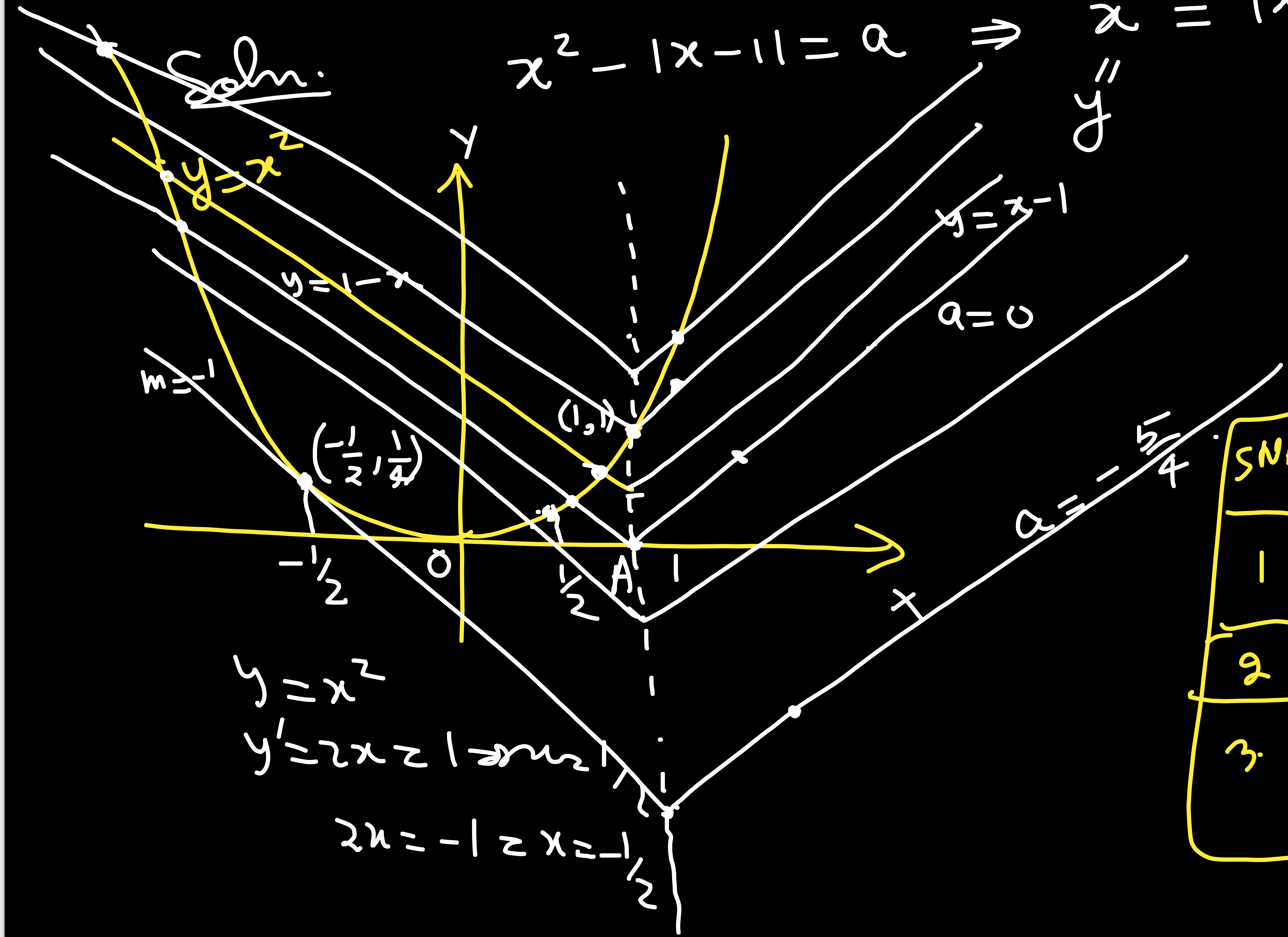
Que. Find number of solutions of the equation $x^2 - |x-1| = a$, where $a \in R$.

Soln.

$$x^2 - |x-1| = a \Rightarrow \underbrace{x^2}_{y} = \underbrace{|x-1| + a}_{y}$$



$y = x^2$

$y' = 2x = 1 \Rightarrow x = \frac{1}{2}$

$2x = -1 \Rightarrow x = -\frac{1}{2}$

$y = 1 - x + a$

$\frac{1}{4} = 1 - \left(-\frac{1}{2}\right) + a \Rightarrow a = \frac{1}{4} - \frac{3}{2} = -\frac{5}{4}.$

S.No.	a	No. of Soln
1	$\left(-\infty, -\frac{5}{4}\right)$	0
2	$-\frac{5}{4}$	1
3.	$\left(-\frac{5}{4}, \infty\right)$	2

Ans

Que. For the quadratic equation $ax^2+bx+c=0$, $a\neq 0$ and $\Delta=b^2-4ac$, $a,b,c\in R$, Let roots are α,β, such that $\alpha+\beta$, $\alpha^2+\beta^2$ & $\alpha^3+\beta^3$ are in G.P. Then which of the following holds :→

(A) $c=0$ (B) $\Delta=0$ ~~(C)~~ $c\cdot\Delta=0$ (D) None of these

Soln

$\alpha+\beta,\ \alpha^2+\beta^2,\ \alpha^3+\beta^3 \longrightarrow$ G.P.

$$(\alpha^2+\beta^2)^2=(\alpha+\beta)\cdot(\alpha^3+\beta^3)$$

$$(\alpha^2+\beta^2)^2=(\alpha+\beta)(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$$

$$(\alpha^2+\beta^2)^2=(\alpha+\beta)^2\cdot(\alpha^2+\beta^2-\alpha\beta)$$

$\alpha^2+\beta^2=p$

$\alpha\beta=q$

$$\cancel{p^2}=(p+2q)\cdot(p-q)=\cancel{p^2}+pq-2q^2$$

$$q(p-2q)=0$$

$$\alpha\beta(\alpha^2+\beta^2-2\alpha\beta)=0 \Rightarrow \alpha\beta\cdot(\alpha-\beta)^2=0$$

$$\frac{c}{a}\cdot\frac{D}{a^2}=0$$

$$\boxed{c\cdot D=0}$$

A ✓

$$|\alpha-\beta|=\frac{\sqrt{D}}{a}$$

$$(\alpha-\beta)^2=\frac{D}{a^2}$$

Que The values of $a\ (\in R)$ such that $x^4 - 2ax^2 + x + a^2 - a = 0$ has 4 - real roots :-

(A) $[1, \infty)$ (B) $[\frac{3}{4}, \infty)$ (C) $[2, \infty)$ (D) $[0, \infty)$.

Sol

$x^4 - 2ax^2 + x + a^2 - a = 0$

$a^2 - a(2x^2+1) + x^4 + x = 0$

$$a = \frac{2x^2+1 \pm \sqrt{\cancel{4x^4} + 4x^2 + 1 - 4(\cancel{x^4} + x)}}{2}$$

$2a = 2x^2 + 1 \pm \sqrt{4x^2 - 4x + 1}$

$2a = 2x^2 + 1 \pm (2x - 1) = 2x^2 + 2x,\ 2x^2 - 2x + 2$

$x^2 + x - a = 0$ or $x^2 - x + 1 - a = 0$.

$D \geq 0 \Rightarrow 1 + 4a \geq 0 \Rightarrow a \in [-\frac{1}{4}, \infty)$

$D \geq 0 \Rightarrow 1 - 4(1-a) \geq 0$

$-\frac{3}{4} + a \geq 0 \Rightarrow a \geq \frac{3}{4}$.

$\cap \rightarrow [\frac{3}{4}, \infty)$

B.

Que Let $\alpha, \beta, \gamma, \delta$ are the roots of $x^4+x^2+1=0$, then the equation whose roots are $\alpha^2, \beta^2, \gamma^2, \delta^2$ is :→

(A) $(x^2-x+1)^2=0$

(B) $(x^2+x+1)^2=0$

(C) $x^4-x^2+1=0$

(D) $x^2-x+1=0$.

Soln.

$x^4+x^2+1=0 \to \alpha, \beta, \gamma, \delta$

$-\alpha \quad -\beta$

$\alpha^4+\alpha^2+1$

$(x^2)^2+(x^2)+1=0$

$x^2=t$

$(t)^2+(t)+1=0 \to \alpha^2, \beta^2, \gamma^2, \delta^2$

$\Rightarrow x^2+x+1=0 \to \alpha^2, \beta^2$.

$(x^2+x+1)^2=0 \to \alpha^2, \beta^2, \gamma^2, \delta^2$.

(B) Ans

Que The product of uncommon real roots of the two polynomials $p(x) = x^4 + 2x^3 - 8x^2 - 6x + 15$ & $q(x) = x^3 + 4x^2 - x - 10$ is

(A) 4 ~~(B) 6~~ (C) 8 (D) 12.

Soln Concept

3-deg $f(x) = 0 \longrightarrow \alpha, \beta, \gamma$ ①

4-deg $g(x) = 0 \longrightarrow \alpha, \beta, \delta, \theta$ ②

4-deg $K_1 f(x) + K_2 g(x) = 0 \longrightarrow \alpha, \beta, \xi, \phi$ ③

(B)

$P(x) = x^4 + 2x^3 - 8x^2 - 6x + 15 \longrightarrow \alpha$

$Q(x) = x^3 + 4x^2 - x - 10 \longrightarrow \alpha$

$R(x) = x \cdot Q(x) - P(x) \Rightarrow 2x^3 + 7x^2 - 4x - 15 = 0 \longrightarrow \alpha.$

$2Q(x) - R(x) \Rightarrow x^2 + 2x - 5 = 0 \longrightarrow \alpha$

$\hookrightarrow D = 4 + 20$
$= 24$

$P(x) \equiv (x^4 + 2x^3 - 8x^2 - 6x + 15)$
$\equiv (x^2 + 2x - 5)(x^2 - 3) \longrightarrow \beta, \gamma$

$Q(x) \equiv (x^3 + 4x^2 - x - 10) \equiv (x^2 + 2x - 5)(x + 2) \longrightarrow \delta$

$R.P. = \beta \cdot \gamma \delta = (-3)(-2) = 6.$

Que The number of real solutions of the system of equations

$$x = \frac{2z^2}{1+z^2}, \quad y = \frac{2x^2}{1+x^2} \quad \& \quad z = \frac{2y^2}{1+y^2} \text{ is.}$$

(A) 1 (B) 2 (C) 3 (D) 4.

Soln

$$x = \frac{2z^2}{1+z^2}, \quad y = \frac{2x^2}{1+x^2}, \quad z = \frac{2y^2}{1+y^2} \longrightarrow (0,0,0).$$

Case-2 If $x \neq 0$ then $z \neq 0$ & $y \neq 0 \Rightarrow x, y, z \in R^+$

$$\cancel{xyz} = \frac{8 \cdot \cancel{x^2y^2z^2}}{(1+x^2)(1+y^2)(1+z^2)}$$

$$\underbrace{\left(x+\frac{1}{x}\right)}_{\geq 2} \cdot \underbrace{\left(y+\frac{1}{y}\right)}_{\geq 2} \cdot \underbrace{\left(z+\frac{1}{z}\right)}_{\geq 2} = 8 \Rightarrow x = y = z = 1$$

$\Rightarrow (x,y,z) \equiv (0,0,0)$ & $(1,1,1)$

L.H.S ≥ 8 & R.H.S. $= 8$

Ans (B).

Que PYQ

Let $a, b, c \in R$ and if $ax^2+bx+c=0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$ then which of the followings is correct :→

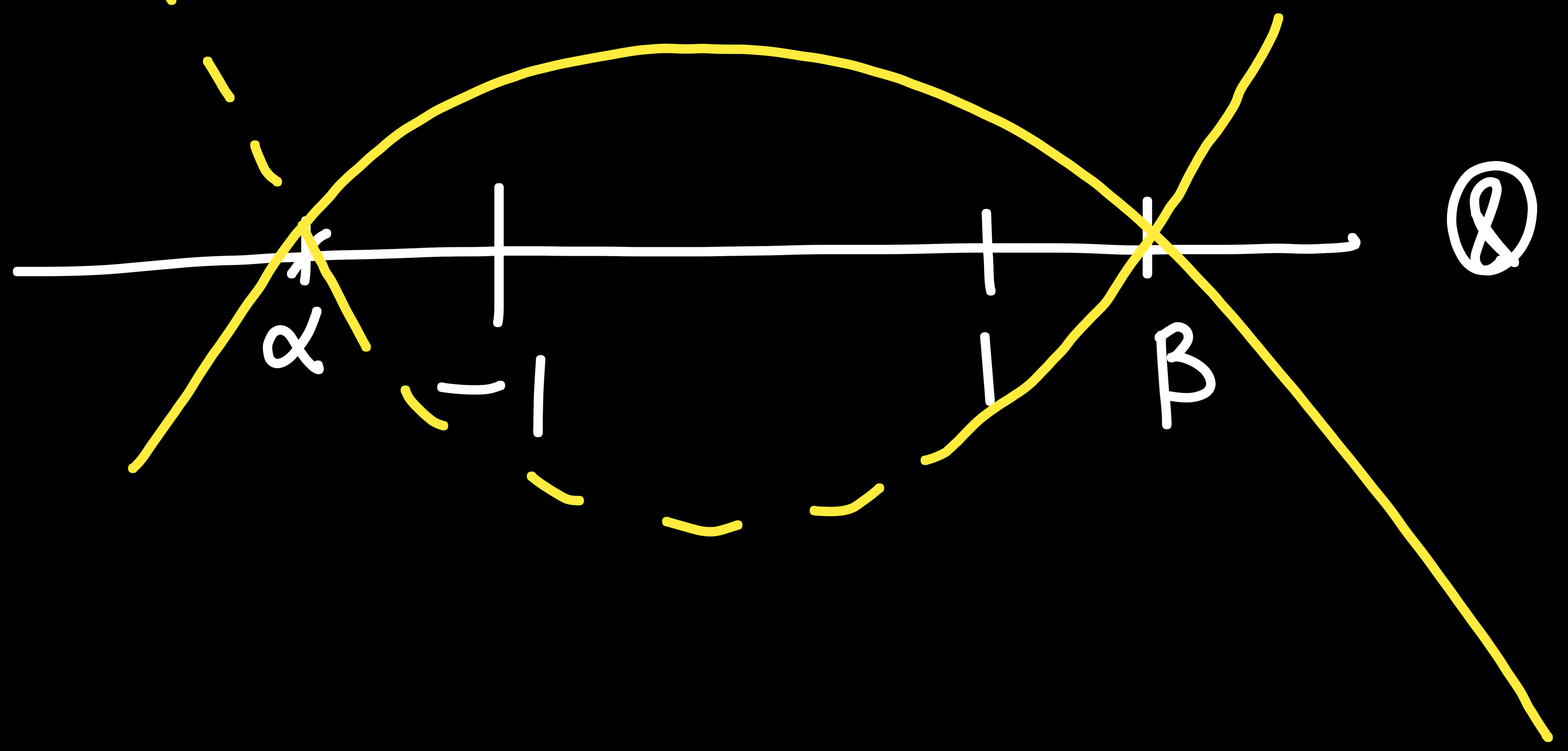
(A) $a + c + |b| < 0$

(B) $1 - \left|\frac{b}{a}\right| + \frac{c}{a} > 0$

(C) $1 - \left|\frac{b}{a}\right| + \frac{c}{a} < 0$

(D) None of these.

Sol



$a \cdot f(-1) < 0 \Rightarrow a(a-b+c) < 0 \Rightarrow 1 - \frac{b}{a} + \frac{c}{a} < 0$

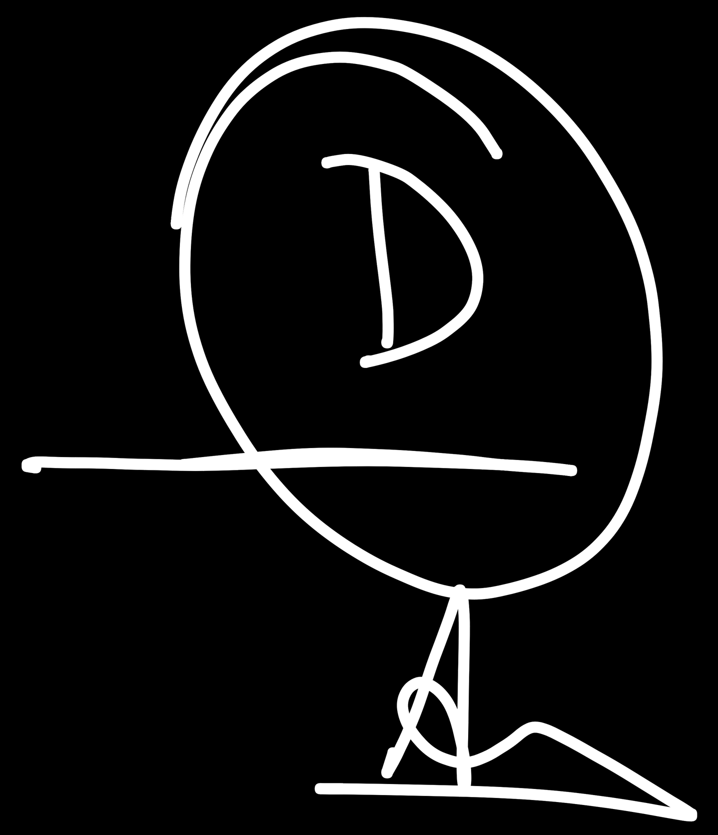
$a \cdot f(1) < 0 \Rightarrow a \cdot (a+b+c) < 0 \Rightarrow 1 + \frac{b}{a} + \frac{c}{a} < 0$

$= 1 + \frac{b}{a} + \frac{c}{a} - \frac{2b}{a}$

0

$\cancel{1 - \left|\frac{b}{a}\right| + \frac{c}{a} < 0}$

$\boxed{1 + \left|\frac{b}{a}\right| + \frac{c}{a} < 0}$ ✓



Que) Let $a, b, c \in R$ and $a \neq 0$ and Let α, β are the roots of the equation $ax^2+bx+c=0$ then express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α and β.

Sol $ax^2+bx+c=0 \rightarrow \alpha, \beta$

$a(\ )^2 + b(\ ) + c = 0$

α

$a^3x^2+abcx+c^3=0.$

$\div c^2$

$\frac{a^3x^2}{c^2} + \frac{ab}{c}x + c = 0.$

$a\left(\frac{ax}{c}\right)^2 + b\left(\frac{ax}{c}\right) + c = 0.$

$\Rightarrow \frac{ax}{c} = \alpha, \beta \Rightarrow x = \frac{c}{a}\alpha, \frac{c}{a}\beta$

$\boxed{x = \alpha^2\beta, \alpha\beta^2}$

Que Let x, y are non-negative integers for which $(xy-7)^2 = x^2+y^2$ then find the sum of all possible values of 'x'.

Soln

$$(xy-7)^2 = x^2+y^2$$

$$x^2y^2 - 14xy + 49 = x^2+y^2$$

$+2xy$

$$(xy)^2 - 12xy + 49 = x^2+y^2+2xy$$

$$(xy-6)^2 + 13 = (x+y)^2$$

$$(x+y)^2 - (xy-6)^2 = 13$$

$$(x+y+xy-6)(x+y-xy+6) = 13.$$

$\hookrightarrow 13 \qquad \hookrightarrow 1$

$1 \qquad 13$

C-1

$$\left.\begin{aligned} x+y+xy-6 &= 13 \\ x+y-xy+6 &= 1 \end{aligned}\right\} \rightarrow \begin{aligned} x+y &= 7 \\ xy-6 &= 6 \\ xy &= 12 \end{aligned}$$

$$t^2 - 7t + 12 = 0 \rightarrow t = 4, 3$$

$x = 4$ & $y = 3$

or $x = 3$ & $y = 4$.

$(4,3)$
$(3,4)$

C-2

$$\left.\begin{aligned} x+y+xy-6 &= 1 \\ x+y-xy+6 &= 13 \end{aligned}\right\} \rightarrow \begin{aligned} x+y &= 7 \\ xy-6 &= -6 \\ xy &= 0 \end{aligned}$$

$(0,7), (7,0)$.

$$\sum x = 3+4+0+7$$

$$= 14 \text{ Ans}$$

Que Let $f(x) = Ax^2 + Bx + C$, where $A, B, C \in R$, if $2A$, $A+B$ and C all are integers then, identify the true statement(s).

(A) $f(x)$ will be a rational number if x is rational. ✓

(B) $f(x)$ will be an integer if $x \in I$. ✓

(C) $f(x)$ will be rational if $x \in I$. ✓

(D) None of these

$\boxed{A, B, C}$

Ans

Sol $f(x) = Ax^2 + Bx + C = \underline{Ax^2 - Ax} + Ax + Bx + C.$

$= Ax(x-1) + (A+B)x + C$

$= (2A)\left(\frac{x(x-1)}{2}\right) + (A+B)x + C = I_1 \cdot \left(\frac{x(x-1)}{2}\right) + I_2 x + I_3$

Que) Find the number of integral roots of $x^2+7x-14(q^2+1)=0$ where q is an integer.

Soln.

$x^2+7x-14(q^2+1)=0$

$x=7y$

$\cancel{49}^{7}y^2+\cancel{49}^{7}y-\cancel{14}_{2}(q^2+1)=0$

$\underbrace{7(y^2+y)}-\underbrace{2(q^2+1)}=0$

$\Rightarrow q^2+1=7k.$ ✗

→ No integral soln

Ans $\boxed{0}$ Ans

q	q^2	q^2+1
$7k$	$7k$	$7k+1$
$7k\pm1$	$7k+1$	$7k+2$
$7k\pm2$	$7k+4$	$7k+5$
$7k\pm3$	$7k+2$	$7k+3$

$(7k\pm2)^2$

$=\underbrace{49k^2\pm28k}_{7k}+4$