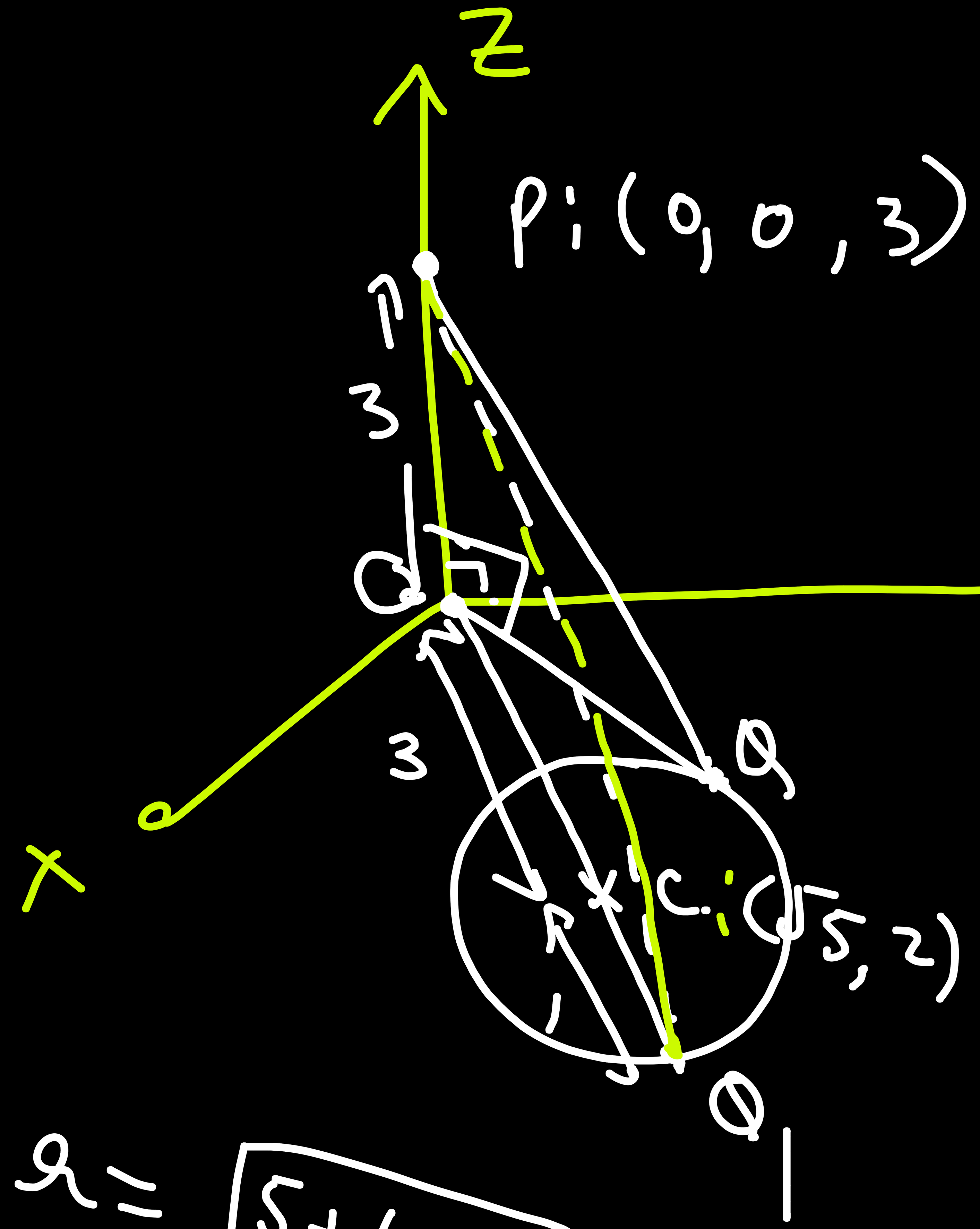
Que Find the maximum distance between the point $P:(0,0,3)$ and the circle $x^2+y^2-2\sqrt{5}x-4y+8=0$, $z=0$.

Soln



$r=\sqrt{5+4-8}$
$=1$

Let Q is any pt on circle and O is origin then

$PQ^2=\underbrace{OP^2}_{=9}+OQ^2$

$\Rightarrow$ for PQ to be maximum, OQ should be maximum.

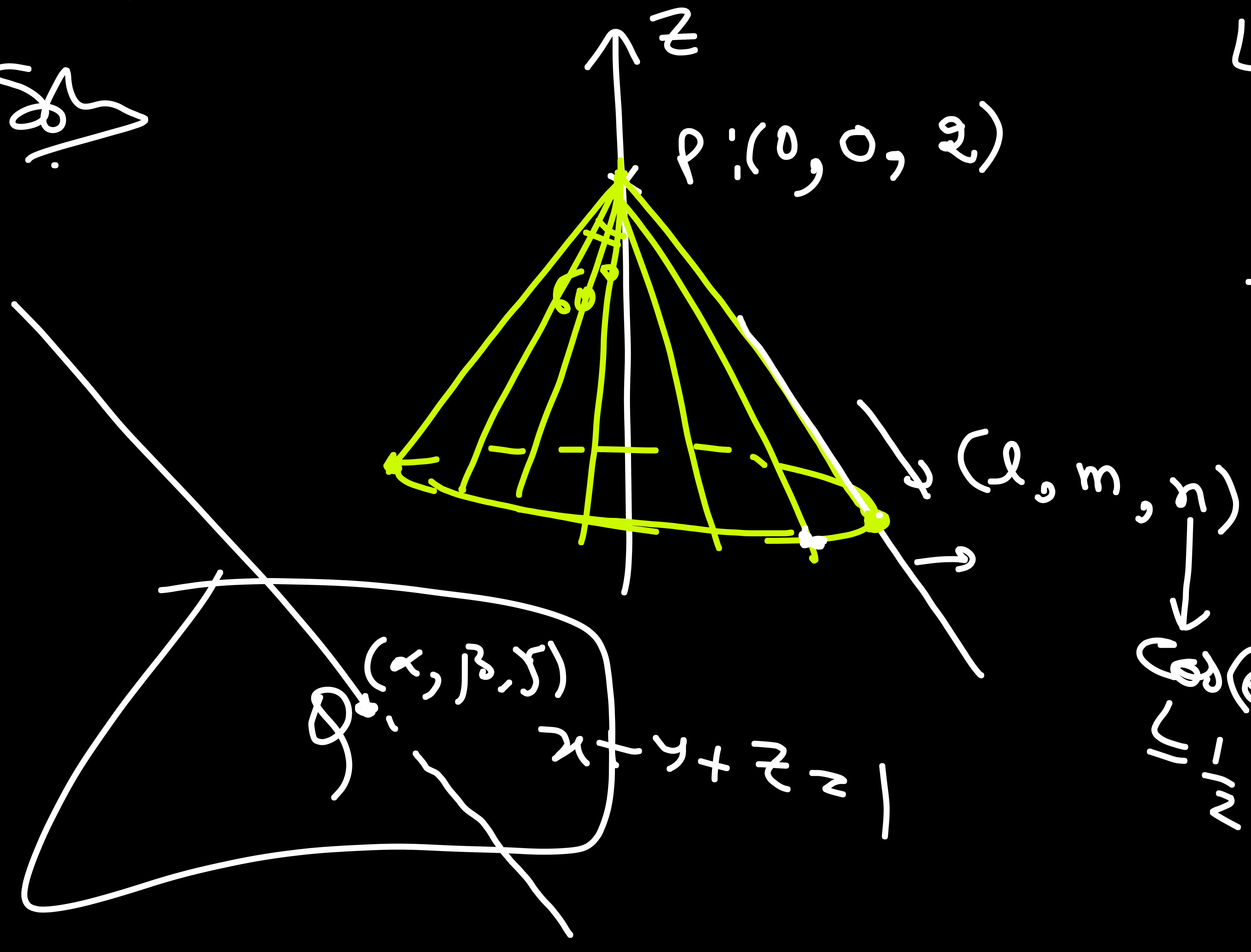
$PQ)_{max.}=\sqrt{OP^2+OQ_1^2}$

$=\sqrt{3^2+4^2}$

$=\boxed{5}$ Ans

Que A variable line passing through the point $P:(0,0,2)$ always makes angle 60° with z-axis then find locus of point of intersection of this variable line and plane $x+y+z=1$.

Sol ⇒



$\cos(60^\circ) = \frac{1}{2}$

Let direction cosine // to variable line is (l, m, n) or $(l, m, \frac{1}{2})$

⇒ Eqn of line is

$$L \equiv \frac{x-0}{l} = \frac{y-0}{m} = \frac{z-2}{1/2}$$

Let pt of x' of line L and given plane is $Q(\alpha, \beta, \gamma)$

$$\frac{\alpha}{l} = \frac{\beta}{m} = \frac{\gamma-2}{1/2} = \lambda \Rightarrow \alpha = l\lambda$$

$$\beta = m\lambda$$

$$\gamma = 2 + \lambda/2$$

also

$$\alpha+\beta+\gamma=1$$

$$l\lambda+m\lambda+2+\frac{\lambda}{2}=1$$

$$\lambda\left(l+m+\frac{1}{2}\right)=-1$$

$$\lambda=\frac{-1}{l+m+1/2}.$$

$$\alpha=l\lambda$$

$$\beta=m\lambda$$

$$\gamma=2+\frac{\lambda}{2}.$$

$$\Rightarrow 2.(\gamma-2)=\lambda$$

$$l^2+m^2+\left(\frac{1}{2}\right)^2=1$$

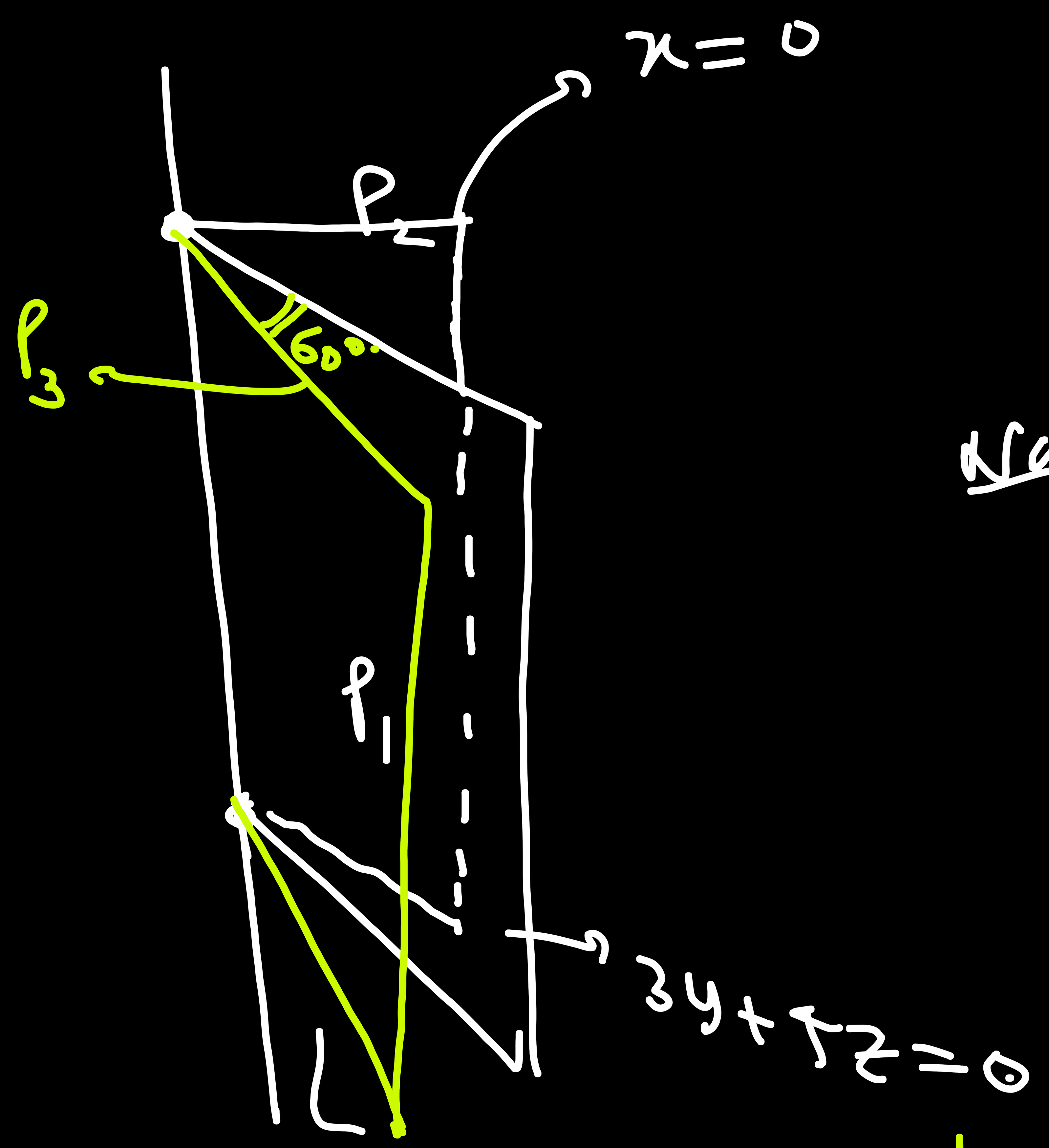
$$\Rightarrow l^2+m^2=\frac{3}{4} \Rightarrow \frac{\alpha^2}{\lambda^2}+\frac{\beta^2}{\lambda^2}=\frac{3}{4}$$

$$\alpha^2+\beta^2=\frac{3}{4}\cdot\lambda^2 \Rightarrow \alpha^2+\beta^2=\frac{3}{\cancel{4}}\,\cancel{4}\cdot(\gamma-2)^2$$

$$\Rightarrow \boxed{x^2+y^2=3(z-2)^2}$$ Ans

Que The plane $3y+4z=0$ is rotated about it's line of intersection with the plane $x=0$ through an angle 60°, then find it's equation in new position.

Sol



$L \rightarrow \bar{n}_1 = (0,3,4)$

Let required plane is $P_3 \equiv P_1 + \lambda P_2 = 0$

$3y+4z+\lambda x = 0 \longrightarrow \bar{n}_3 \equiv (\lambda, 3, 4)$

Now $\angle$ b/w P_1 & $P_3 = 60^\circ \Rightarrow \angle$ b/w $\bar{n}_1$ & $\bar{n}_3 = 60^\circ$.

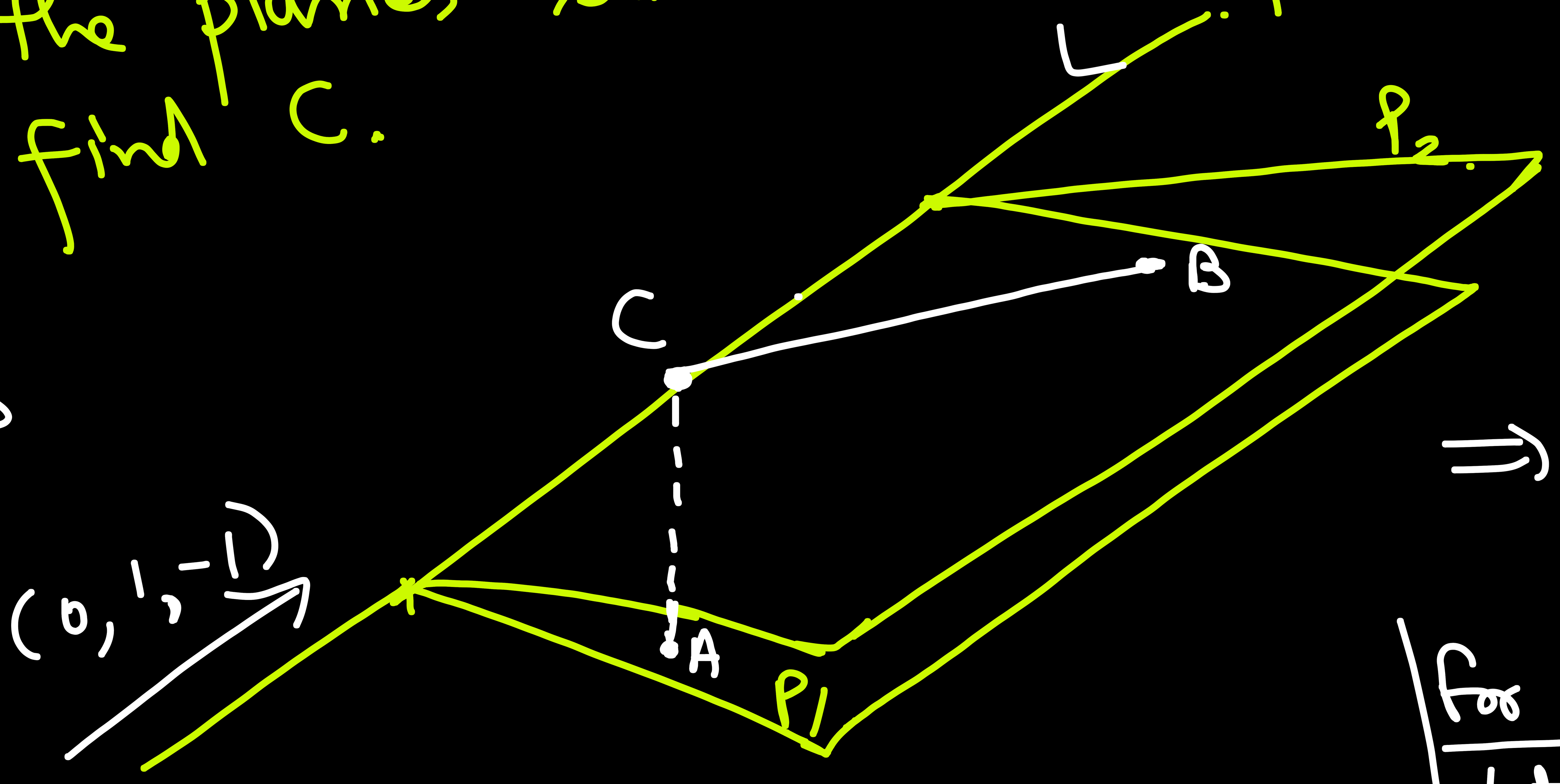
$\cos(60^\circ) = |\hat{n}_1 \cdot \hat{n}_2| \Rightarrow \pm\frac{1}{2} = \frac{\bar{n}_1 \cdot \bar{n}_2}{n_1 n_2}$

$\pm\frac{1}{2} = \frac{(0,3,4)\cdot(\lambda,3,4)}{5 \cdot \sqrt{\lambda^2+25}} = \frac{\cancel{25}\ 5}{\cancel{5}\sqrt{\lambda^2+25}}$

$\lambda^2+25 = 100 \Rightarrow \lambda^2 = 75 \Rightarrow \lambda = \pm 5\sqrt{3}$

R-Plane $= \boxed{3y+4z \pm 5\sqrt{3}\,x = 0}$ Ans

Que A : $(3,-1,-1)$ and B : $(3,1,0)$ lie on $x+y+z=1$ and $2x-y-z=5$ respectively. If C is a point lying on both the planes such that the perimeter of ΔABC is minimum then find C.



Sol

Acc. to question we need to minimize $AB+BC+CA$

$\Rightarrow$ We need to minimize $\underline{AC+BC}$ (as AB is const)

$P_1 \equiv x+y+z=1 \rightarrow \bar{n}_1=(1,1,1)$

$P_2 \equiv 2x-y-z=5 \rightarrow \bar{n}_2=(2,-1,-1)$

$\bar{n}_1 \times \bar{n}_2 \equiv (0,3,-3)$

$\equiv (0,1,-1)$

For 1 - pt on L.

let $z=0$

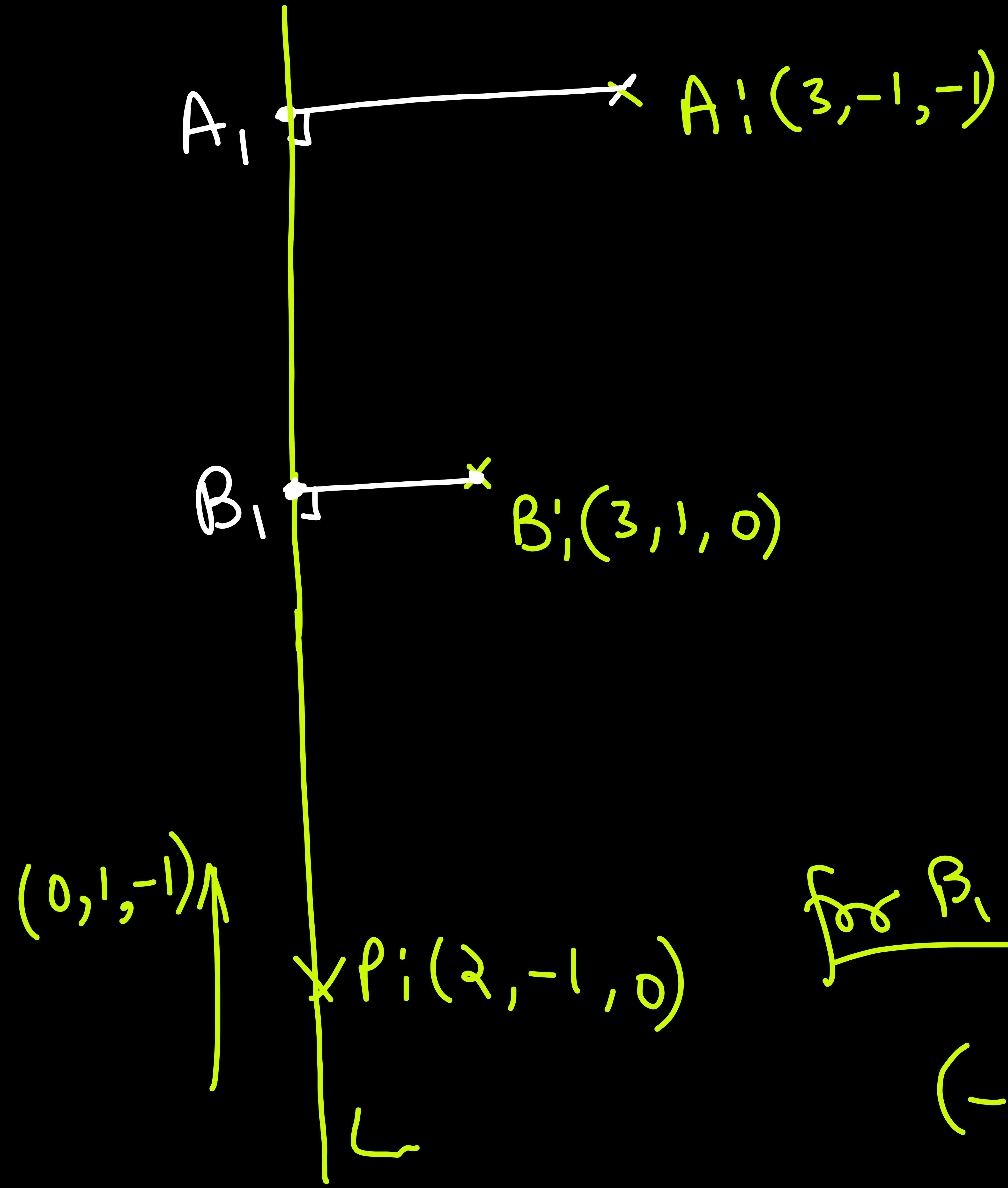
$x+y=1$

$2x-y=5$ (+)

$x=2$

$y=-1$

$\Rightarrow$ A pt on L can be taken as $P_1(2,-1,0)$.



$$L \equiv \frac{x-2}{0} = \frac{y+1}{1} = \frac{z}{-1} = \lambda$$

any pt on L can be taken as
$(2, \lambda-1, -\lambda)$.

for A_1 $\quad \overline{A_1A} \cdot (0,1,-1) = 0$.

let A_1 is $(2, \lambda-1, -\lambda)$

$3, -1, -1)$

$(-1, \lambda, -\lambda+1)\cdot(0,1,-1) = 0$

$\lambda + \lambda - 1 = 0 \Rightarrow \lambda = 1/2$

$A_1: \left(2, -\frac{1}{2}, -\frac{1}{2}\right)$.

for B_1

$\overline{BB_1} \cdot (0,1,-1) = 0$

let B_1 is

$(2, \lambda-1, -\lambda)$

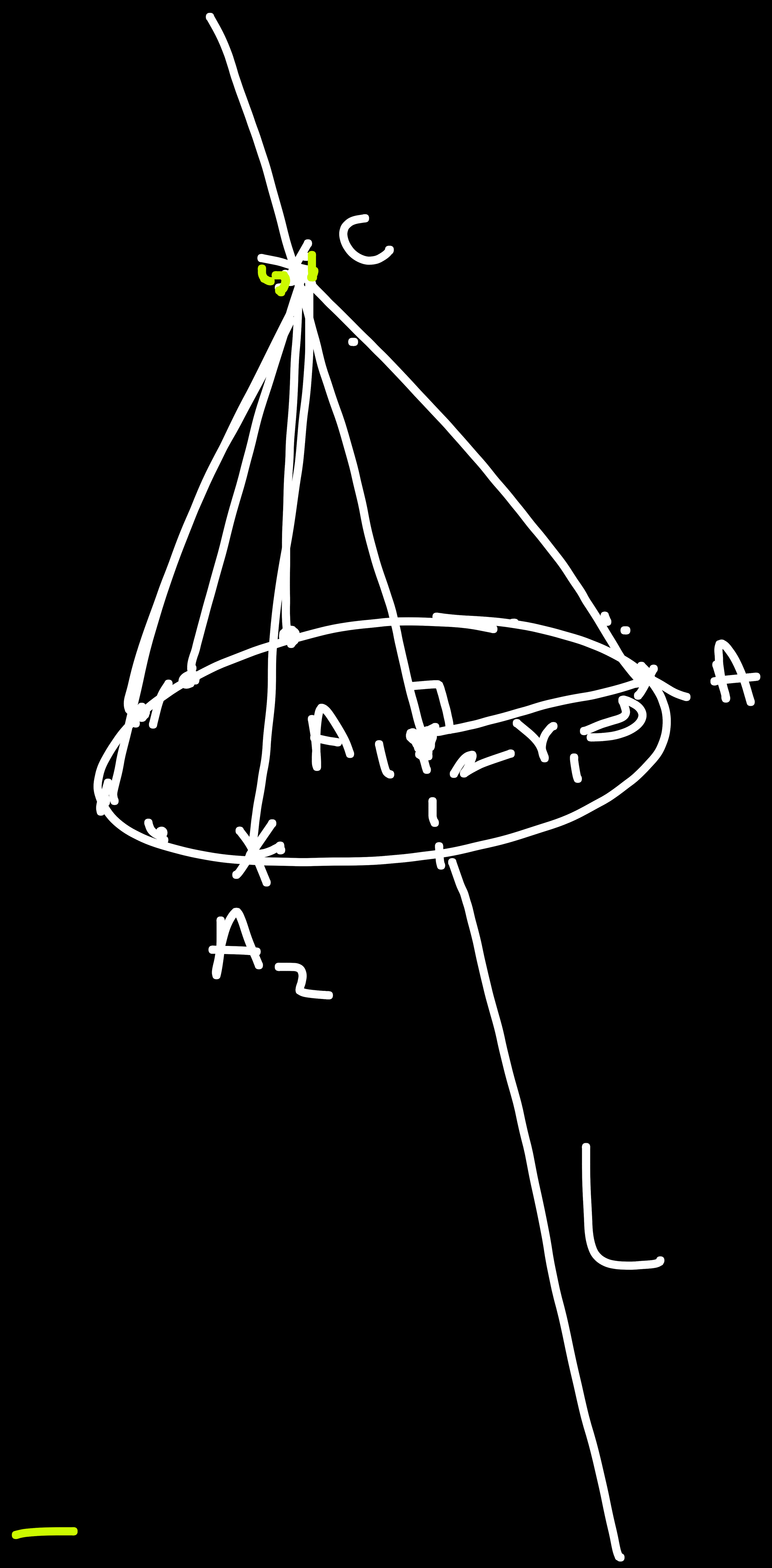
$-\;3, 1, 0$

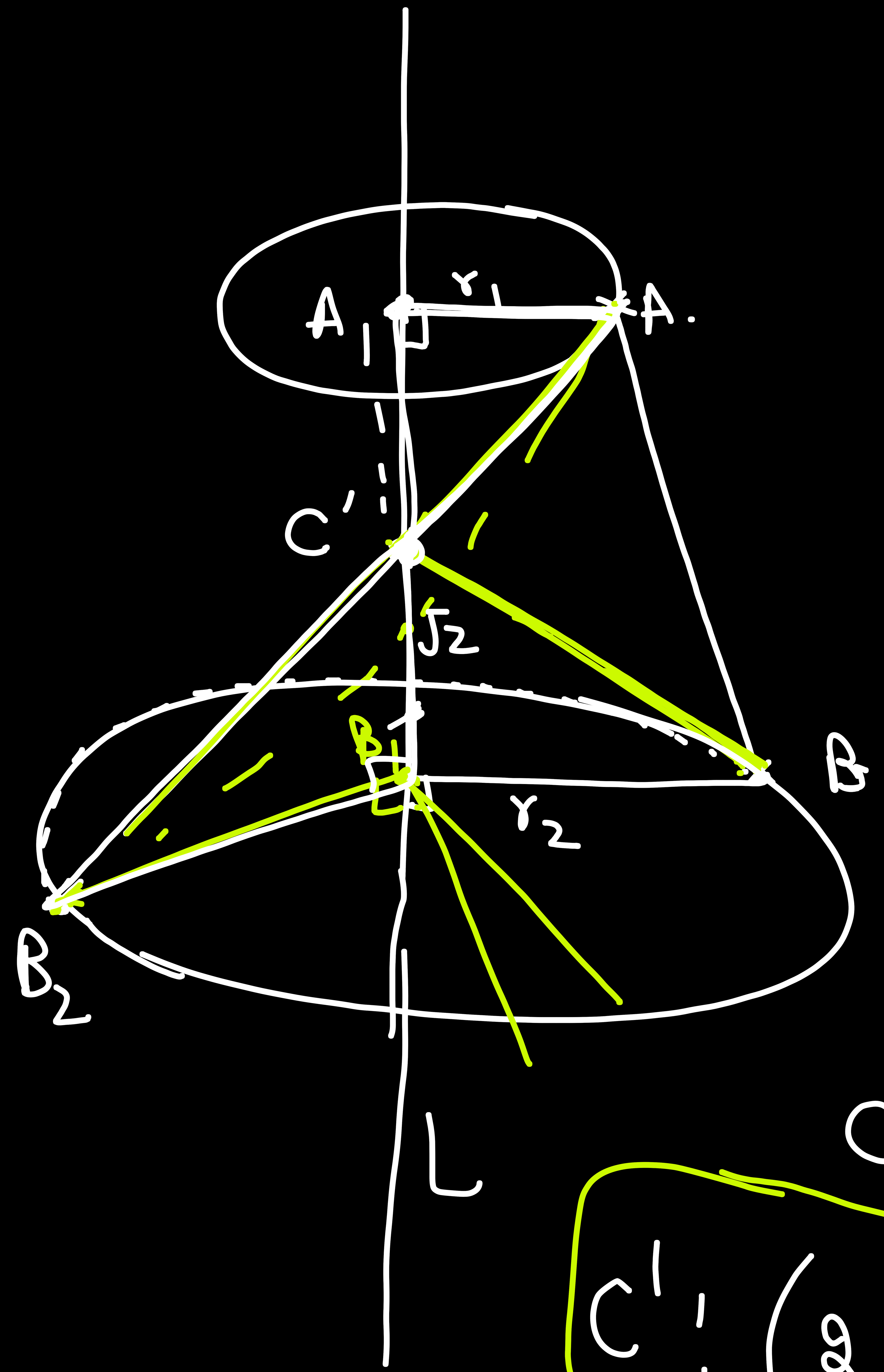
$(-1, \lambda-2, -\lambda)$

$(-1, \lambda-2, -\lambda)\cdot(0,1,-1) = 0$

$\lambda - 2 + \lambda = 0 \Rightarrow \lambda = 1$

$B_1: (2, 0, -1)$.





$$A : (3,-1,-1) \qquad B = (3,1,0)$$

$$A_1 : \left(2, -\tfrac{1}{2}, -\tfrac{1}{2}\right) \qquad B_1 = (2,0,-1)$$

$$r_1 = AA_1 = \sqrt{1+\tfrac{1}{4}+\tfrac{1}{4}} = \sqrt{\tfrac{3}{2}}$$

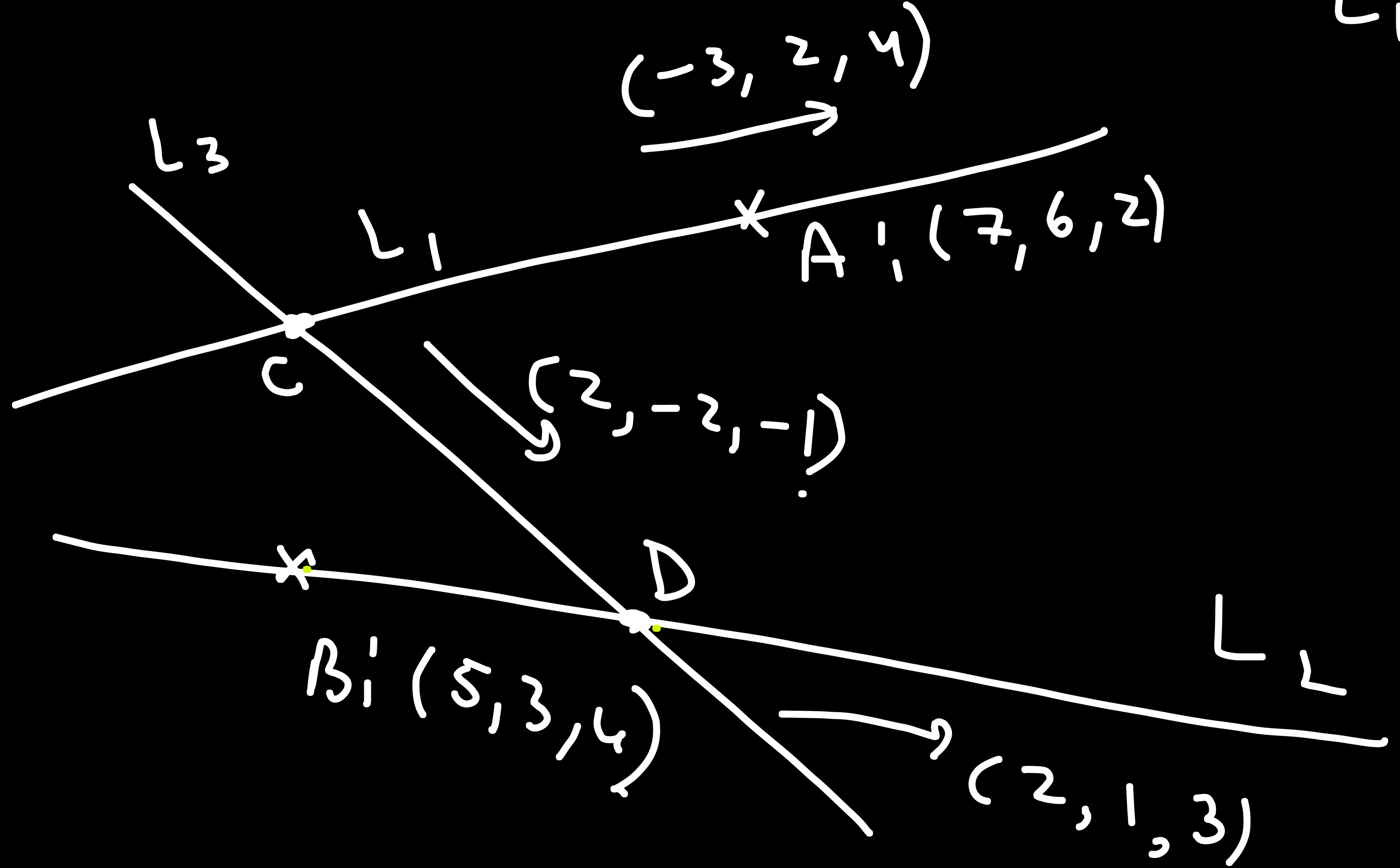
$$r_2 = BB_1 = \sqrt{1+1+1} = \sqrt{3}$$

$$\frac{A_1C'}{B_1C'} = \frac{r_1}{r_2} = \frac{\sqrt{3/2}}{\sqrt{3}} = \frac{1}{\sqrt{2}}.$$

$$C' : \left(\frac{2\sqrt{2}+1}{\sqrt{2}+1}, \frac{\sqrt{2}\left(-\frac{1}{2}\right)+1\cdot 0}{\sqrt{2}+1}, \frac{\sqrt{2}\left(-\frac{1}{2}\right)+1\cdot(-1)}{\sqrt{2}+1} \right)$$

$$\boxed{C' : \left(2, -\frac{1}{\sqrt{2}(\sqrt{2}+1)}, -\frac{1}{\sqrt{2}}\right)}$$

Que. Let line L_1 has direction ratios $(-3, 2, 4)$ and passes through A: $(7, 6, 2)$ and line L_2 has d.r. $(2, 1, 3)$ and passes through B: $(5, 3, 4)$. If line L_3 has d.r. $(2, -2, -1)$ intersects L_1 & L_2 at C and D then find CD.



$$L_1 \equiv \frac{x-7}{-3} = \frac{y-6}{2} = \frac{z-2}{4} = \lambda$$

C: $(7-3\lambda,\ 6+2\lambda,\ 2+4\lambda)$

D: $(5+2\mu,\ 3+\mu,\ 4+3\mu)$

$$\overrightarrow{CD} = \left(\underset{2}{-2+2\mu+3\lambda},\ \underset{-2}{-3+\mu-2\lambda},\ \underset{-1}{2+3\mu-4\lambda}\right)$$

$-2+2\mu+3\lambda = 3-\mu+2\lambda$ & $-3+\mu-2\lambda = 4+6\mu-8\lambda$

$3\mu+\lambda = 5$ $\qquad$ $6\lambda-5\mu = 7$

6. $3\mu + \lambda = 5$

$-5\mu + 6\lambda = 7$

$23\mu = 23$

$\mu = 1$

$\lambda = 2.$

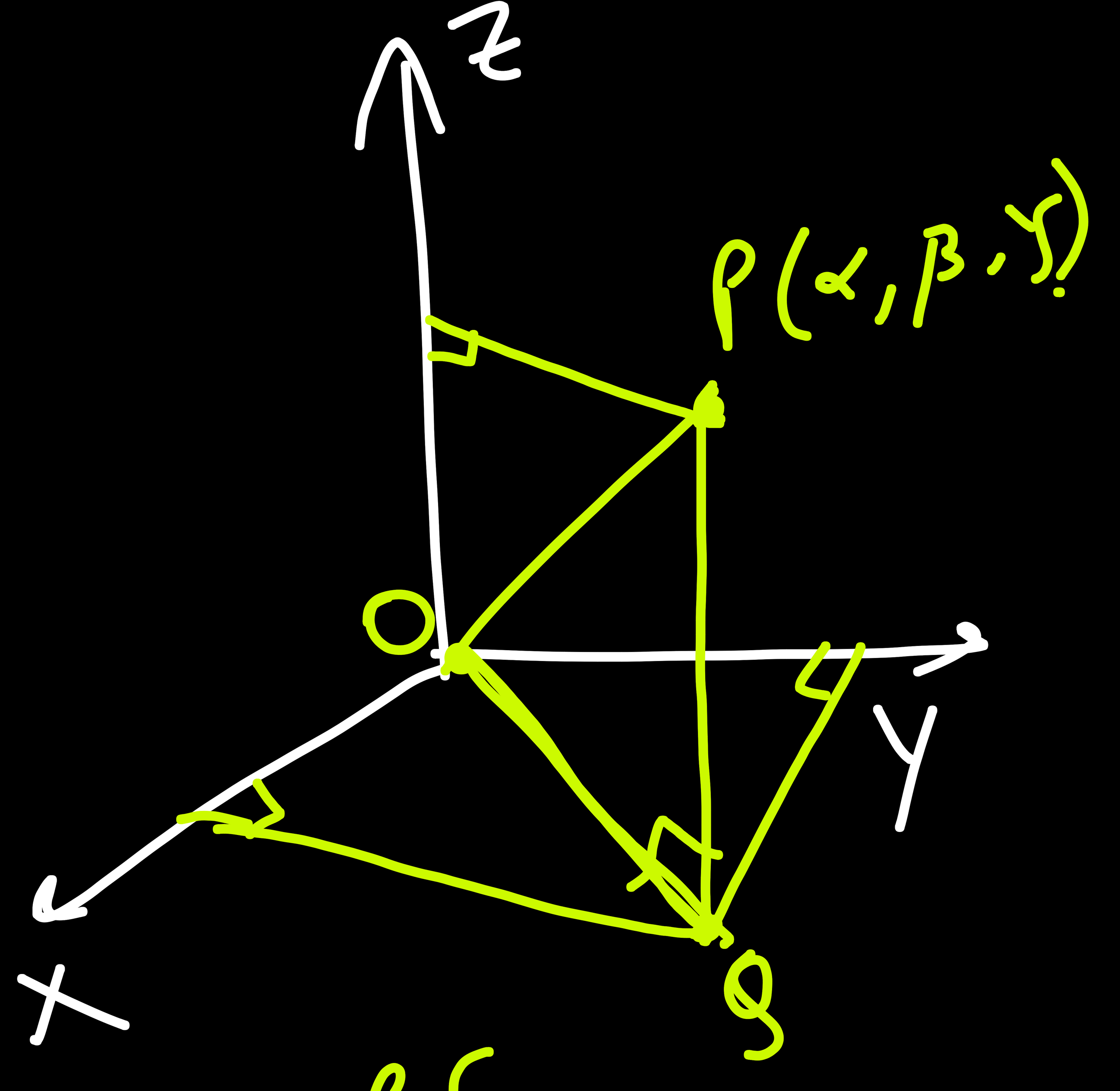
$\overline{CD} \equiv (-2+2\mu+3\lambda, -3+\mu-2\lambda, 2+3\mu-4\lambda)$

$\equiv (6, -6, -3)$

$|CD| = \sqrt{6^2+6^2+3^2}$

$= 3.\times 3 = \boxed{9}$

Que: Find the points whose distances from Y-Z, Z-X and X-Y planes are in A.P. and whose distances from x, y, z axes are $\sqrt{13}$, $\sqrt{10}$ and $\sqrt{5}$ respectively.



Sol: Let distances are $a-d,\ a,\ a+d$

then pt can be taken as $(\pm(a-d), \pm a, \pm(a+d))$

$$\sqrt{13} = \sqrt{a^2+(a+d)^2} \Rightarrow 2a^2+2ad+d^2 = 13$$
$$\sqrt{10} = \sqrt{(a-d)^2+(a+d)^2} \Rightarrow 2a^2+2d^2 = 10.$$
$$\sqrt{5} = \sqrt{(a-d)^2+a^2} \Rightarrow 2a^2-2ad+d^2 = 5$$
$$\Rightarrow 2a^2+d^2 = 9$$
$$ad = 2$$

$$a^2+d^2 = 5 \Rightarrow a^2 + \frac{4}{a^2} = 5$$
$$(a^2)^2 - 5a^2 + 4 = 0 \Rightarrow a^2 = 1, 4$$
$$a = \cancel{\pm 1}, \pm 2.$$

distance of P from Z axis $= OQ$
$$= \sqrt{OP^2 - PQ^2}$$
$$= \sqrt{\alpha^2+\beta^2+\gamma^2-\gamma^2}$$
$$= \sqrt{\alpha^2+\beta^2}$$

$ad = 2$

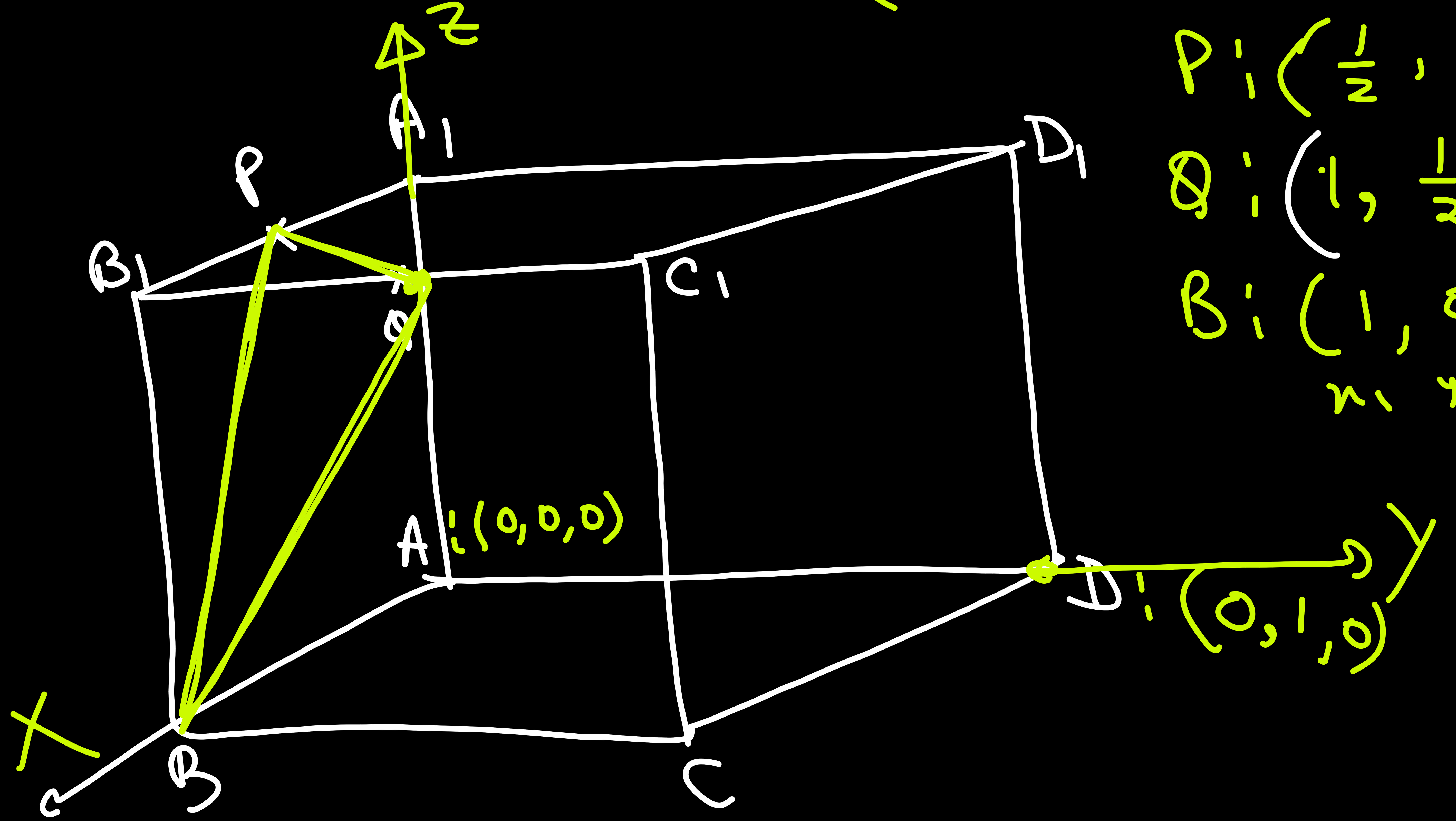
$\cancel{a = \pm 1 \Rightarrow d = \pm 2}$

$a = \pm 2 \Rightarrow d = \pm 1$

$(\pm 1, \pm 2, \pm 3)$

$\hookrightarrow 8 \mid 16$

Que) ABCD, $A_1B_1C_1D_1$ is a cube of side length 1. If P, Q are the midpts of the edges B_1A_1 & B_1C_1 respectively then find distance of vertex D from PBQ. (given AA_1, BB_1, CC_1 & DD_1 are edges mutually //)



P : $\left(\frac{1}{2}, 0, 1\right)$

Q : $\left(1, \frac{1}{2}, 1\right)$

B : $(1, 0, 0)$

$x_1 \quad y_1 \quad z_1$

Eqn of Plane PBQ

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y & z \\ 0 & 1/2 & 1 \\ -1/2 & 0 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}(x-1) - \frac{1}{2}y + \frac{1}{4}z = 0$$

$$\Rightarrow \frac{1}{2}(x-1) - \frac{1}{2}y + \frac{1}{4}z = 0$$

$$2(x-1) - 2y + z = 0$$

$$2x - 2y + z - 2 = 0$$

$2x - 2y + z - 2 = 0$

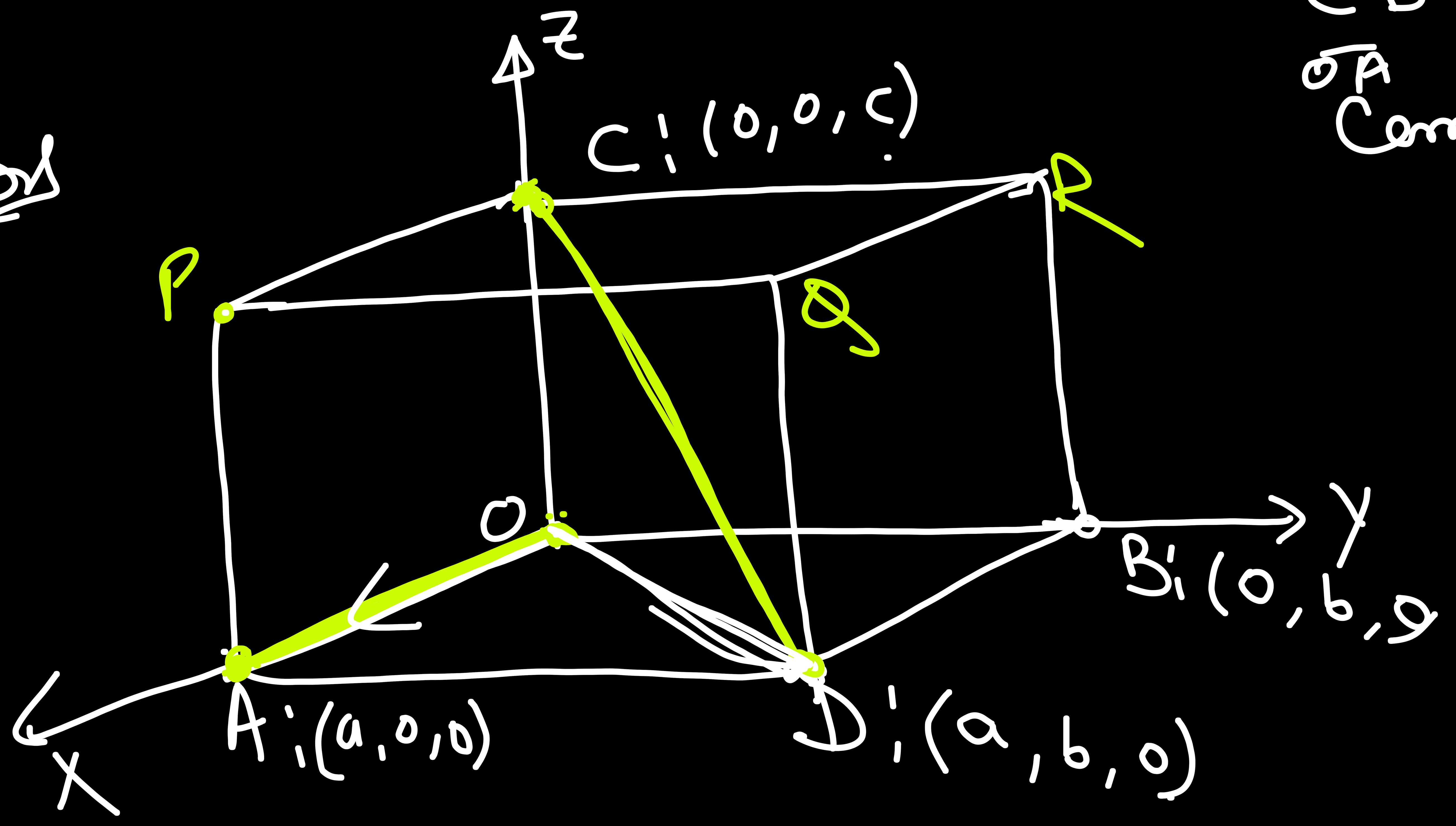
$D : (0, 1, 0)$

R. distance

$= \dfrac{|-2-2|}{\sqrt{4+4+1}} = \boxed{\dfrac{4}{3}}$ Ans

Que) Let OA, OB, OC are coinitial edges of a cuboid and l, m, n be the shortest distances between the sides OA, OB, OC and their respective skew body diagonals to them respectively then find $\dfrac{\sum l^{-2}}{\sum OA^{-2}}$.

Sol



$\overline{CD} = (a, b, -c)$

$\overline{OA} = (a, 0, 0)$

Common normal to CD & OA will be

$\equiv \overline{CD} \times \overline{OA} \equiv (0, -ac, -ab)$

$\overline{n}_1 \equiv \left(0, \frac{1}{b}, \frac{1}{c}\right)$

Shortest distance b/w OA & CD

$= l =$ Projection of OD on $\overline{n}_1$

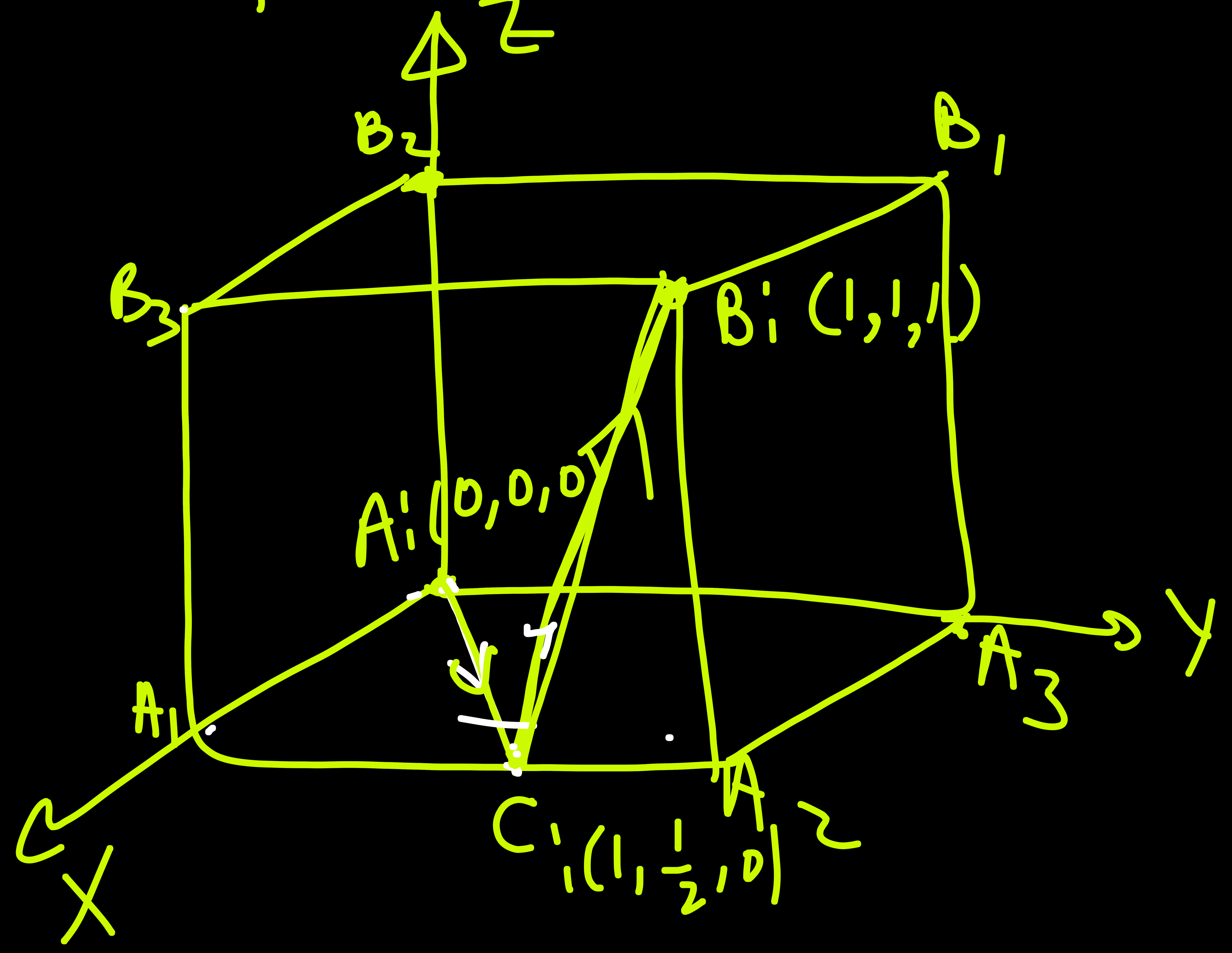
$$= \frac{|\overline{OD} \cdot \overline{n}_1|}{|\overline{n}_1|} = \frac{\left|(a, b, 0) \cdot \left(0, \frac{1}{b}, \frac{1}{c}\right)\right|}{\sqrt{\frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{b^2} + \frac{1}{c^2}}}$$

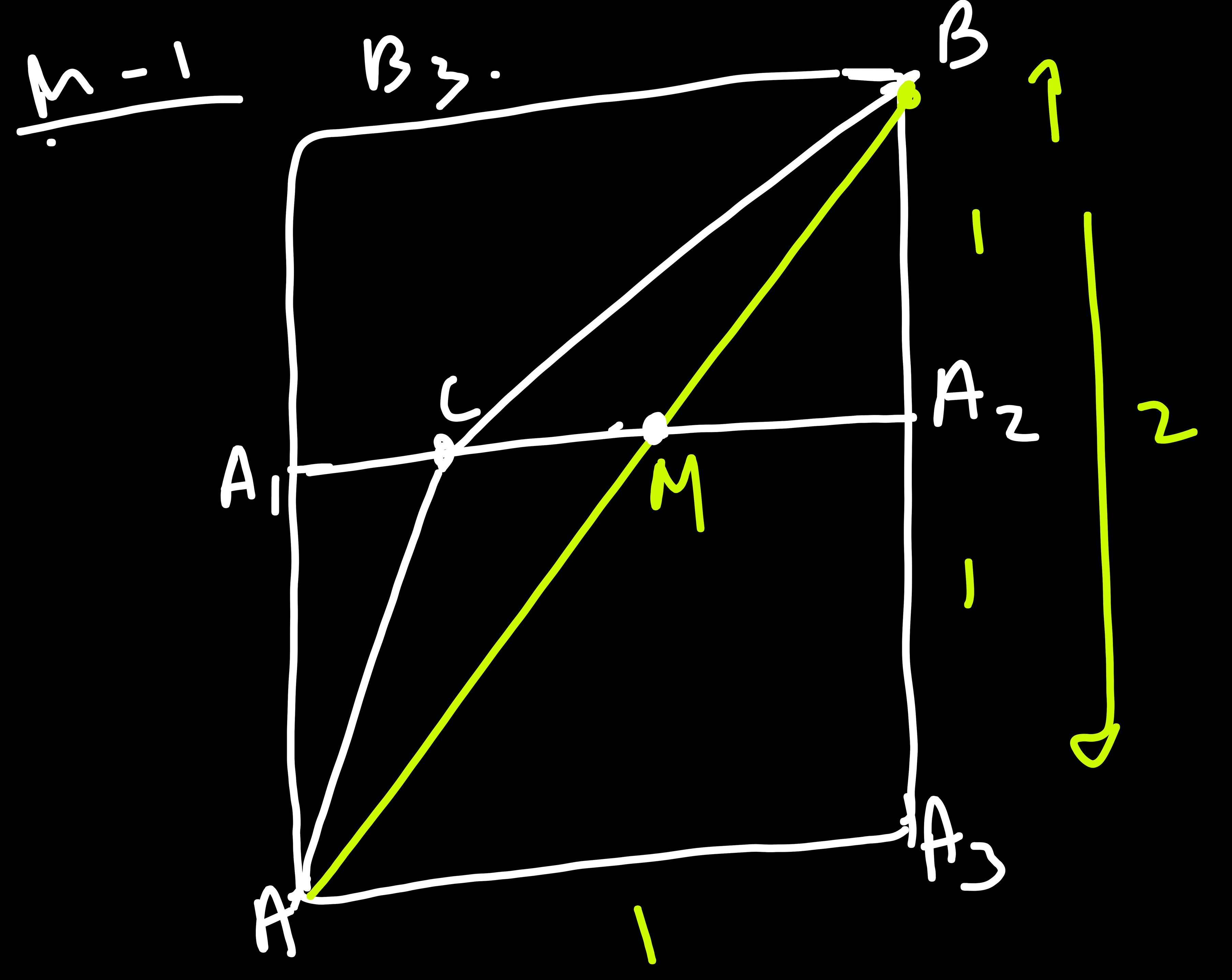
$$\frac{1}{l^2} = \frac{1}{b^2} + \frac{1}{c^2}$$

$$\sum \frac{1}{l^2} = \sum \frac{1}{b^2} + \frac{1}{c^2} = 2 \sum \frac{1}{a^2}$$

$$\Rightarrow \frac{\sum \frac{1}{l^2}}{\sum \frac{1}{a^2}} = \boxed{2.}$$

Que Let a cube has sides along x-y-z axes and two of the opposite vertices are $A:(0,0,0)$ and $B:(1,1,1)$. An insect moves from A to B by crawling on the surface by traveling minimum distance. Find the possible direction cosines of the directions of it's movements.





$\sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$

For min. distance

C must be mid pt of A_1A_2.

$\Rightarrow AC = (1, \frac{1}{2}, 0)$

dir. cosine $= (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0)$

$\overline{CB} = (0, \frac{1}{2}, 1)$

dir cosine $= (0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$

Que Let direction cosines (l, m, n) of two lines L_1 & L_2 satisfy $al+bm+cn=0$ and $fmn+gnl+hlm=0$. If L_1 & L_2 are mutually perpendicular then find $\sum \frac{f}{a}$.

$$al+bm+cn=0 \xrightarrow{\div n} ax+by+c=0$$

$$f(mn)+gnl+hlm=0 \xrightarrow{\div n^2} fy+gx+hxy=0.$$

$$-f\left(\frac{ax+c}{b}\right)+gx+hx\left(-\left(\frac{ax+c}{b}\right)\right)=0.$$

$\hookrightarrow x_1, x_2$

$$(l, m, n) \equiv \left(\frac{l}{n}, \frac{m}{n}, 1\right)$$

$$\equiv (x, y, 1)$$

Dir_1 $\quad$ Dir_2

$(x_1, y_1, 1)$ $\quad$ $(x_2, y_2, 1)$

$$x_1 \cdot x_2 = \frac{-\frac{fc}{b}}{-\frac{ah}{b}} = \frac{f \cdot c}{a \cdot h}$$

||^{ly}

$$y_1 y_2 = \frac{g \cdot c}{b \cdot h}$$

$\rightarrow$ Consider

$x_1x_2 + y_1y_2 + 1$

$x_1x_2 + y_1y_2 + 1 = 0$ if directions are mutually $\perp$'r

$$\frac{cf}{ah} + \frac{cg}{bh} + 1 = 0$$

$\cdot \frac{h}{c}$

$$\boxed{\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0.}$$

Que Let Line L_1 $x = ay + b$, $z = cy + d$ and L_2 $a'y + b' = x$ and $z = c'y + d'$ are perpendicular to each other then find the required condition.

Sol

$L_1 \rightarrow x = ay + b$ & $z = cy + d$

$\Downarrow$

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

$L_2 \rightarrow x = a'y + b'$ & $z = c'y + d'$

$\Downarrow$

$$\frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$$

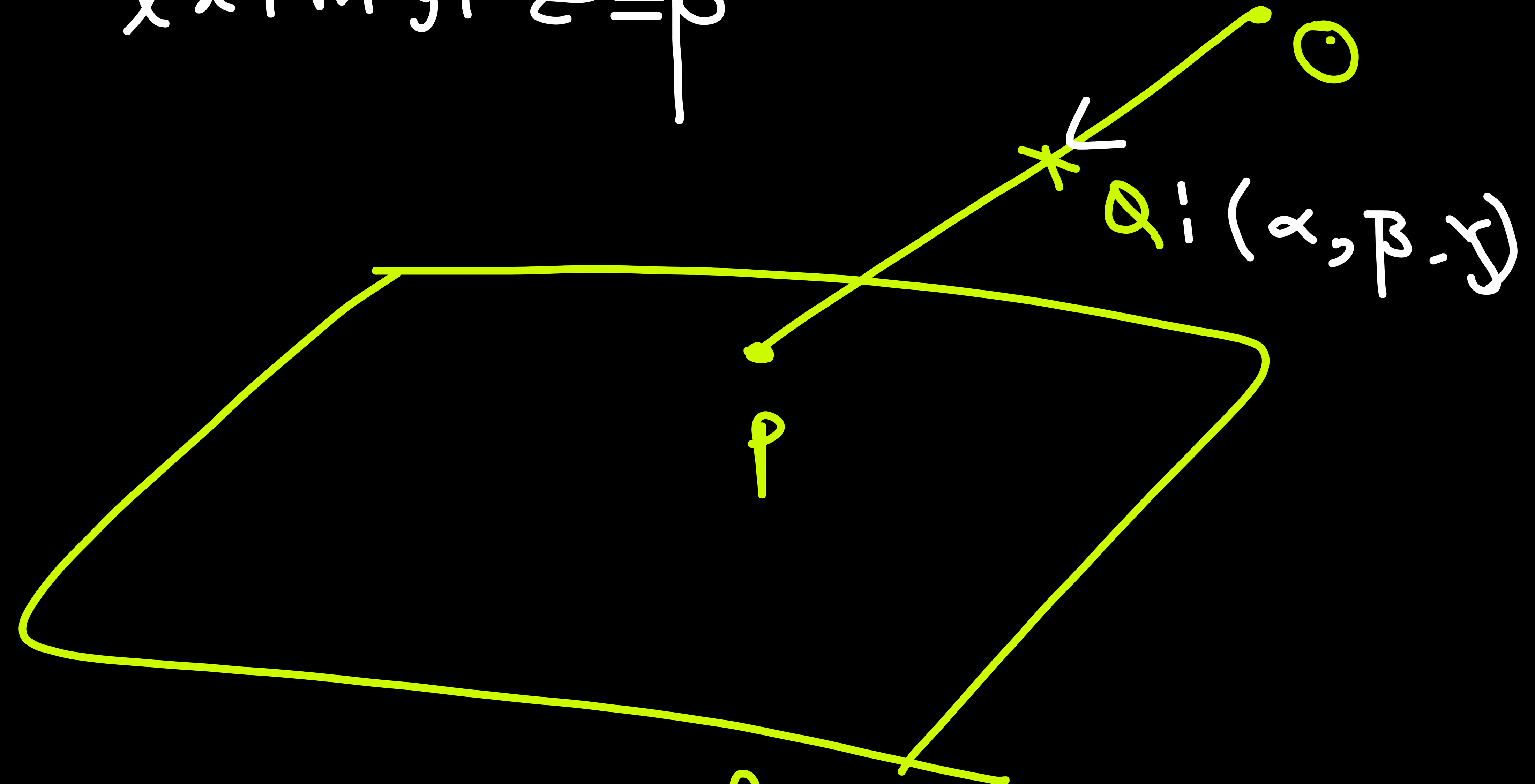
$\perp r$

$(a, 1, c) \cdot (a', 1, c') = 0.$

$$\Rightarrow \boxed{aa' + cc' + 1 = 0} \text{ Ans}$$

Que Let P be any point on the plane $lx+my+z=p$ and Q be a point on line OP such that $OP.OQ=p^2$ then find the locus of Q.

Sol $lx+my+z=p$



$OP.OQ=p^2.$

$$\vec{OP}=\frac{\vec{OQ}}{OQ}\cdot OP=\frac{p^2}{OQ^2}\cdot\vec{OQ}$$

$$=\left(\frac{p^2}{\alpha^2+\beta^2+\gamma^2}\right)(\alpha,\beta,\gamma)$$

$\vec{OP}$ will satisfy $lx+my+z=p$.

$$l\left(\frac{\alpha p^2}{\Sigma\alpha^2}\right)+m\left(\frac{p^2\beta}{\Sigma\alpha^2}\right)+\frac{\gamma p^2}{\Sigma\alpha^2}=p.$$

$(\alpha,\beta,\gamma)\to(x,y,z)$

$$\boxed{lpx+mpy+pz=x^2+y^2+z^2}$$