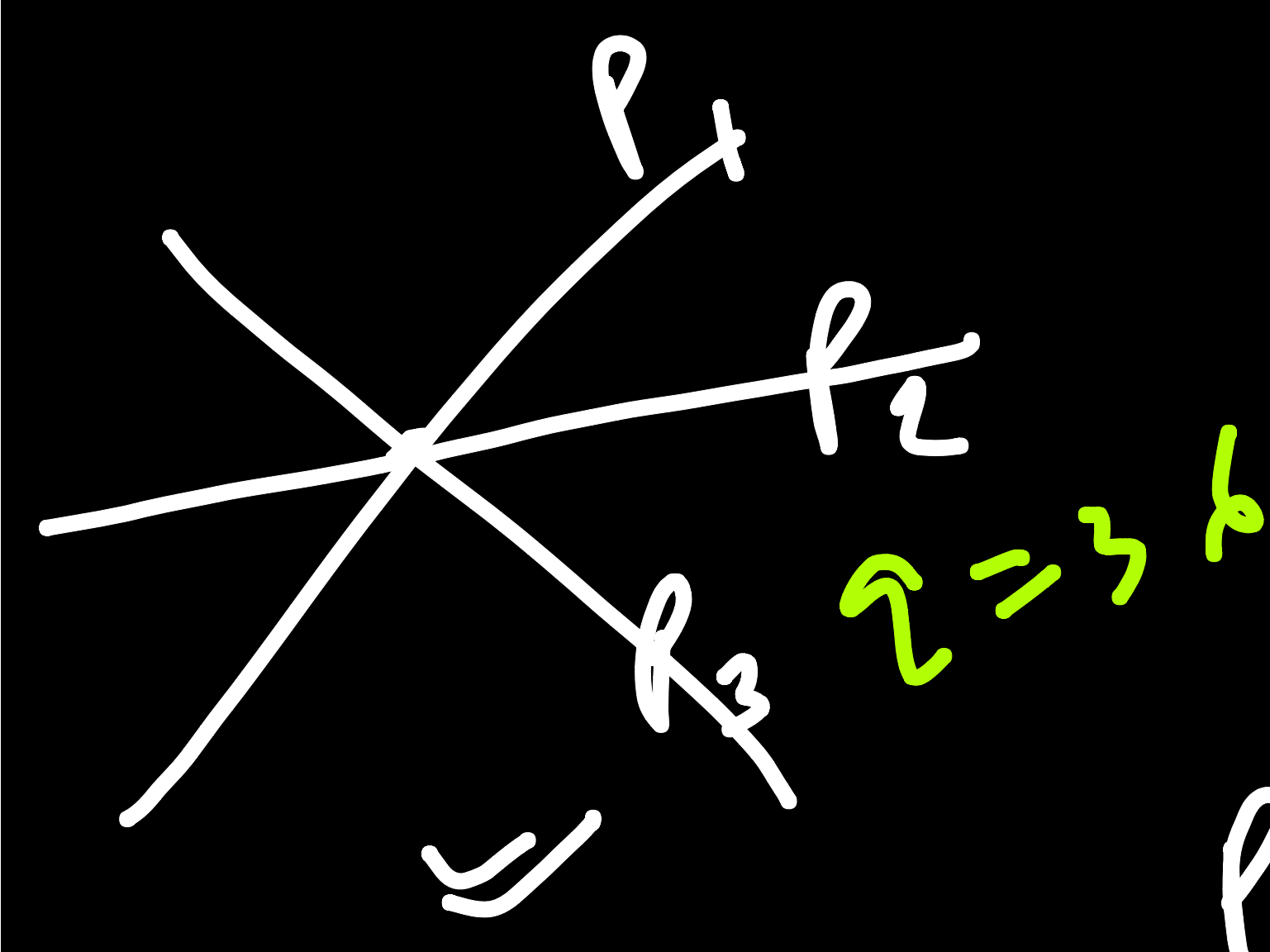
For a unique value of p and q , the system of equations given by
 $x + y + z = 6$
 $x + 2y + 3z = 14$
 $2x + 5y + pz = q$
has infinitely many solutions, then the value of $(p + q)$ is equal to
(A) 14 (B) 24 (C) 34 (D) 44

Sol

$$\begin{aligned}x + y + z &= 6 \\x + 2y + 3z &= 14 \\2x + 5y + pz &= q\end{aligned}$$

$$\Delta = 0 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & p \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 3 & p-2 \end{vmatrix}$$
$$= p - 2 - 6 \Rightarrow \boxed{p = 8}$$

$$\begin{aligned}x + y + z &= 6 \rightarrow p_1 \\x + 2y + 3z &= 14 \rightarrow p_2 \\2x + 5y + 8z &= q \rightarrow p_3\end{aligned}$$



~~$p_3 = \lambda p_1 + \mu p_2 = 0$~~

$$\begin{aligned}\lambda + \mu &= 2 \\x + 2\mu &= 5\end{aligned}$$
$$\mu = 3, \lambda = -1$$
$$q = 14 \times 3 - 6 = 36$$
$$q = 36$$

$$p + q = 8 + 36 = \boxed{44}$$

~~D~~

Ques Consider a system of equation $ax_1 + x_2 + x_3 = 1$, $x_1 + ax_2 + x_3 = 1$ and $x_1 + x_2 + ax_3 = 1$ then

- (A) if $a = 2$, then the system has unique solution.
 (B) if $a = 1$, then the system has infinite solutions.
 (C) if $a = -2$, then the system has no solution.
 (D) if $a = 2$, then the system has infinite solutions.

Sol

$$\begin{aligned} ax_1 + x_2 + x_3 &= 1 \\ x_1 + ax_2 + x_3 &= 1 \\ x_1 + x_2 + ax_3 &= 1 \end{aligned}$$

$$\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3 - 3a + 2$$

$$= a^3 + 2 - (3a)$$

$$= (a-1)(a^2 + a - 2)$$

$$= (a-1)^2(a+2)$$

$a = 1$

$$x_1 + x_2 + x_3 = 1 \Rightarrow \text{infinite sol}$$

$a = -2$

$$\begin{aligned} -2x_1 + x_2 + x_3 &= 1 \\ x_1 - 2x_2 + x_3 &= 1 \\ x_1 + x_2 - 2x_3 &= 1 \end{aligned}$$

$$\lambda r_1 + \mu r_2 = r_3$$

$$-2\lambda + \mu = 1$$

$$\lambda - 2\mu = 1$$

$$\begin{aligned} -3\lambda &= 3 \\ \lambda &= -1 \\ \mu &= -1 \end{aligned}$$

No sol

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$aei + dhc + b \cdot f \cdot g - (gec + hfa + ibd)$$

$$\begin{vmatrix} r_1 & r_2 & r_3 \\ r_1 & r_2 & r_3 \\ r_1 & r_2 & r_3 \end{vmatrix}$$

Ques

Let $a > 0, d > 0$. Find the value of the determinant

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{(a+d)} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{a+d} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{a+2d} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

$$= \frac{1}{a(a+d)^2(a+2d)^3(a+3d)^4(a+4d)} \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(a+3d) & a+3d & a+d \\ (a+3d)(a+4d) & a+4d & a+2d \end{vmatrix}$$

$$= \frac{2d}{()}$$

$$= \frac{2d}{()}$$

$$\begin{vmatrix} (a+d)(a+2d) & \downarrow a \\ (a+2d)(a+3d) & \downarrow a+d \\ (a+3d)(a+4d) & \downarrow a+2d \end{vmatrix}$$

$$\downarrow$$

$$\begin{vmatrix} (a+d)(a+2d) & 1 & a \\ (a+d)2d & 0 & d \\ (a+3d)2d & 0 & d \end{vmatrix}$$

$$= \frac{2d^3 \cdot 2d}{()} = \frac{4 \cdot d^4}{a \cdot (a+d)^2 (a+2d)^3 \cdot (a+3d)^2 (a+4d)}$$

Que

If α , β and γ are such that $\alpha + \beta + \gamma = 0$, then prove that

$$\begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & 1 & \cos \alpha \\ \cos \beta & \cos \alpha & 1 \end{vmatrix} = 0.$$

Sol

$$\begin{aligned} & \begin{vmatrix} S^2(\alpha) + C^2(\alpha) & \cos(\alpha + \beta) & \cos(\alpha + \gamma) \\ \cos(\alpha + \beta) & \beta^2(\beta) + C^2(\beta) & \cos(\beta + \gamma) \\ \cos(\alpha + \gamma) & \cos(\beta + \gamma) & C^2(\gamma) + S^2(\gamma) \end{vmatrix} \\ &= \begin{vmatrix} S(\alpha) & C(\alpha) & 0 \\ S(\beta) & C(\beta) & 0 \\ S(\gamma) & C(\gamma) & 0 \end{vmatrix} \cdot \begin{vmatrix} S(\alpha) & C(\alpha) & 0 \\ S(\beta) & C(\beta) & 0 \\ \beta(\gamma) & C(\gamma) & 0 \end{vmatrix} \\ &= 0. \end{aligned}$$

Ques In a ΔABC , Evaluate
$$\begin{vmatrix} \sin(2A) & \sin(C) & \sin(B) \\ \sin(C) & \sin(2B) & \sin(A) \\ \sin(B) & \sin(A) & \sin(2C) \end{vmatrix}$$

Sol

$$= \begin{vmatrix} 2\sin(A)\cos(A) & \sin(A+B) & \sin(A+C) \\ \sin(A+B) & - & - \\ \sin(A+C) & - & - \\ \sin(A) & \cos(A) & 0 \\ \sin(B) & \cos(B) & 0 \\ \sin(C) & \cos(C) & 0 \end{vmatrix} \cdot \begin{vmatrix} \cos(A) & \sin(A) & 0 \\ \cos(B) & \sin(B) & 0 \\ \cos(C) & \sin(C) & 0 \end{vmatrix} = \boxed{0}$$

Ques Find the maximum possible value of a 3×3 determinant, whose all elements are either 1 or -1 .

$\rightarrow \Delta$ must be a multiple of '4'.

Sol

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= aei + dhc + gbf - gec - hfa - dbi$$

$$\begin{matrix} \pm 1 & \pm 1 & \pm 1 & \pm 1 & \pm 1 & \pm 1 \\ | & | & | & | & | & | \\ 1.1.1 & 1.1.1 & -1.1.1 & -1.1.1 & \cancel{1.1.1} & \cancel{1.1.1} \end{matrix}$$

max. value = 4.

$$= a \begin{vmatrix} e & f \\ g & h \end{vmatrix} - a \begin{vmatrix} b & c \\ h & i \end{vmatrix} + a \begin{vmatrix} b & c \\ e & f \end{vmatrix}$$

$$= a \left(\left(e - \frac{bf}{a} \right) \left(h - \frac{cg}{a} \right) - \left(f - \frac{cd}{a} \right) \left(h - \frac{bg}{a} \right) \right)$$

Ques

If

$$ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2 = ax_3^2 + by_3^2 + cz_3^2 = \lambda$$

and $ax_1x_2 + by_1y_2 + cz_1z_2 =$

$$ax_2x_3 + by_2y_3 + cz_2z_3 = ax_3x_1 + by_3y_1 + cz_3z_1$$

$$= \mu \text{ \& } \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = (\lambda - \mu) \left(\frac{\lambda + \theta\mu}{abc} \right)^{\frac{1}{\beta}}$$

then the value of $\theta + \beta$ is

- A) 4 B) 3 C) 2 D) 1

$$\begin{aligned} ax_1^2 + by_1^2 + cz_1^2 &= ax_2^2 + by_2^2 + cz_2^2 \\ &= ax_3^2 + by_3^2 + cz_3^2 = \lambda \end{aligned}$$

$$\begin{aligned} ax_1x_2 + by_1y_2 + cz_1z_2 &= \text{---} \\ &= \mu \end{aligned}$$

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}^2 = \frac{1}{abc} \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \begin{vmatrix} ax_1 & by_1 & cz_1 \\ ax_2 & by_2 & cz_2 \\ ax_3 & by_3 & cz_3 \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} \lambda & \mu & \mu \\ \mu & \lambda & \mu \\ \mu & \mu & \lambda \end{vmatrix}$$

$$\begin{aligned} &= \frac{1}{abc} \left(\lambda^3 + 2\mu^3 - 3\lambda\mu^2 \right) \\ &= \frac{1}{abc} (\lambda - \mu) (\lambda^2 - 2\mu^2 + \lambda\mu) \\ &= \frac{1}{abc} (\lambda - \mu) (\lambda - \mu) (\lambda + 2\mu) \\ &= \frac{(\lambda - \mu)^2 \cdot (\lambda + 2\mu)}{abc} \end{aligned}$$

A

Ques

The determinant

$$\Delta = \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix}$$

is divisible by

- A) x B) x^2 C) x^3 D) x^4

Sol

$$\Delta = \frac{1}{abc} \begin{vmatrix} a(a^2+x^2) & a^2b & a^2c \\ a & b^2 & b^2c \\ a & bc^2 & c(c^2+x^2) \end{vmatrix}$$

$$= \begin{vmatrix} a^2+x^2 & a^2 & a^2 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix} = (a^2+b^2+c^2+x^2) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$

$$= (a^2+b^2+c^2+x^2) \begin{vmatrix} 1 & 0 & 0 \\ b^2 & x^2 & 0 \\ c^2 & 0 & x^2 \end{vmatrix}$$

$$= (a^2+x^2) \cdot x^4$$

A, B, C, D

D