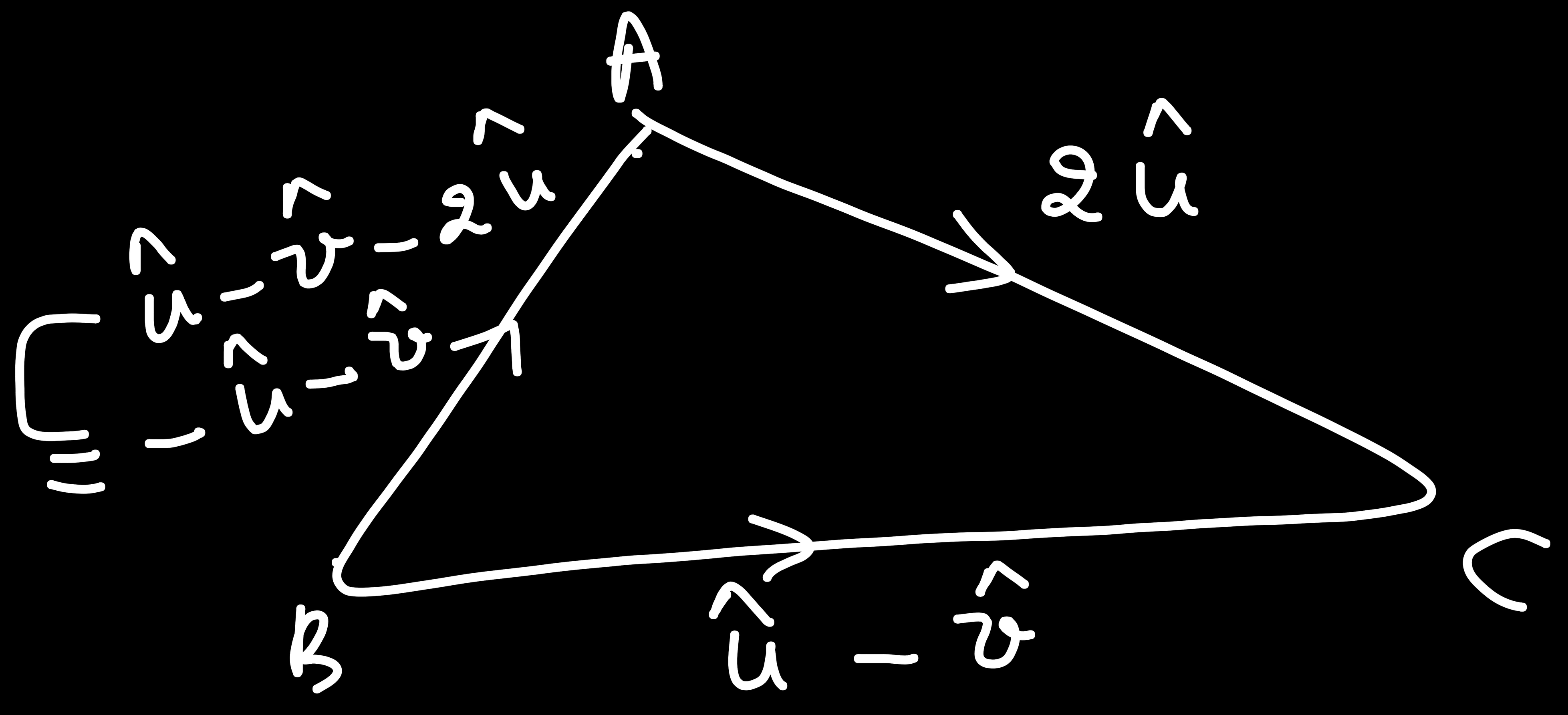
Que

If in a triangle ABC, $\overrightarrow{BC} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$ where $|\vec{u}| \neq |\vec{v}|$, then show that $1 + \cos 2A + \cos 2B + \cos 2C = 0$.



$\overline{AB} = \hat{u} + \hat{v}$

$\overline{BC} = \hat{u} - \hat{v}$

$\rightarrow \overline{AB} \cdot \overline{BC}$

$= (\hat{u} + \hat{v}) \cdot (\hat{u} - \hat{v})$

$= (\hat{u})^2 - (\hat{v})^2$

$= 1 - 1 = 0 \Rightarrow AB \perp BC$

$\rightarrow B = \pi/2.$

Now L.H.S. $= 1 + \Sigma \cos(2A)$

$= 1 + (-1 - 4\,\underbrace{\Pi \cos(A)}_{\rightarrow 0})$

$= 1 - 1 = 0$

H.P.

Que

If $\bar{u}$ and $\bar{v}$ are two non-collinear unit vectors such that $|\hat{u}\times\hat{v}| = \left|\dfrac{\hat{u}-\hat{v}}{2}\right|$, find the value of $|\hat{u}\times(\hat{u}\times\hat{v})|^2$.

Sol

$$|\hat{u}\times\hat{v}| = \left|\frac{\hat{u}-\hat{v}}{2}\right|$$

$$1.1.\sin(\theta) = \frac{1}{2}\sqrt{|\hat{u}|^2+|\hat{v}|^2-2\,\hat{u}\cdot\hat{v}}$$

$$= \frac{1}{2}\sqrt{1+1-2\cdot 1\cdot\cos\theta}$$

$$= \frac{1}{2}\sqrt{2(1-\cos(\theta))}$$

$$= \frac{1}{2}\sqrt{2\cdot 2\sin^2(\theta/2)}$$

$$\sin(\theta) = \sin\left(\frac{\theta}{2}\right)$$

$$2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right) = \sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \cancel{\sin\left(\frac{\theta}{2}\right)=0} \text{ or } \cos\left(\frac{\theta}{2}\right) = \frac{1}{2} \Rightarrow \frac{\theta}{2} = \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$$

Now

$$|\hat{u}\times(\hat{u}\times\hat{v})|^2$$

$$= \left(|\hat{u}|\cdot|\hat{u}\times\hat{v}|\cdot\sin(90^\circ)\right)^2$$

$$= \left(1\cdot 1.1.\sin(\theta)\cdot\right)^2 = \sin^2(\theta)$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 = \boxed{\frac{3}{4}} \text{ Ans}$$

One

Four points P, A, B and C with position vectors as $\bar{r}, \bar{a}, \bar{b}$ and $\bar{c}$ respectively are in a plane such that P, A, B are collinear and satisfy $|\bar{a}| = |\bar{b}| = |\bar{c}|$ and $\bar{r}.\bar{c} = |\bar{c}|^2$. Prove that $|\bar{r}-\bar{a}||\bar{r}-\bar{b}| = |\bar{r}-\bar{c}|^2$.

Sol

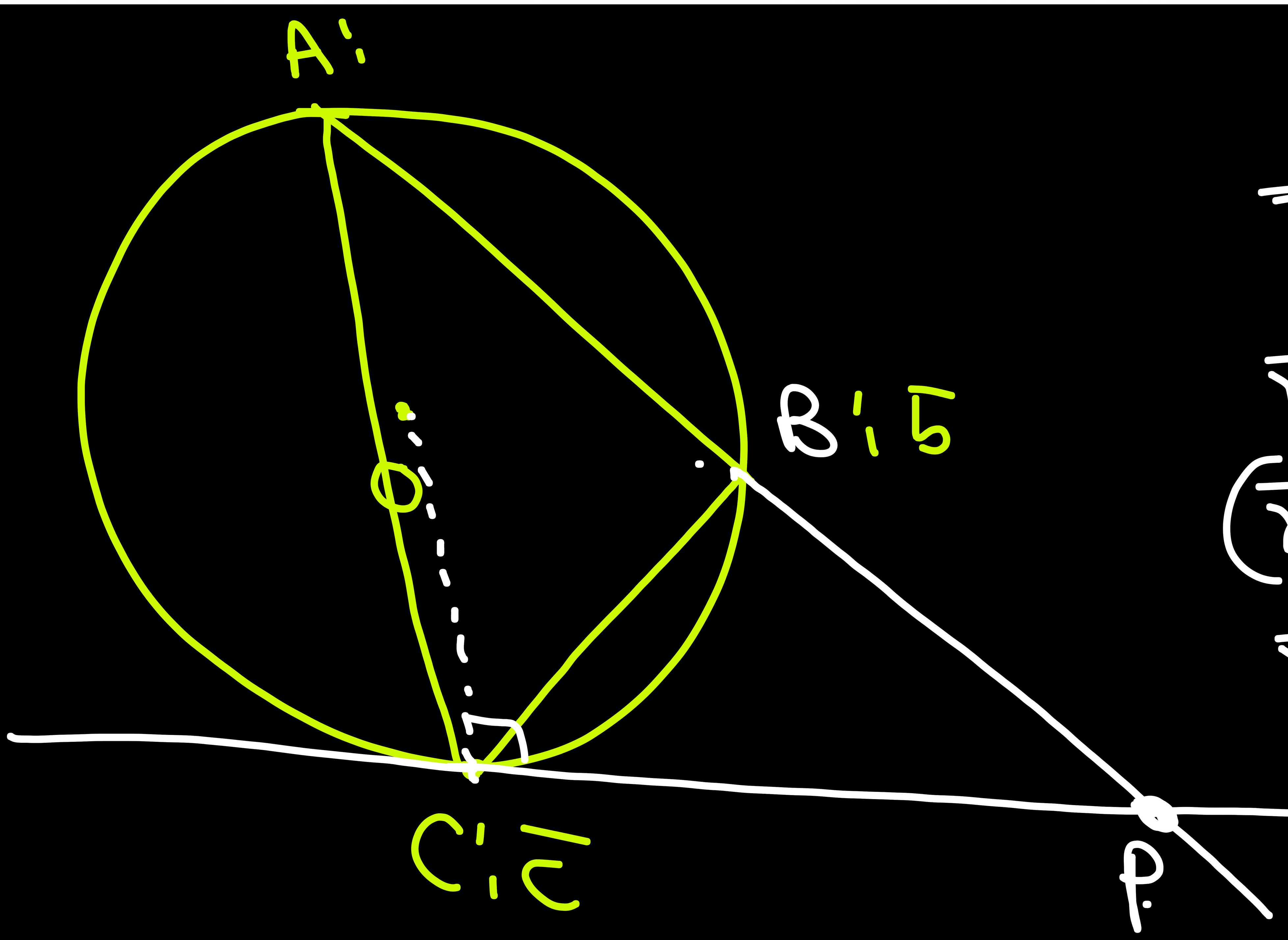
$P : \bar{r}$, $A : \bar{a}$, $B : \bar{b}$, $C : \bar{c}$

P, A, B $\longrightarrow$ Collinear.

$|\bar{a}| = |\bar{b}| = |\bar{c}|$

$\Rightarrow$ Origin is circumcentre of ΔABC.

$\bar{r}\cdot\bar{c} = c^2$.



Given

$\bar{r}\cdot\bar{c} = c^2 = \bar{c}\cdot\bar{c}$

$\bar{r}\cdot\bar{c} - \bar{c}\cdot\bar{c} = 0$

$(\bar{r}-\bar{c})\cdot\bar{c} = 0$

$\bar{r}-\bar{c} \perp \bar{c}$

$PC \perp OC$.

$\Downarrow$

P will lie on tangent of 'C'

$\Rightarrow$ P must be point of X' of AB & tangent at C.

$PA \cdot PB = PC^2 \Rightarrow |\bar{r}-\bar{a}| \cdot |\bar{r}-\bar{b}| = |\bar{r}-\bar{c}|^2$.

H.P.

One

$\hat{w}$ is a unit vector perpendicular to the plane of two other unit vectors $\hat{u}$ and $\hat{v}$. Three unit vectors $\hat{x}, \hat{y}$ and $\hat{z}$ are along the angle bisectors of $\hat{u}, \hat{v}$; $\hat{v}, \hat{w}$ and $\hat{u}, \hat{w}$ respectively. If $\left[\hat{x}\times\hat{y}\ \ \hat{y}\times\hat{z}\ \ \hat{z}\times\hat{x}\right]=\frac{1}{2}$ then find the angle between $\hat{u}$ and $\hat{v}$.

Sol $\hat{w} \perp \hat{u}, \hat{v}$, Let $\angle$ b/w $\hat{u}$ & $\hat{v} = \theta$

$\hat{x} \xrightarrow[\text{bisector of}]{\text{along angular}} \hat{u}, \hat{v} \Rightarrow \hat{x} = \dfrac{\hat{u}+\hat{v}}{2\cos(\theta/2)}$

$\hat{y} \xrightarrow{\text{''}} \hat{v}, \hat{w} \Rightarrow \hat{y} = \dfrac{\hat{v}+\hat{w}}{\sqrt{2}}$

$\hat{z} \xrightarrow{\text{''}} \hat{u}, \hat{w} \Rightarrow \hat{z} = \dfrac{\hat{u}+\hat{w}}{\sqrt{2}}$

$\hat{w} = \dfrac{\hat{u}\times\hat{v}}{\sin(\theta)}$

Given

$\frac{1}{2} = [\hat{x}\times\hat{y}\ \ \hat{y}\times\hat{z}\ \ \hat{z}\times\hat{x}]$

$= [\hat{x}\ \ \hat{y}\ \ \hat{z}]^2.$

$[\hat{x}\ \ \hat{y}\ \ \hat{z}] = \left[\dfrac{\hat{u}+\hat{v}}{2\cos(\frac{\theta}{2})}\ \ \dfrac{\hat{v}+\hat{w}}{\sqrt{2}}\ \ \dfrac{\hat{u}+\hat{w}}{\sqrt{2}}\right]$

$= \dfrac{1}{4\cos(\theta/2)}[\hat{u}+\hat{v}\ \ \hat{v}+\hat{w}\ \ \hat{u}+\hat{w}]$

$= \dfrac{2}{4\cos(\theta/2)}[\hat{u}\ \ \hat{v}\ \ \hat{w}]$

$(\hat{u}\times\hat{v})\cdot\hat{w}$

$= \dfrac{1}{2\cos(\theta/2)}$

$$[\hat{x} \ \hat{y} \ \hat{z}] = \frac{1}{2\cos(\theta/2)} (\hat{u} \times \hat{v}) \cdot \hat{\omega} \qquad \text{when } \hat{\omega} = \frac{\hat{u} \times \hat{v}}{\sin(\theta)}$$

$$[\hat{x} \ \hat{y} \ \hat{z}] = \frac{1}{2\cos(\theta/2)} \sin(\theta) \underbrace{(\hat{\omega} \cdot \hat{\omega})}_{1}$$

$$[\hat{x} \ \hat{y} \ \hat{z}] = \sin(\theta/2)$$

Now

$$\frac{1}{2} = [\hat{x} \ \hat{y} \ \hat{z}]^2 = \sin^2(\theta/2) \Rightarrow 1 = 2\sin^2(\theta/2)$$

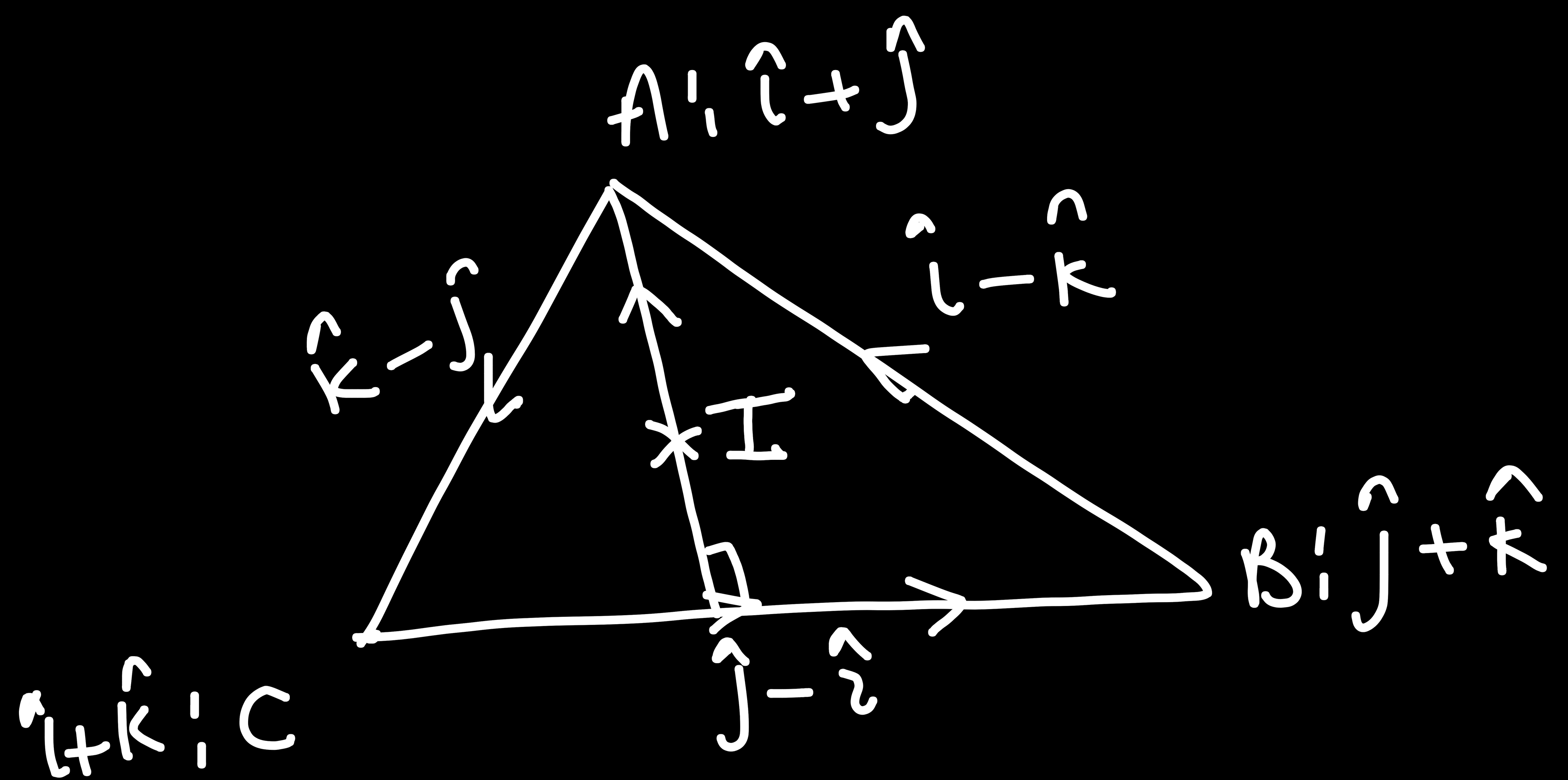
$$\Downarrow$$

$$2\sin^2(\theta/2) - 1 = 0$$

$$\cos(\theta) = 0 \Rightarrow \boxed{\theta = \pi/2} \ \text{Ans}$$

One

The position vectors of the vertices A, B and C of a triangle are $\hat{i}+\hat{j}$, $\hat{j}+\hat{k}$ and $\hat{i}+\hat{k}$ respectively. Find a unit vector $\hat{r}$ lying in the plane of ΔABC and perpendicular to IA, where I is the incentre of the triangle.



Clearly $AB = BC = CA = \sqrt{2}$

$\Rightarrow$ Δ is an equilateral Δ.

$\Rightarrow$ I will be G.

$\Rightarrow$ BC will be $\perp$r to IA

$\Rightarrow$ Required unit vector

$$= \pm \frac{(\hat{j}-\hat{i})}{\sqrt{2}}.$$

Ans

Que

A and B are two points in space with position vector $\bar{a}$ and $\bar{b}$ respectively. Find the value of λ such that the system of equation $|3\bar{r} - 2\bar{a} - \bar{b}| = |\bar{a} - \bar{b}|$ and $[\bar{r} - \lambda\bar{a} - (1-\lambda)\bar{b}]\cdot(\bar{a} - \bar{b}) = 0$ does not have any solution.

$$|3\bar{r} - 2\bar{a} - \bar{b}| = |\bar{a} - \bar{b}| \quad \text{---} \quad (1)$$

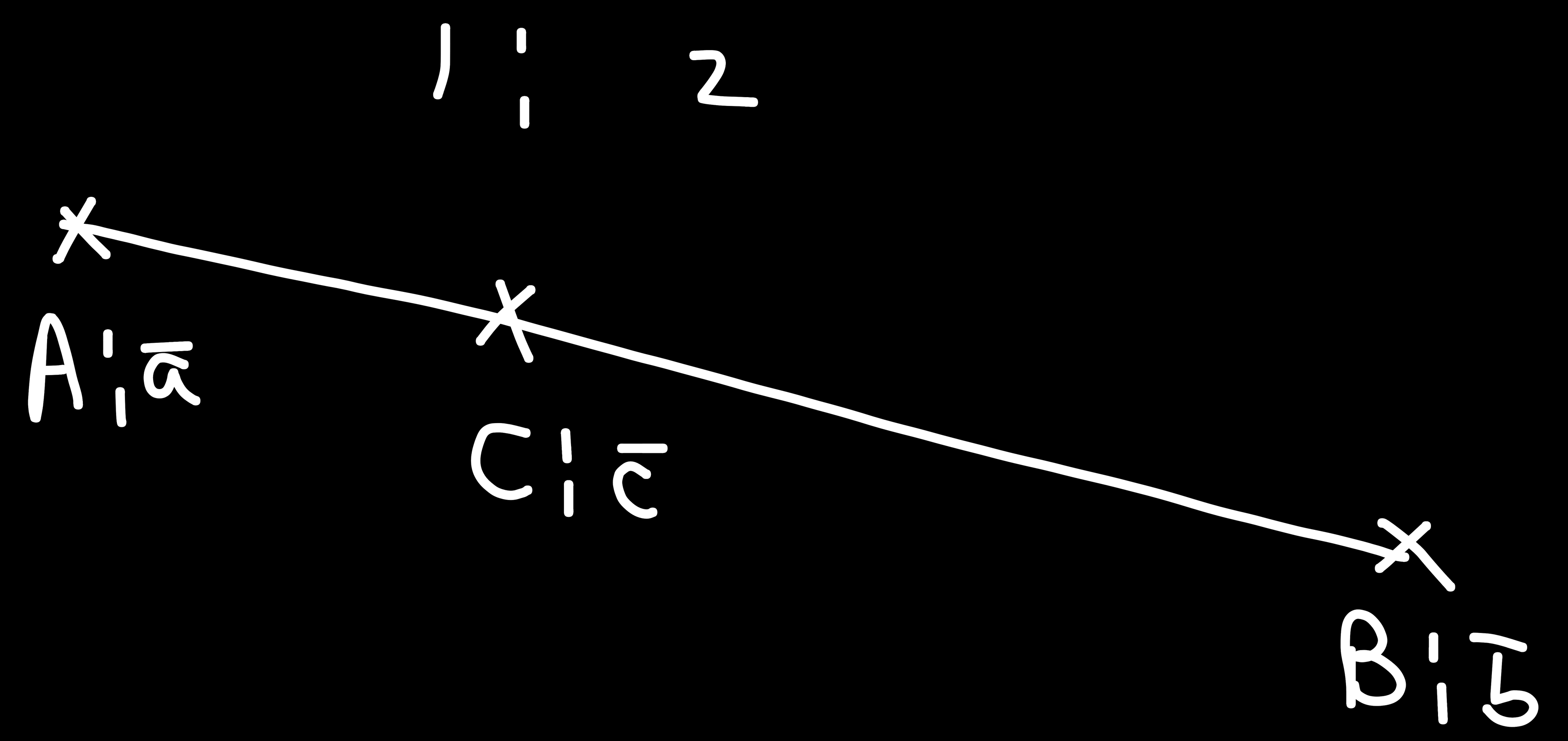
$$\Rightarrow \left|\bar{r} - \left(\frac{2\bar{a} + \bar{b}}{3}\right)\right| = \left|\frac{\bar{a} - \bar{b}}{3}\right|$$

p.v. of a pt on AB dividing it in the ratio 1 : 2.

$\bar{c}$

$$\Rightarrow |\bar{r} - \bar{c}| = \frac{AB}{3}$$

$\Rightarrow$ $\bar{r}$ lies on a sphere of radius $\frac{AB}{3}$ and centre at C : $\bar{c}$



$$(\bar{r} - \lambda\bar{a} - (1-\lambda)\bar{b})\cdot(\bar{a} - \bar{b}) = 0 \quad \text{---} \quad (2)$$

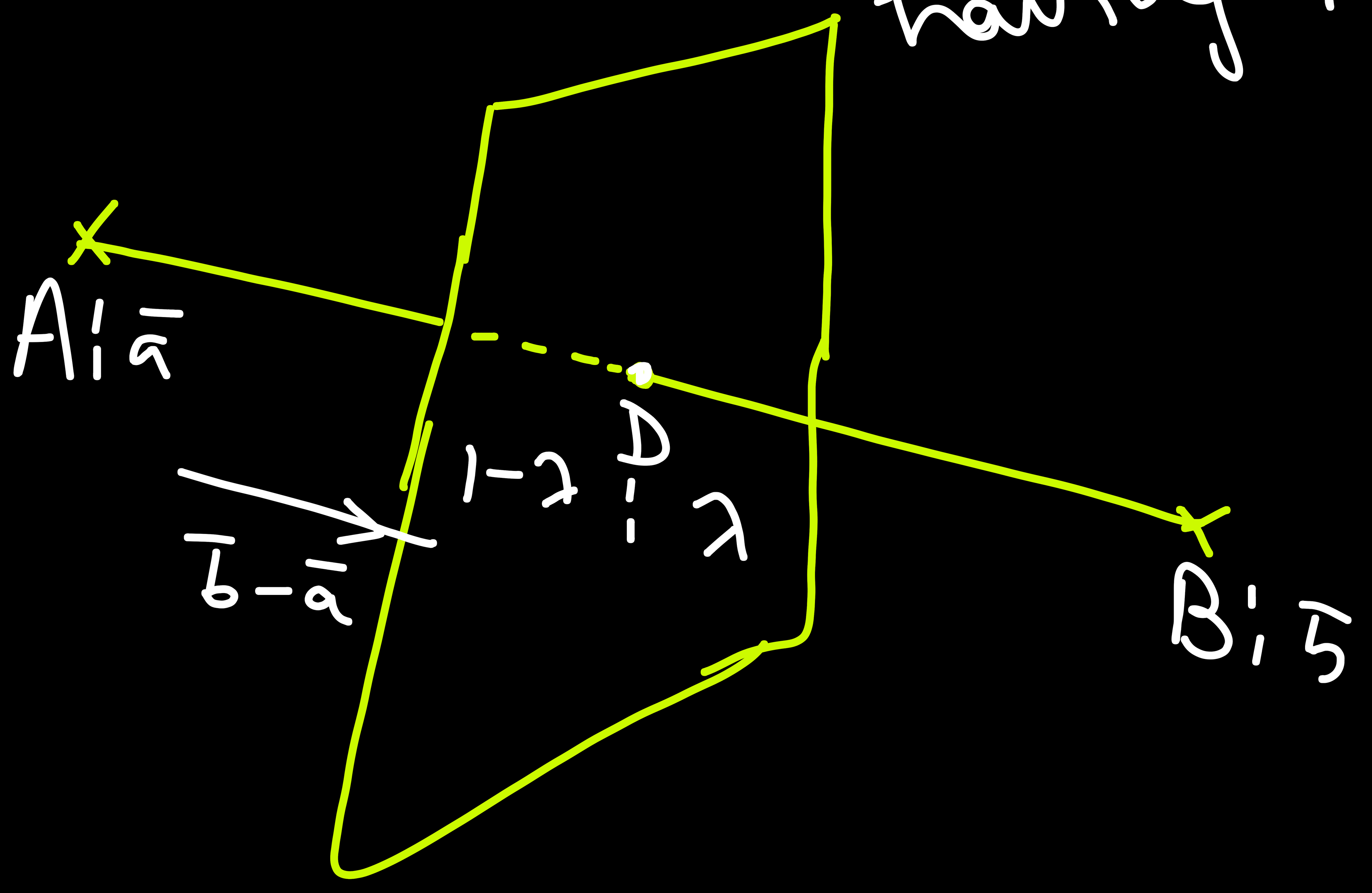
$$\left(\bar{r} - \left(\frac{\lambda\bar{a} + (1-\lambda)\bar{b}}{\lambda + (1-\lambda)}\right)\right)\cdot(\bar{a} - \bar{b}) = 0$$

$\Rightarrow$ Let it is p.v. of a pt D which will divide AB in the ratio $1-\lambda : \lambda$.

$$\Rightarrow (\bar{r} - \bar{d})\cdot(\bar{a} - \bar{b}) = 0$$

$(\bar{r} - \bar{d}) \cdot \underbrace{(\bar{a} - \bar{b})}_{= \overrightarrow{BA}} = 0$

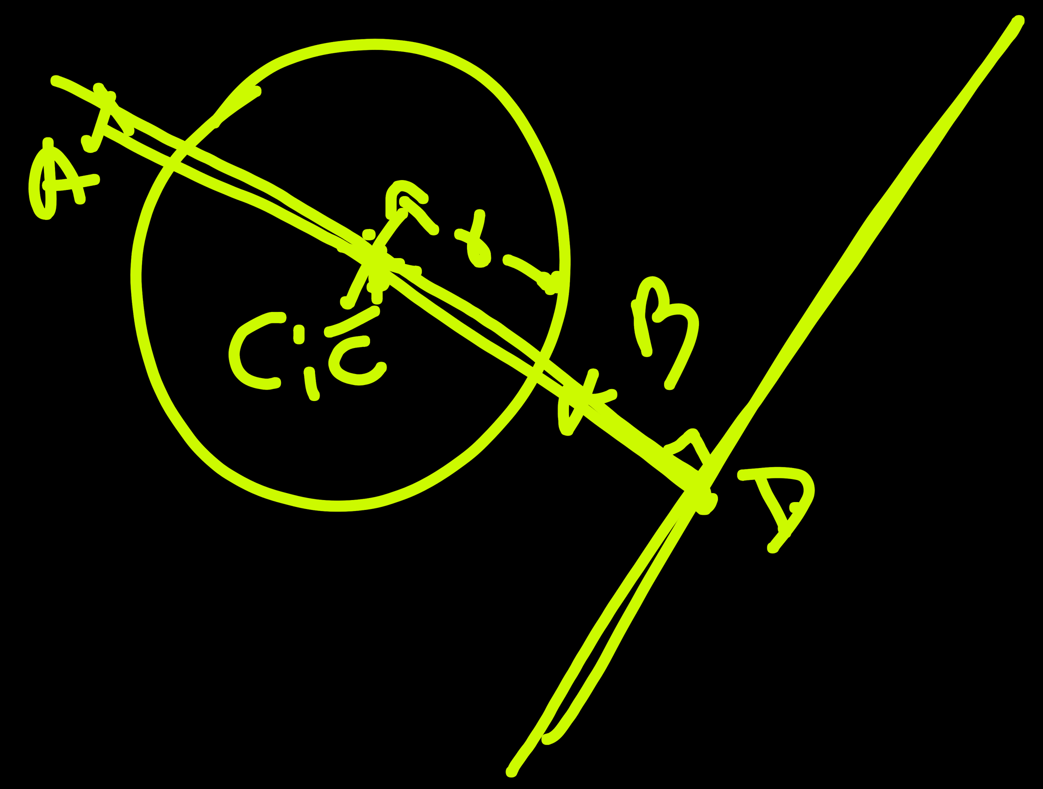
$\hookrightarrow$ Equation of plane passing through $D : \bar{d}$ and having normal as AB.



All the pts which satisfy ① will lie on a sphere ⊛

all the pts which satisfy ② will lie on a plane.

So if system of eqn has no solution then plane and sphere must not intersect



$CD > r$

$|\bar{d} - \bar{c}| > \bar{r}$

$$|\bar{d}-\bar{c}| > r$$

$$\Rightarrow \left|\lambda\bar{a}+(1-\lambda)\bar{b}-\left(\frac{2\bar{a}+\bar{b}}{3}\right)\right| > \left|\frac{\bar{a}-\bar{b}}{3}\right|$$

$$\Rightarrow |(3\lambda-2)\bar{a}+(2-3\lambda)\bar{b}| > |\bar{a}-\bar{b}|$$

$$\Rightarrow |(3\lambda-2)(\bar{a}-\bar{b})| > |\bar{a}-\bar{b}|$$

$$\Rightarrow |(3\lambda-2)|\cdot\cancel{|\bar{a}-\bar{b}|} > \cancel{|\bar{a}-\bar{b}|}$$

$$|3\lambda-2| > 1$$

$$3\lambda-2<-1 \quad \text{or} \quad 3\lambda-2>1$$

$$\lambda<1/3 \quad \text{or} \quad \lambda>1 \Rightarrow \lambda\in\left(-\infty,\frac{1}{3}\right)\cup(1,\infty)$$

One

The position vectors of the vertices A, B, C of a triangle are $\bar{a}, \bar{b}$ and $\bar{c}$ respectively, where $\bar{c} = \bar{a} \times \bar{b}$ and $\bar{a}$ and $\bar{b}$ are non–collinear vectors. If $\bar{d}$, the position vector of the centroid of the triangle ABC, makes equal angles 'α' with the vectors $\bar{a}, \bar{b}$ and $\bar{c}$, then prove that

(i) $|\bar{a}| = |\bar{b}|$.

(ii) the value of α is $\cos^{-1}\frac{1}{\sqrt{3}}$ if $\bar{a}.\bar{b} = 0$.

Sol

$A : \bar{a}, \quad B : \bar{b} \quad \& \quad C : \bar{c} = \bar{a} \times \bar{b}$

$$\bar{d} = \frac{\bar{a} + \bar{b} + \bar{a} \times \bar{b}}{3}$$

$$\bar{d} \cdot \bar{a} = d\, a \cos(\alpha) = \frac{a^2 + \bar{a} \cdot \bar{b}}{3} \quad \text{— (1)}$$

$$\bar{d} \cdot \bar{b} = d\, b \cos(\alpha) = \frac{\bar{a} \cdot \bar{b} + b^2}{3} \quad \text{— (2)}$$

$$\bar{d} \cdot (\bar{a} \times \bar{b}) = d\, |\bar{a} \times \bar{b}| \cos(\alpha) = \frac{|\bar{a} \times \bar{b}|^2}{3}$$

$$\Rightarrow d \cos(\alpha) = \frac{|\bar{a} \times \bar{b}|}{3} \quad \text{— (3)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{a}{b} = \frac{a^2 + \bar{a} \cdot \bar{b}}{\bar{a} \cdot \bar{b} + b^2}$$

$$\Rightarrow a(ab\,c(\theta) + b^2) = b(a^2 + ab\cos(\theta))$$

÷ ab

$$\Rightarrow a\,c(\theta) + b = a + b\,c(\theta)$$

$$\Rightarrow (a-b)\,c(\theta) - (a-b) = 0$$

$$(a-b)(c(\theta) - 1) = 0$$

$$\Rightarrow \boxed{a = b}$$

Proved part (i).

$$\bar{d} = \frac{\bar{a} + \bar{b} + \bar{a}\times\bar{b}}{3}$$

$$|\bar{d}| = \frac{1}{3}\sqrt{a^2 + b^2 + a^2b^2\sin^2(\theta) + 2\,\bar{a}\cdot\bar{b}}$$

$$|\bar{d}| = \frac{1}{3}\sqrt{a^2 + a^2 + a^4 s^2(\theta) + 2a^2\cos(\theta)}$$

$$|\bar{d}|\cos(\alpha) = \frac{|\bar{a}\times\bar{b}|}{3}$$

$$\frac{1}{c(\alpha)} = \frac{\sqrt{2a^2(1-c) + a^4 s(\theta)}}{ab\sin(\theta)}$$

$$\frac{1}{c(\alpha)} = \frac{\sqrt{2a^2(1-c(\theta)) + a^4 s(\theta)}}{a^2 s(\theta)}$$

If $\bar{a}\cdot\bar{b} = 0$ then

$s(\theta) = 1$ & $c(\theta) = 0$

$$\frac{1}{c(\alpha)} = \frac{\sqrt{2a^2 + a^4}}{a^2} \quad (4)$$

$$(2) \Rightarrow \quad d\,b\cos(\alpha) = \frac{\bar{a}\cdot\bar{b} + b^2}{3}$$

$$(3) \Rightarrow \quad d\cos(\alpha) = \frac{|\bar{a}\times\bar{b}|}{3}$$

let $\boxed{a = b = \lambda}$

$$b = \frac{\bar{a}\cdot\bar{b} + b^2}{|\bar{a}\times\bar{b}|}$$

$$\Rightarrow \lambda\,\cancel{\lambda}\cdot s(\theta) = \cancel{\lambda}\,c(\theta) + \cancel{\lambda^2}$$

$$\lambda = \frac{1 + c(\theta)}{s(\theta)} \xrightarrow[\text{then}]{\text{If } \bar{a}\cdot\bar{b}=0} \lambda = 1$$

$$(4) \Rightarrow \quad c(\alpha) = \frac{a^2}{\sqrt{2a^2 + a^4}} = \frac{1}{\sqrt{\frac{2}{a^2} + 1}} = \frac{1}{\sqrt{3}}$$

$\rightarrow \cos(\alpha) = \frac{1}{\sqrt{3}}$

$\Rightarrow \boxed{\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)}$. H.P.

If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are three mutually perpendicular unit vectors and $\bar{d}$ is a vector such that $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ are non–coplanar. If $[\bar{d}\ \bar{b}\ \bar{c}] = [\bar{d}\ \bar{a}\ \bar{b}] = [\bar{d}\ \bar{c}\ \bar{a}] = 1$. Find $\bar{d}$ in terms of $\bar{a}$, $\bar{b}$ and $\bar{c}$.

Que

Sol

$\bar{a}, \bar{b}, \bar{c} \rightarrow$ mutually $\perp$'r unit vectors.

$$[\bar{d}\ \bar{b}\ \bar{c}] = [\bar{d}\ \bar{a}\ \bar{b}] = [\bar{d}\ \bar{c}\ \bar{a}] = 1$$

Let

$$\bar{d} = \lambda_1 \bar{a} + \lambda_2 \bar{b} + \lambda_3 \bar{c}$$

$\cdot\ \bar{b} \times \bar{c}$

$$[\bar{d}\ \bar{b}\ \bar{c}] = \lambda_1 [\bar{a}\ \bar{b}\ \bar{c}]$$

$$\Rightarrow 1 = \lambda_1 [\bar{a}\ \bar{b}\ \bar{c}]$$

$$\lambda_1 = \frac{1}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

$|||^{ly}$ $\lambda_2 = \lambda_3 = \dfrac{1}{[\bar{a}\ \bar{b}\ \bar{c}]}$

$$\Rightarrow \bar{d} = \frac{1}{[\bar{a}\ \bar{b}\ \bar{c}]} (\bar{a} + \bar{b} + \bar{c}).$$

as $\bar{a}, \bar{b}, \bar{c}$ are mutually $\perp$'r unit vectors

$\angle$ b/w $\bar{a}$ & $\bar{b}$

$$\Rightarrow [\bar{a}\ \bar{b}\ \bar{c}] = a\,b\,c \sin(\theta) \cos(\alpha)$$

$a = 1$, $b = 1$, $c = 1$, $\theta = 90^\circ$, $\cos(\alpha) = \pm 1$

$\angle$ b/w $\bar{a} \times \bar{b}$ & $\bar{c}$

$$= \pm 1$$

$$\boxed{\bar{d} = \pm (\bar{a} + \bar{b} + \bar{c})}$$

Ans

One

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, find the scalars p, q and r in terms of θ.

Soln $\bar{a}, \bar{b}, \bar{c}$ → non-coplanar
→ unit vectors
→ Equally inclined with one another at ∠ 'θ'.

$\Rightarrow \bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{c}$
$= \bar{c} \cdot \bar{a}$
$= \cos(\theta)$

$a^2 = b^2 = c^2 = 1$

$\bar{a} \times \bar{b} + \bar{b} \times \bar{c} = p\bar{a} + q\bar{b} + r\bar{c}$

$\cdot(\bar{b} \times \bar{c})$

$(\bar{a} \times \bar{b}) \cdot (\bar{b} \times \bar{c}) + (\bar{b} \times \bar{c}) \cdot (\bar{b} \times \bar{c}) = p\,\bar{a} \cdot (\bar{b} \times \bar{c})$

$\Rightarrow (\bar{a} \cdot \bar{b})(\bar{b} \cdot \bar{c}) - (\bar{a} \cdot \bar{c})(\bar{b} \cdot \bar{b}) + |\bar{b} \times \bar{c}|^2 = p[\bar{a}\,\bar{b}\,\bar{c}]$

$\cos^2(\theta) \cdot - \cos(\theta) + \sin^2(\theta) = p[\bar{a}\ \bar{b}\ \bar{c}]$

$1 - \cos(\theta) = p[\bar{a}\ \bar{b}\ \bar{c}] \Rightarrow p = \dfrac{1 - \cos(\theta)}{[\bar{a}\ \bar{b}\ \bar{c}]}$

$\bar{c} \times \bar{a} \cdot$

$(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{a}) + (\bar{b} \times \bar{c}) \cdot (\bar{c} \times \bar{a}) = q[\bar{b}\ \bar{c}\ \bar{a}]$

$2(\cos^2(\theta) - \cos(\theta)) = q[\bar{a}\ \bar{b}\ \bar{c}]$

$\Rightarrow q = \dfrac{2\cos(\theta)(\cos(\theta) - 1)}{[\bar{a}\ \bar{b}\ \bar{c}]}$

$$\bar{a}\times\bar{b}+\bar{b}\times\bar{c}=p\bar{a}+q\bar{b}+r\bar{c}$$

$$\bar{a}\times\bar{b}\cdot \quad |\bar{a}\times\bar{b}|^2+(\bar{a}\times\bar{b})\cdot(\bar{b}\times\bar{c})=r[\bar{c}\ \bar{a}\ \bar{b}]$$

$$\sin^2(\theta)+\cos^2(\theta)-\cos(\theta)=r[\bar{c}\ \bar{a}\ \bar{b}]$$

$$(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})$$

$$=(\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{d})-(\bar{a}\cdot\bar{d})\cdot(\bar{b}\cdot\bar{c})$$

$$\Rightarrow r=\frac{1-\cos(\theta)}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

Now we know

$$[\bar{a}\ \bar{b}\ \bar{c}]^2=\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}$$

$$=\begin{vmatrix}1 & c & c\\ c & 1 & c\\ c & c & 1\end{vmatrix}=(1+2c)\begin{vmatrix}1 & c & c\\ 1 & 1 & c\\ 1 & c & 1\end{vmatrix}$$

$$=(1+2c)\begin{vmatrix}1 & c & c\\ 0 & 1-c & 0\\ 0 & 0 & 1-c\end{vmatrix}$$

$$=(1+2c)(1-c)^2$$

$$\Rightarrow [\bar{a}\ \bar{b}\ \bar{c}]^2=(1+2\cos(\theta))(1-\cos(\theta))^2$$

$$\rightarrow [\bar{a}\ \bar{b}\ \bar{c}]=\pm(1-\cos(\theta))\cdot\sqrt{(1+2\cos(\theta))}$$

$$\Rightarrow \quad p = \frac{1-\cos(\theta)}{[\bar{a}\ \bar{b}\ \bar{c}]} = r$$

$$[\bar{a}\ \bar{b}\ \bar{c}] = \pm\sqrt{1+2\cos(\theta)}\,(1-\cos(\theta))$$

$$q = \frac{-2\cos(\theta)(1-\cos(\theta))}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

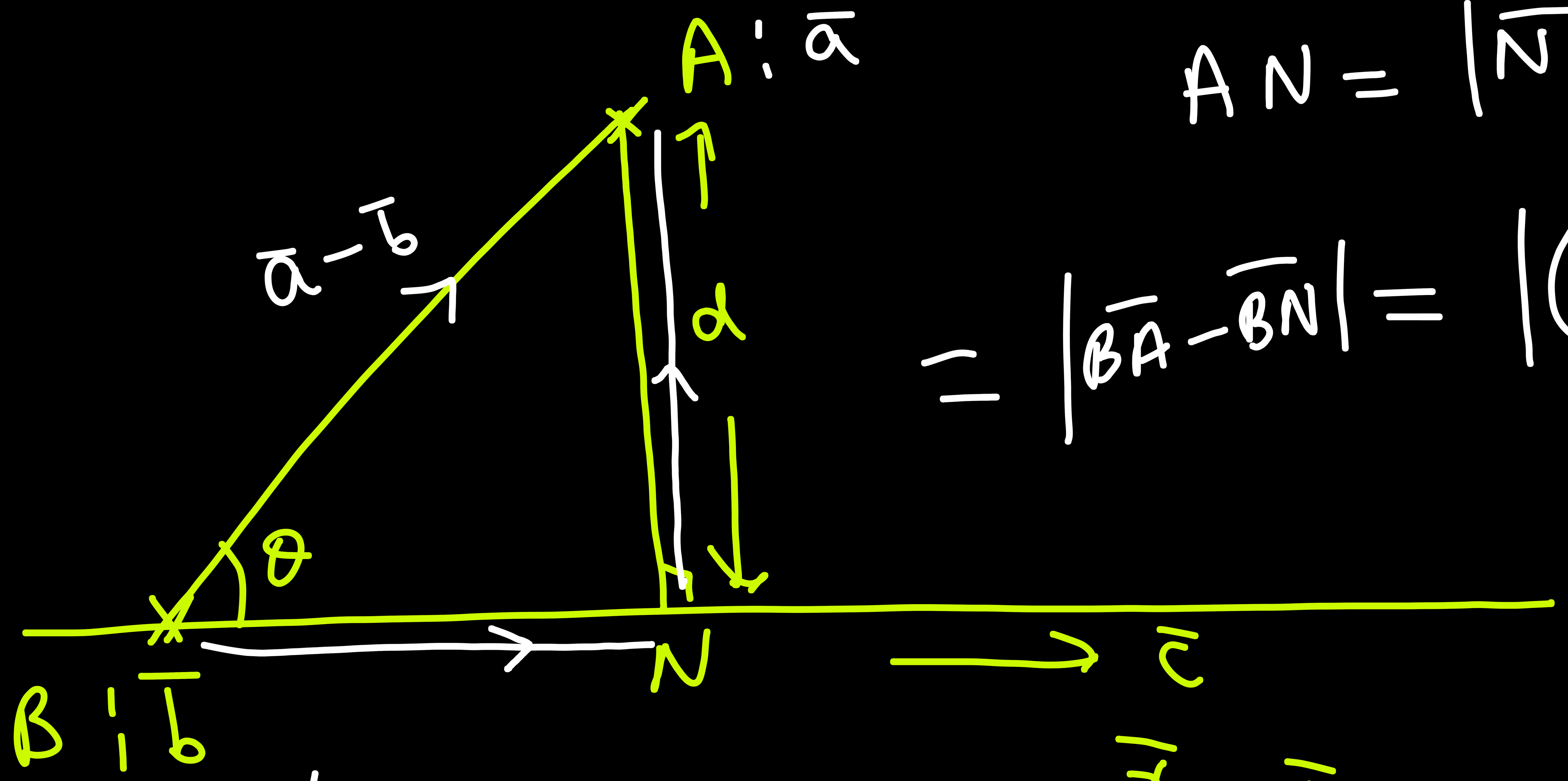
$$(p,\ q,\ r) \equiv \pm\frac{1}{\sqrt{1+2\cos(\theta)}}\left(1, -2\cos(\theta), 1\right)$$

$$= \left(\frac{1}{\sqrt{1+2\cos(\theta)}}, \frac{-2\cos(\theta)}{\sqrt{1+2\cos(\theta)}}, \frac{1}{\sqrt{1+2\cos(\theta)}}\right)$$

$$\text{or} \quad \left(-\frac{1}{\sqrt{1+2\cos(\theta)}}, \frac{2\cos(\theta)}{\sqrt{1+2\cos(\theta)}}, -\frac{1}{\sqrt{1+2\cos(\theta)}}\right) \quad \text{Ans}$$

Ques

Show that the perpendicular distance of a point $A(\bar a)$ from the line $r = b + tc$ is $\left|\bar b + \frac{(\bar a - \bar b)\bar c}{|\bar c|^2}\bar c - \bar a\right|$.



$\bar r = \bar b + t\,\bar c$

$AN = |\overline{NA}| = |\perp\text{'r component of } \overline{BA} \text{ w.r.t } \bar c|$

$= |\overline{BA} - \overline{BN}| = |(\bar a - \bar b) - \parallel\text{-component of } \overrightarrow{BA} \text{ w.r.t } \bar c|$

$= \left|(\bar a - \bar b) - \frac{((\bar a - \bar b)\cdot \bar c)\,\bar c}{c^2}\right|$

$= \left|\bar b - \bar a + \frac{((\bar a - \bar b)\cdot \bar c)\,\bar c}{c^2}\right|$. H.P.

$= |$

$d = AB\sin(\theta)$

$= \frac{|(\bar a - \bar b)\times \bar c|}{|\bar c|}$

$\parallel$ component of $\bar p$ along $\bar q$

$= (\bar p\cdot\hat q)\,\hat q$

$= \frac{(\bar p\cdot\bar q)\,\bar q}{q^2}$

Alt

$$d = AB\sin(\theta) = \frac{|(\bar{a}-\bar{b})\times\bar{c}|}{|\bar{c}|} = \frac{|((\bar{a}-\bar{b})\times\bar{c})\times\bar{c}|}{|\bar{c}|^2}$$

$$= \left|\left(\frac{(\bar{a}-\bar{b})\cdot\bar{c}}{c^2}\right)\bar{c} - \frac{\cancel{c^2}(\bar{a}-\bar{b})}{\cancel{c^2}}\right|$$

H.P.

Que

Let $\hat{x}, \hat{y}$ and $\hat{z}$ be unit vectors such that $\hat{x}+\hat{y}+\hat{z}=\vec{a}$, $\hat{x}\times(\hat{y}\times\hat{z})=\vec{b}$, $(\hat{x}\times\hat{y})\times\hat{z}=\vec{c}$

$\vec{a}.\hat{x}=\frac{3}{2}$, $\vec{a}.\hat{y}=\frac{7}{4}$ and $|\vec{a}|=2$. Find $\hat{x}, \hat{y}$ and $\hat{z}$ in terms of $\hat{a}, \hat{b}$ and $\hat{c}$.

Sol Given

$\hat{x}+\hat{y}+\hat{z}=\bar{a}$ — ① $\xrightarrow{\cdot \bar{a}} \frac{3}{2}+\frac{7}{4}+\hat{z}\cdot\bar{a}=\bar{a}\cdot\bar{a}=2$

$\hat{z}\cdot\bar{a}=2-\frac{13}{4}=-\frac{5}{4}.$

$\hat{x}\times(\hat{y}\times\hat{z})=\hat{b} \Rightarrow (\hat{x}\cdot\hat{z})\hat{y}-(\hat{x}\cdot\hat{y})\hat{z}=\hat{b}$ — ②

$(\hat{x}\times\hat{y})\times\hat{z}=\hat{c} \Rightarrow (\hat{x}\cdot\hat{z})\hat{y}-(\hat{y}\cdot\hat{z})\hat{x}=\hat{c}$ — ③

$\bar{a}\cdot\hat{x}=\frac{3}{2}$

$\bar{a}\cdot\hat{y}=\frac{7}{4}$

$|\bar{a}|=2$

Now again. ① $\Rightarrow \hat{x}+\hat{y}+\hat{z}=\bar{a}$

Let $\hat{x}\cdot\hat{y}=\alpha$

$\hat{y}\cdot\hat{z}=\beta$

$\hat{z}\cdot\hat{x}=\gamma$

$\xrightarrow{\cdot\hat{x}} 1+\alpha+\gamma=\bar{a}\cdot\hat{x}=3/2$

$\Rightarrow \alpha+\gamma=1/2=\frac{2}{4}$

$\xrightarrow{\cdot\hat{y}} \alpha+1+\beta=\frac{7}{4} \Rightarrow \alpha+\beta=\frac{3}{4}$

$\xrightarrow{\cdot\hat{z}} \gamma+\beta+1=-\frac{5}{4} \Rightarrow \gamma+\beta=-\frac{9}{4}.$

$$\alpha+\gamma=\frac{2}{4}$$
$$\alpha+\beta=\frac{3}{4}$$
$$\gamma+\beta=-\frac{9}{4}$$
$$+\quad \alpha+\beta+\gamma=-\frac{1}{2}=-\frac{2}{4}$$

$$\beta=-1$$
$$\gamma=-\frac{5}{4}$$
$$\alpha=\frac{7}{4}.$$

$$\left.\begin{aligned} \hat{x}+\hat{y}+\hat{z}&=\hat{a}\\ -\frac{5}{4}\hat{y}+\frac{7}{4}\hat{z}&=\hat{b}\\ -\hat{x}-\frac{5}{4}\hat{y}\qquad&=\hat{c}\end{aligned}\right\}+\rightarrow$$

$$-\frac{1}{4}\hat{y}+\hat{z}=\hat{a}+\hat{c}$$
$$-\frac{5}{4}\hat{y}+\frac{7}{4}\hat{z}=\hat{b}$$

$$\frac{13}{4}\hat{z}=5\hat{a}+5\hat{c}-\hat{b}$$

$$\boxed{\hat{z}=\frac{4}{13}(5\hat{a}+5\hat{c}-\hat{b})}$$