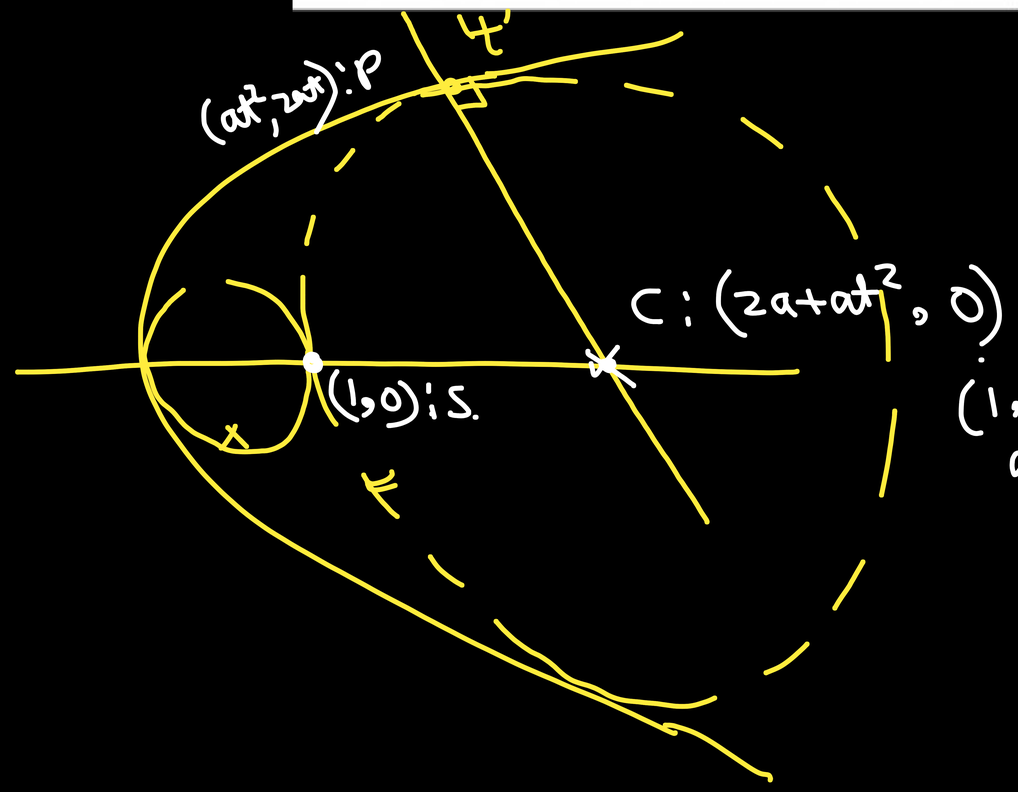
Que

Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is

(A) 8 (B) 4 (C) 2 (D) 5



radius of circle is $PC = \sqrt{(2a)^2 + (2at)^2}$.

$(1,0)$, $a=1$

$$(x - 2a - at^2)^2 + y^2 = (2a)^2 + (2at)^2.$$

$$(1 - 2 - t^2)^2 + 0 = 4 + 4t^2.$$

$$(1+t^2)^{\cancel{2}} = 4\cancel{(1+t^2)} \Rightarrow 1 + t^2 = 4$$

$$t^2 = 3.$$

$$r = \sqrt{4(1+t^2)} = \sqrt{4 \times 4} = \boxed{4}$$

B

Que

The locus of the vertices of the parabolas represented by the equation

$$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$$

(a is a non-zero parameter) is

A) a circle

B) a parabola

C) a straight line

D) $xy = c^2$ form where 'c' is constant

Sol

$$y = \left(\frac{a^3}{3}\right)x^2 + \left(\frac{a^2}{2}\right)x - 2a$$

$A = \frac{a^3}{3}$

$B = a^2/2$

$C = -2a$

$$D = \frac{a^4}{4} + 4 \cdot \frac{a^3}{3} \cdot 2a$$

$$= a^4\left(\frac{1}{4} + \frac{8}{3}\right) = \frac{35}{12}a^4.$$

$$V: \left(\underbrace{-\frac{B}{2A}}_{=x}, \underbrace{-\frac{D}{4A}}_{=y}\right)$$

$$x = -\frac{a^2}{2} \cdot \frac{3}{2a^3} = -\frac{3}{4a}.$$

$$y = -\frac{35}{\cancel{12}\,4} \cdot \frac{a^4}{4a^3} \times \cancel{3} = -\frac{35}{16} \cdot a.$$

$$\boxed{xy = \frac{35 \times 3}{64} = \frac{105}{64}}$$

D

$$\text{Product} = -\frac{4f}{a} = \cancel{t_1}\left(-t_1 - \frac{2}{t_1}\right)\cdot\left(\frac{2}{\cancel{t_1}}\right)$$

$$\frac{2f}{a} = -\left(t_1 + \frac{2}{t_1}\right)$$

$$\frac{4f^2}{a^2} = \underbrace{t_1^2 + \frac{4}{t_1^2}} + 4 = -\left(2 + \frac{4a + 2g}{a}\right) + 4$$

$$-g \to x$$
$$-f \to y$$

$$\frac{4y^2}{a^2} = 2 - \frac{4a - 2x}{a} = -2 + \frac{2x}{a}.$$

$$\boxed{2y^2 = ax - a^2.}$$

$$2t_1x - (t_1^2-1)y - 2at_1 = 0$$

$T_1(at_1t_2, a(t_1+t_2))$

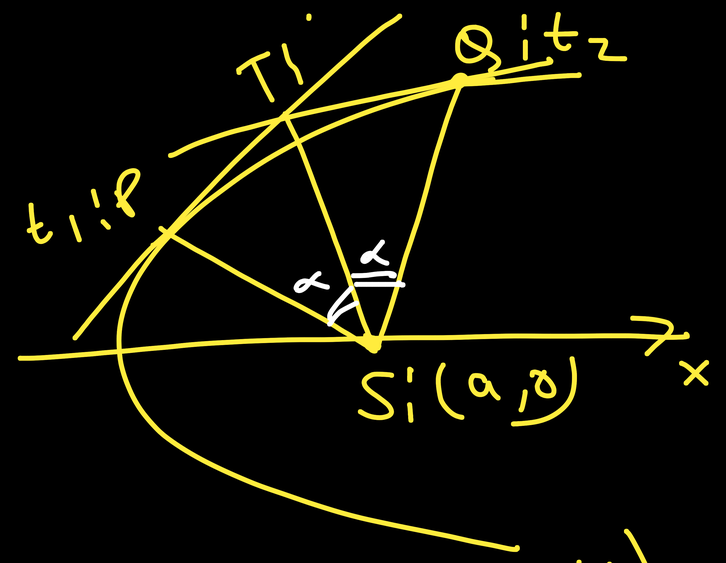
$$p_1 = \frac{\left|2t_1 a t_1 t_2 - (t_1^2-1)\cdot a(t_1+t_2) - 2a t_1\right|}{\sqrt{4t_1^2 + (t_1^2-1)^2}}$$

$$p_1 = \frac{a\left|\cancel{2}t_1^2t_2 - t_1^3 + t_1 - \cancel{t_1^2t_2} + t_2 - 2t_1\right|}{\sqrt{(t_1^2+1)^2}}$$

$$= \frac{a\left|t_1^2t_2 - t_1^3 + t_2 - t_1\right|}{t_1^2+1} = \frac{a\left|t_1^2(t_2-t_1) + 1\cdot(t_2-t_1)\right|}{(t_1^2+1)}$$

$$= \frac{a\cancel{(t_1^2+1)}\,|t_2-t_1|}{\cancel{(t_1^2+1)}} = a|t_2-t_1|$$

$$p_2 = a|t_2-t_1| \Rightarrow ST \text{ is angular bisector. } \underline{H.P.\ (i)}$$



$T(at_1t_2, a(t_1+t_2))$

$SP = a(1+t_1^2)$

$SQ = a(1+t_2^2)$

$SP \cdot SQ = a^2(1+t_1^2)(1+t_2^2)$

$\boxed{ST^2 = SP \cdot SQ}$

H.P. (ii)

$ST^2 = (at_1t_2 - a)^2 + (a(t_1+t_2) - 0)^2$

$= a^2((t_1t_2 - 1)^2 + (t_1+t_2)^2)$

$= a^2(t_1^2t_2^2 + 1 + t_1^2 + t_2^2)$

$= a^2(1+t_1^2)(1+t_2^2)$

(iii) $\frac{ST}{SP} = \frac{SQ}{ST}$ & $\alpha = \alpha$.

$\Downarrow$

$\Delta SPT \sim STQ$

H.P. (iii)

Que

If l, m are variable real numbers such that $3l^2+4m^2-lm-3m=0$, then variable line $lx+my=1$ always touches a fixed parabola,

[redacted]

① **Vertex of the parbola is**

A) $\left(\frac{1}{3},-\frac{4}{3}\right)$ B) $\left(-\frac{1}{3},\frac{4}{3}\right)$

C) $\left(\frac{4}{3},-\frac{1}{3}\right)$ D) $\left(-\frac{4}{3},\frac{1}{3}\right)$

② **Focus of the parabola is**

A) $\left(\frac{1}{3},-\frac{1}{3}\right)$ B) $\left(-\frac{1}{3},\frac{7}{3}\right)$

C) $\left(-\frac{1}{3},\frac{1}{3}\right)$ D) $\left(-\frac{7}{3},\frac{1}{3}\right)$

③ **Directrix of the parabola is**

A) $3y+7=0$ B) $3y+4=0$

C) $3y-4=0$ D) $3y-7=0$

$3l^2+4m^2-lm-3m=0$

$lx+my=1$

$l=\frac{1-my}{x}$

① → B

② → C

③ → D

Concept of Envelope

$y=mx+\frac{a}{m}$

$\downarrow (h,k)$

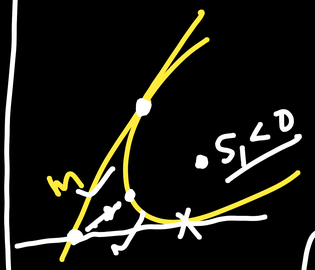
$k=mh+\frac{a}{m}$

$hm^2-km+a=0$

$D>0$

$k^2-4ah>0$

$S_1>0$



$D=0$

$$3l^2+4m^2-3m-lm=0$$

$$l=\frac{1-my}{x} \rightarrow 3\left(\frac{1+m^2y^2-2my}{x^2}\right)+4m^2-3m+m\left(\frac{my-1}{x}\right)=0$$

$$3+3m^2y^2-6my+4x^2m^2-3x^2m+xym^2-mx=0$$

$$m^2(3y^2+4x^2+xy)-m(6y+3x^2+x)+3=0$$

$$\rightarrow D=0 \Rightarrow \cancel{36y^2}+9x^4+x^2+6x^3+\cancel{12xy}+36yx^2$$

$$-12(\cancel{3y^2}+4x^2+\cancel{xy})=0$$

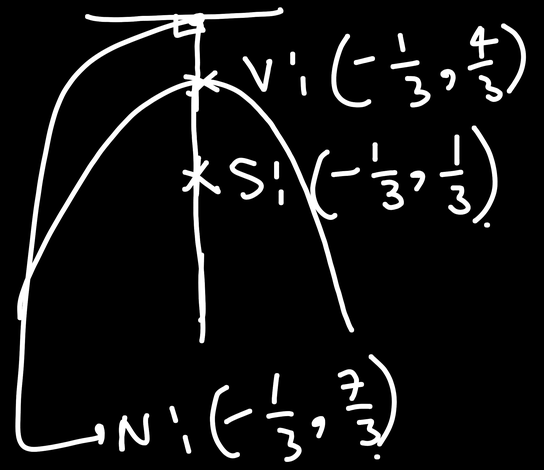
$$9x^4+x^2+6x^3+36x^2y-48x^2=0$$

$$\frac{1}{x^2}$$

$$9x^2+1+6x+36y-48=0 \quad \div 9$$

$$(3x+1)^2=-36\left(y-\frac{48}{36}\right)$$

$$\left(x+\frac{1}{3}\right)^2=-4\left(y-\frac{4}{3}\right)$$



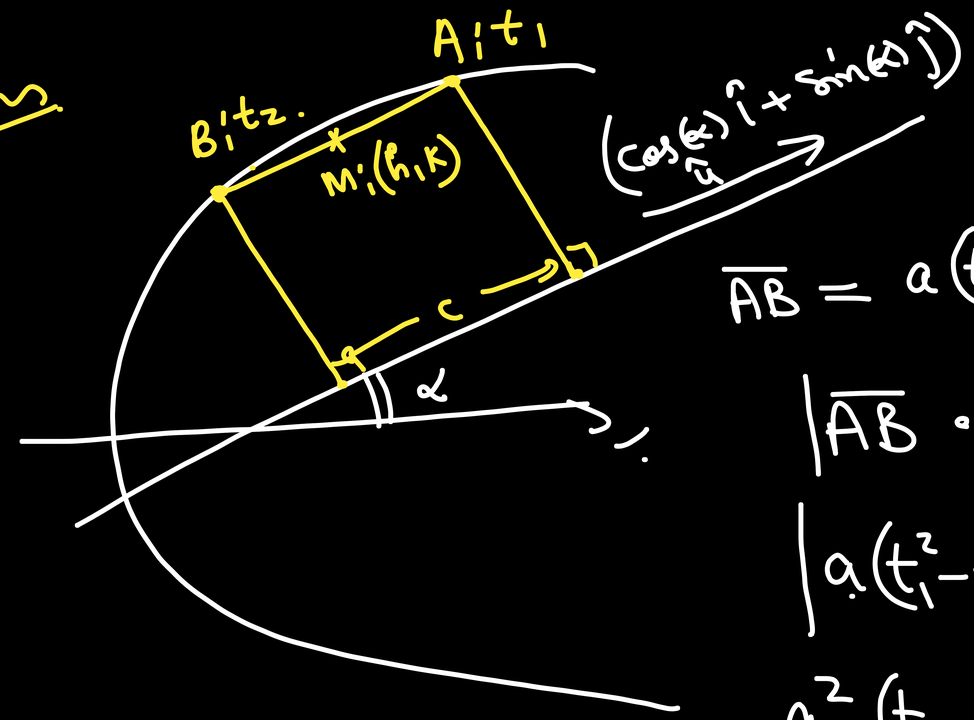
$y=7/3 \rightarrow$ Directrix

Que

A series of chords is drawn so that their projections on the straight line which is inclined at an angle α to the axis are of constant length c. Prove that the locus of their middle point is the curve

$$(y^2 - 4ax)(y\cos\alpha + 2a\sin\alpha)^2 + a^2c^2 = 0$$

Soln



$$h = \frac{a(t_1^2+t_2^2)}{2}, \quad k = a(t_1+t_2)$$

$$k^2 = a^2(t_1^2+t_2^2+2t_1t_2)$$

$$k^2 = a(2h) + 2a^2 t_1t_2.$$

$$(t_1-t_2)^2 = t_1^2+t_2^2-2t_1t_2.$$

$$\overline{AB} = a(t_1^2-t_2^2)\hat{i} + 2a(t_1-t_2)\hat{j}$$

$$|\overline{AB}\cdot\hat{u}| = c.$$

$$|a(t_1^2-t_2^2)\cdot\cos(\alpha) + 2a(t_1-t_2)\cdot\sin(\alpha)| = c.$$

$$a^2(t_1-t_2)^2\left((t_1+t_2)\cos(\alpha) + 2\sin(\alpha)\right)^2 = c^2.$$

$$a^2(t_1-t_2)^2\left((t_1+t_2)\cos(\alpha)+2\sin(\alpha)\right)^2=c^2$$

$$(t_1-t_2)^2=(t_1^2+t_2^2)-2t_1t_2$$

$$=\frac{2h}{a}-\cancel{2}\left(\frac{k^2-2ah}{\cancel{2}a^2}\right)$$

$$=\frac{2ah-k^2+2ah}{a^2}.$$

$$\cancel{a^2}\frac{(4ah-k^2)}{\cancel{a^2}}\left(\frac{k}{a}\cos(\alpha)+2\sin(\alpha)\right)^2=c^2$$

$$\Downarrow \quad k\longrightarrow y,\quad h\longrightarrow x$$

$$(4ax-y^2)\left(y\cos(\alpha)+2a\sin(\alpha)\right)^2=a^2c^2$$

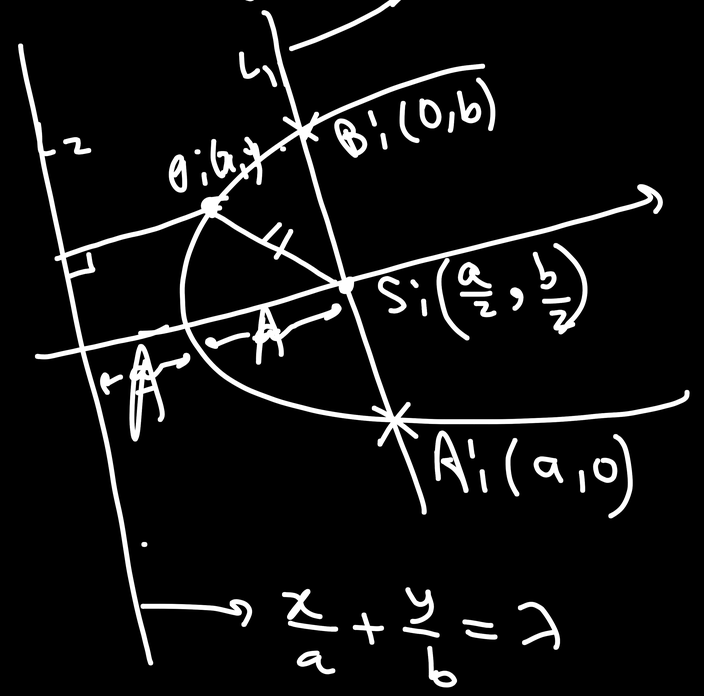
$$(y^2-4ax)\left(y\cos(\alpha)+2a\sin(\alpha)\right)^2+a^2c^2=0.$$

Que

A line AB makes intercepts of length a and b on the coordinate axes. Find the equation of the parabola passing through A, B and [redacted] if AB is the shortest focal chord of the parabola. Given A, B lie on X-axis & Y-axis resp.

$AB \equiv \frac{x}{a} + \frac{y}{b} = 1.$

Shortest focal chord
$\Rightarrow$ AB is L.R.



$4A = \sqrt{a^2+b^2}$

$2A = \frac{\sqrt{a^2+b^2}}{2}$

Distance b/w L_1 & L_2

$$\frac{|\lambda-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{\sqrt{a^2+b^2}}{2} \Rightarrow \lambda = 1 \pm \frac{a^2+b^2}{2ab}$$

$$= \pm\left(\frac{a^2+b^2}{2ab} \pm 1\right)$$

$$= \pm\left(\frac{(a\pm b)^2}{2ab}\right)$$

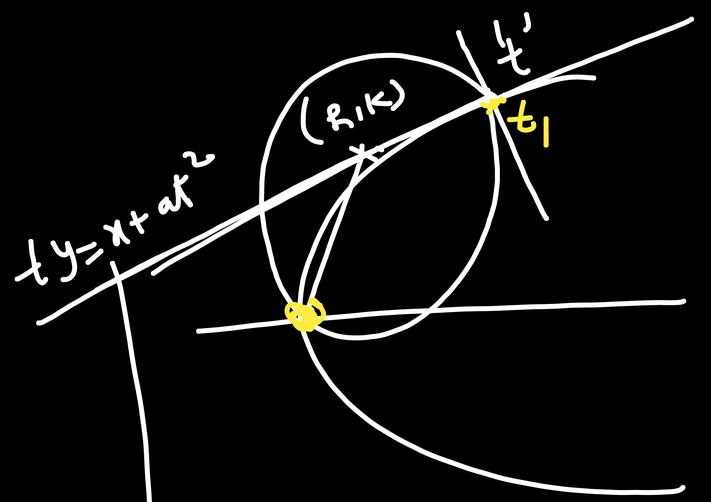
$PS^2 = PL^2$

$$\boxed{(x-\tfrac{a}{2})^2+(y-\tfrac{b}{2})^2 = \frac{\left(\frac{x}{a}+\frac{y}{b} \mp \frac{(a\pm b)^2}{2ab}\right)^2}{\frac{1}{a^2}+\frac{1}{b^2}}}$$

#

Que

Find the locus of centres of a family of circles passing through the vertex of the parabola $y^2 = 4ax$ and cutting the parabola orthogonally at the other point of intersection.



$$(x-h)^2 + (y-k)^2 = \left(\sqrt{h^2+k^2}\right)^2$$

$$x^2 + y^2 - 2hx - 2ky = 0.$$

$(at^2, 2at)$ $\rightarrow$ $a^2t^4 + 4at^2 - 2hat^2 - 4akt = 0.$

$\rightarrow$ $at^3 + (4-2h)t - 4k = 0 \rightarrow t_1, \ldots$ ②

$tk = h + at^2$

$at^2 - kt + h = 0 \rightarrow t_1$ ①

$\rightarrow$ $at^3 - kt^2 + ht$

② $- t \cdot$ ①

$(4-2h)t - 4k + kt^2 - ht = 0.$

$kt^2 + (4-3h)t - 4k = 0$ — ③

$$at^2 - kt + h = 0 \quad \text{—— (1)}$$

$$kt^2 + (4-3h)t - 4k = 0 \quad \text{—— (3)}$$

C.R.C.

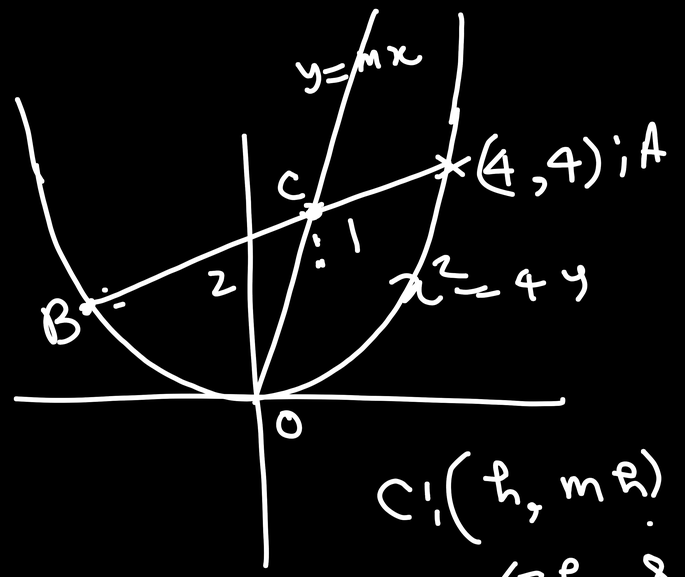
$$(hk + 4ak)^2 = \left(a(4-3h) + k^2\right)\left(4k^2 - h(4-3h)\right)$$

$$(h, k) \longrightarrow (x, y)$$

$$\boxed{y^2(x+4a)^2 = \left(y^2 + 4a - 3ax\right)\left(4y^2 - x(4-3x)\right)}$$

Que

If two chords drawn from the point (4, 4) to the parabola $x^2 = 4y$ are divided by line $y = mx$ in the ratio 1 : 2 then find the interval in which m lies.



$C'(h, mh)$

$B'\left(\frac{3h-8}{3-2}, \frac{3mh-8}{3-2}\right)$

$\equiv (3h-8, 3mh-8)$

$x^2=4y$

$(3h-8)^2 = 4(3mh-8)$

$9h^2+64-48h = 12mh-12.$

$9h^2-(48+12m)h+76=0.$

$D>0$

$\cancel{12^2}(m+4)^2-\cancel{4}\cdot\cancel{9}\cdot 76>0$

$19\times\cancel{4}$

$(m+4)^2>19$

$|m+4|>\sqrt{19}$

$\Rightarrow m\in(-\infty,-4-\sqrt{19})\cup(-4+\sqrt{19},\infty)$