Que. Find area of the figure enclosed by
$5x^2+6xy+2y^2+7x+6y+6=0$.

Sol

$2y^2+6(x+1)y+5x^2+7x+6=0$.

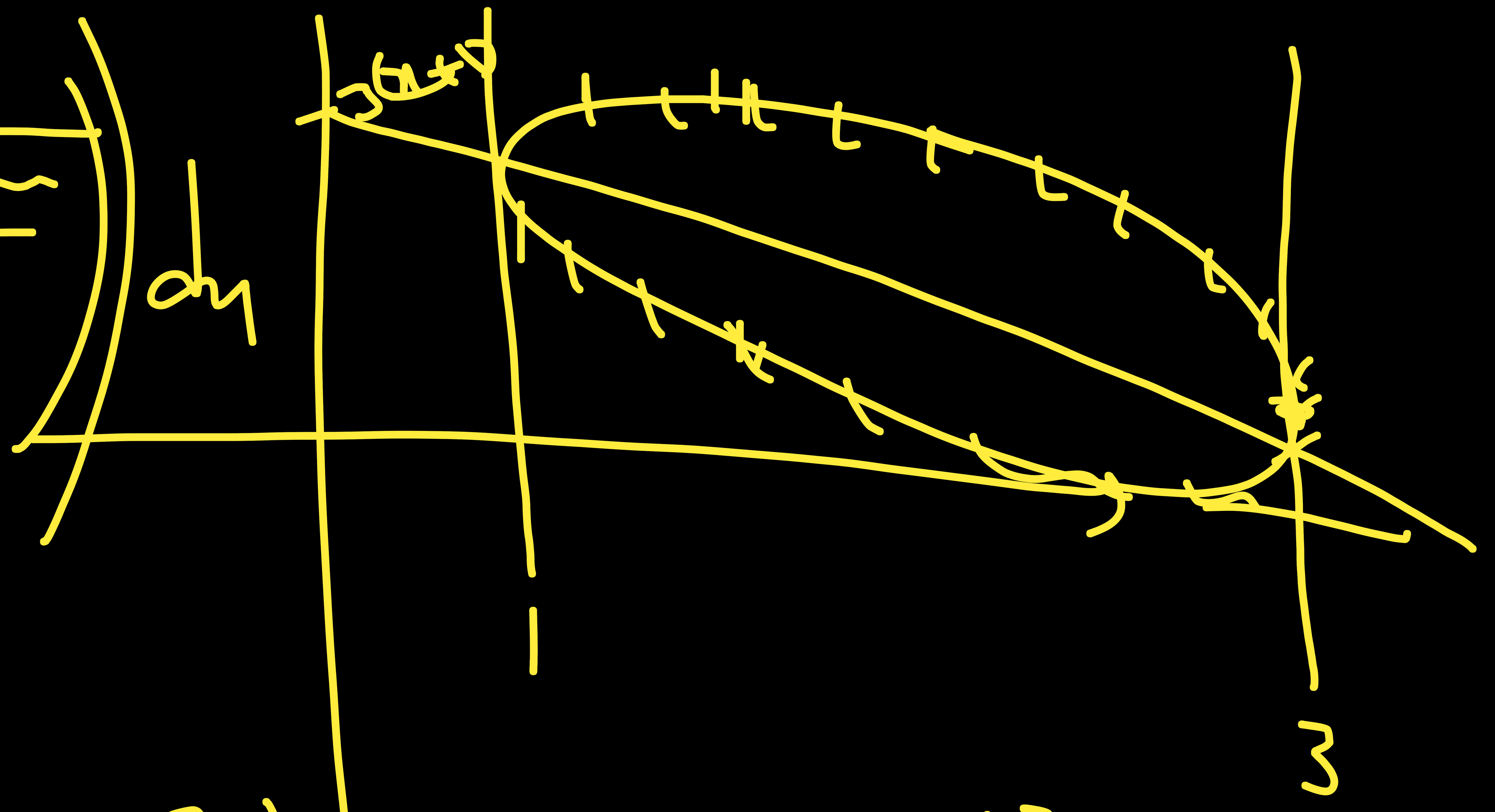
$$y=\frac{-6(x+1)\pm\sqrt{36(x^2+2x+1)-8(5x^2+7x+6)}}{2\cdot 2}$$

$$=\frac{-3(x+1)\pm\sqrt{-x^2+4x-3}}{2}=\frac{-3(x+1)\pm\sqrt{-(x^2-4x+3)}}{2}$$

$x^2-4x+3\le 0$
$\Rightarrow x\in[1,3]$

$$y=\frac{-3(x+1)+\sqrt{(x-1)(3-x)}}{2} \quad \& \quad y=\frac{-3(x+1)-\sqrt{(x-1)(3-x)}}{2}$$

$$\text{R. Area} = \int_1^3 \left(\left(\frac{-3(x+1)+\sqrt{\sim}}{2} \right) - \left(\frac{-3(x+1)-\sqrt{\sim}}{2} \right) \right) dx$$



$$= \int_1^3 \sqrt{(x-1)(3-x)} \; dx$$

$$x = 3\sin^2(\theta) + \cos^2(\theta) \longrightarrow x-1 = 2\sin^2(\theta), \quad 3-x = 2\cos^2(\theta)$$

$$dx = 2(3\, s.c - s c)\, d\theta$$

$$= 2(2\, s.c.\, d\theta) = 2\sin(2\theta)\, d\theta$$

$$x=1 \rightarrow s^2 = 3s^2 \Rightarrow \theta = 0$$

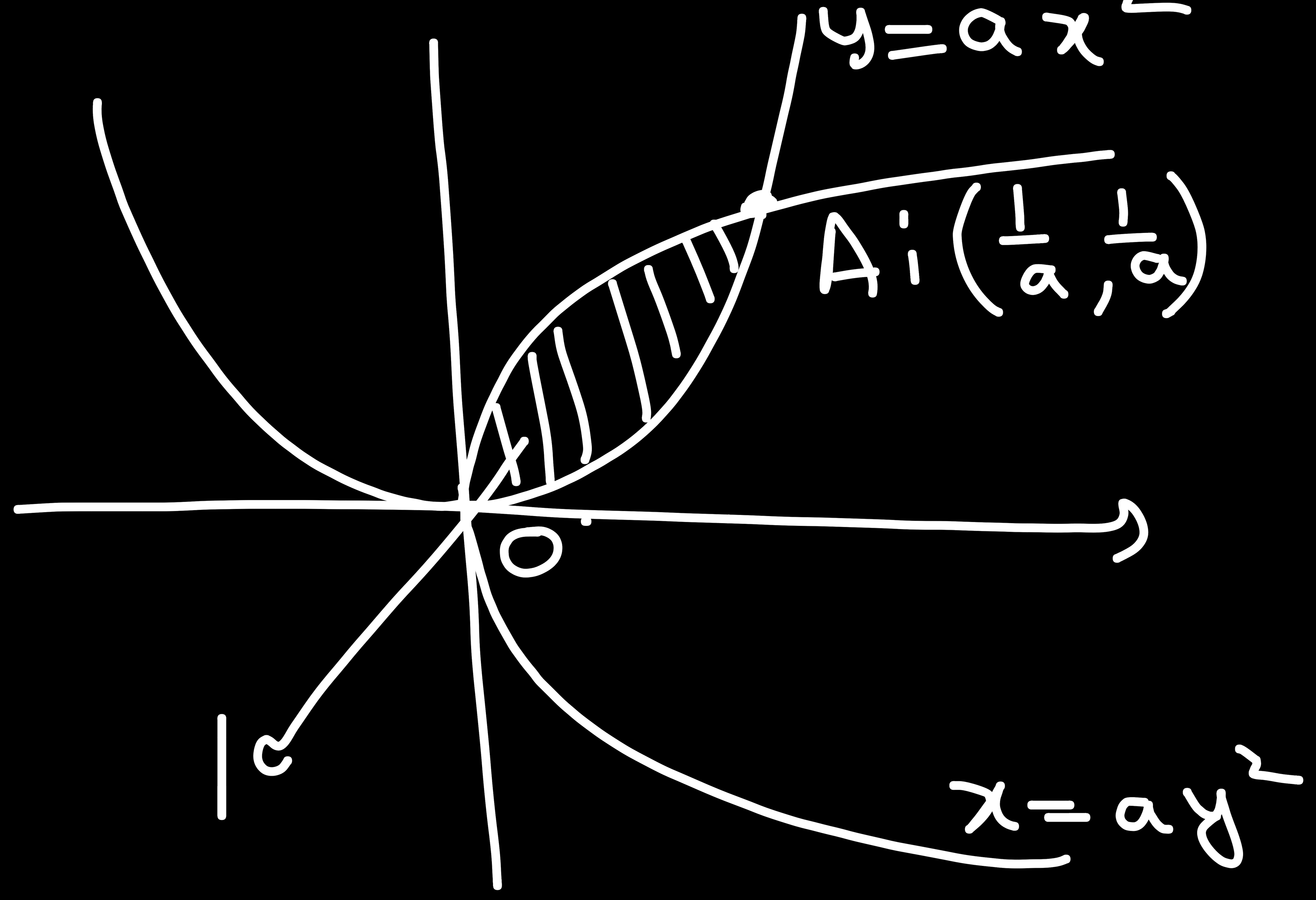
$$x=3 \rightarrow 3c^2 = c^2 \Rightarrow \theta = \frac{\pi}{2}$$

$$= 2\int_0^{\pi/2} \sqrt{2s^2 \cdot 2c^2} \cdot 2s.c.d\theta$$

$$= 2\int_0^{\pi/2} \sin^2(2\theta)\, d\theta = \frac{2}{2}\int_0^{\pi/2} (1-\cos(4\theta))\, d\theta = \frac{2}{2}\left(\frac{\pi}{2} - \left[\frac{\sin(4\theta)}{4} \right]_0^{\pi/2} \right) = \frac{\pi}{2}.$$

Que If area enclosed by $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1-square unit then find 'a'.

Soln



for A

$y = a(ay^2)^2$

$y = a^3y^4 \Rightarrow y = \frac{1}{a}$

$$\text{Area} = 1 = \int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx$$

$$1 = \frac{2}{3}\frac{x^{3/2}}{\sqrt{a}}\Bigg]_0^{1/a} - \frac{a}{3}\left[x^3\right]_0^{1/a}$$

$$1 = \frac{2}{3}\frac{1}{a^2} - \frac{\cancel{a}}{3}\cdot\frac{1}{\cancel{a^3}a^2} = \frac{1}{3a^2}$$

$$a^2 = \frac{1}{3} \Rightarrow \boxed{a = \frac{1}{\sqrt{3}}}$$ Ans

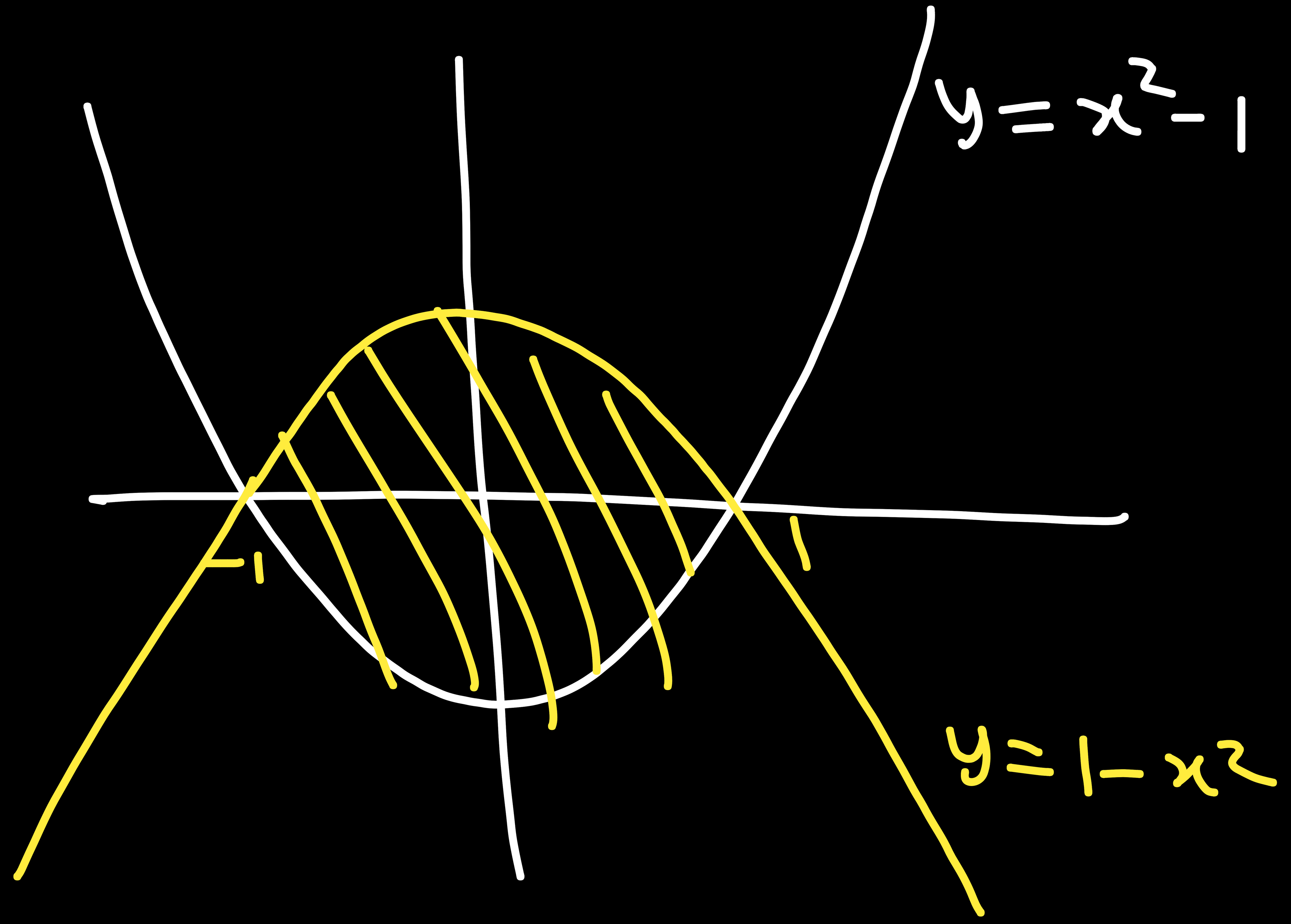
Que: Find the area enclosed by $x^4+1 = 2x^2+y^2$.

Sol:

$$x^4 - 2x^2 + 1 = y^2$$

$$(x^2-1)^2 = y^2$$

$$\Rightarrow \quad y = \pm(x^2-1)$$



$$R.A. = 2\int_{-1}^{1}(1-x^2)\,dx$$

$$= 4\int_{0}^{1}(1-x^2)\,dx$$

$$= 4\left[x - \frac{x^3}{3}\right]_0^1 = 4\left(1-\frac{1}{3}\right) = \boxed{8/3}$$

Ans

Que Let $T_n = \sum_{2n}^{3n-1} \frac{r}{r^2+n^2}$ & $S_n = \sum_{2n+1}^{3n} \frac{r}{r^2+n^2}$, for $n \in N$, then identify correct statement(s):

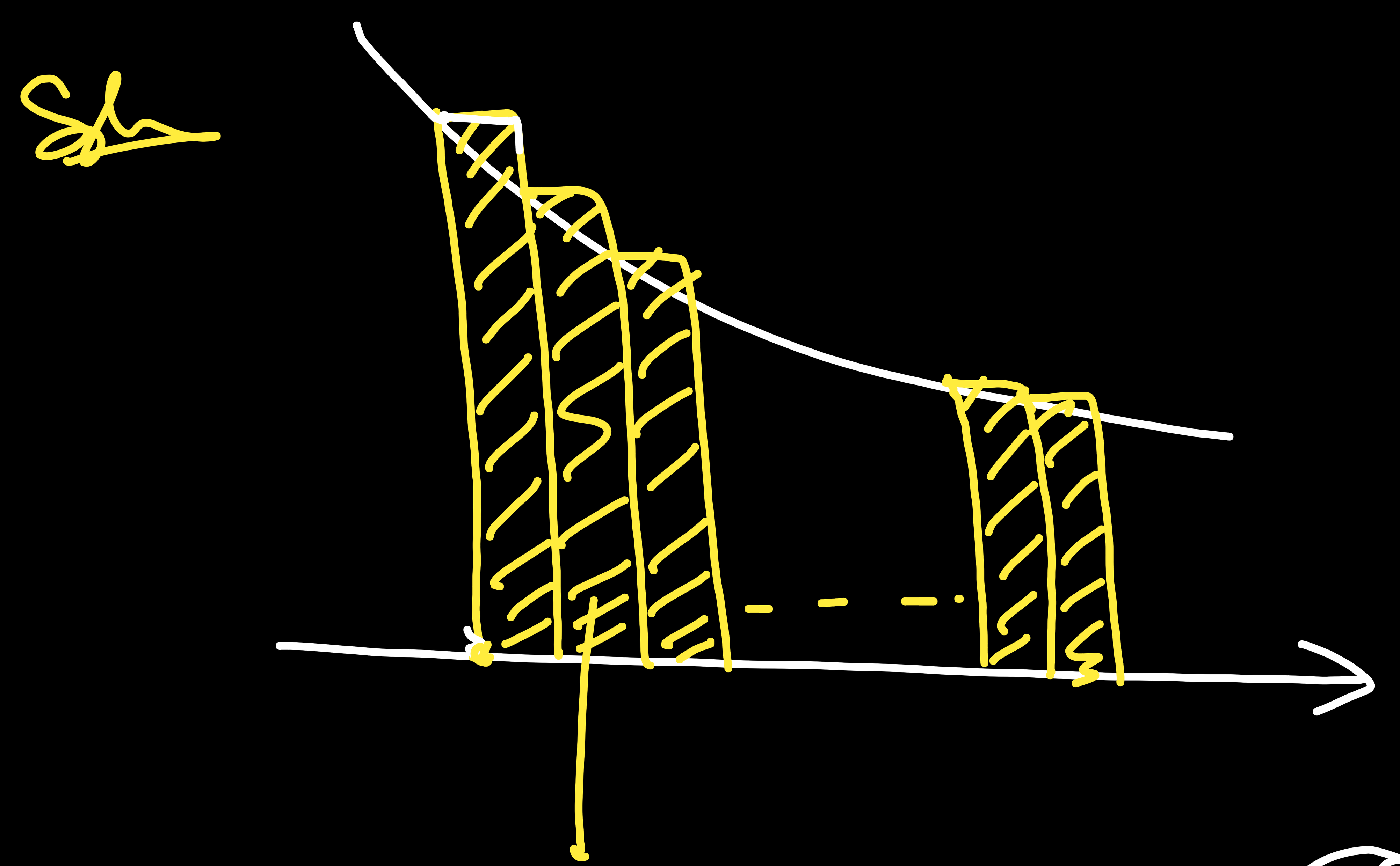
(A) $T_n > \frac{1}{2}\ln(2)$ (B) $S_n < \frac{1}{2}\ln(2)$ (C) $T_n < \frac{1}{2}\ln(2)$ (D) $S_n > \frac{1}{2}\ln(2)$.

$$\lim_{n\to\infty} \sum_{2n}^{3n-1} \frac{r}{r^2+n^2} = \lim_{n\to\infty} \frac{1}{n}\sum_{2n}^{3n-1} \frac{r/n}{\left(\frac{r}{n}\right)^2+1}$$

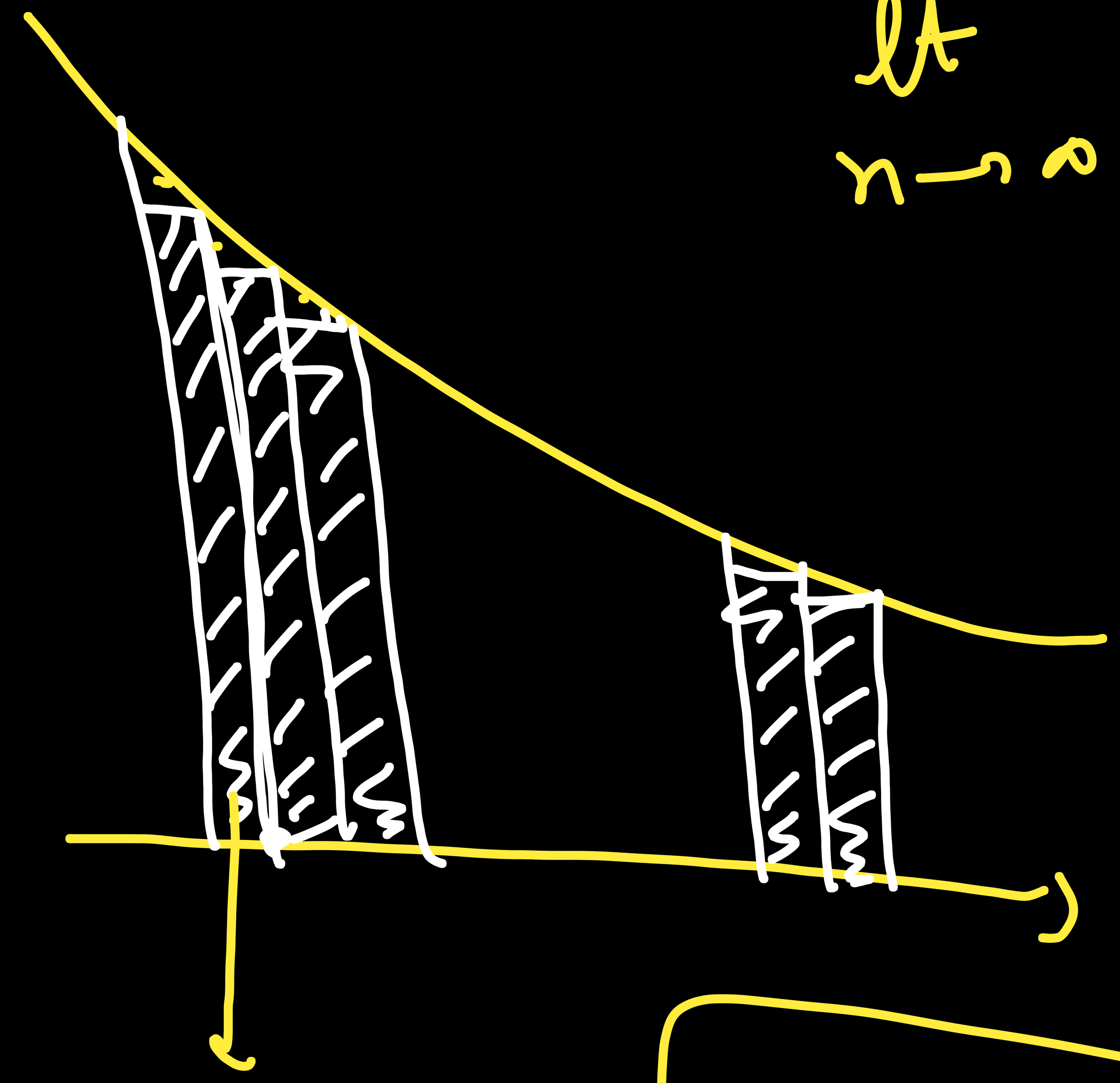
$$= \int_2^3 \left(\frac{x}{x^2+1}\right) dx$$

$$= \frac{1}{2}\ln(x^2+1)\Big]_2^3$$

$$I = \frac{1}{2}\ln\left(\frac{10}{5}\right) = \frac{1}{2}\ln(2)$$



Area = T_n



(A), (B) S_n

$$\boxed{S_n < I < T_n}$$

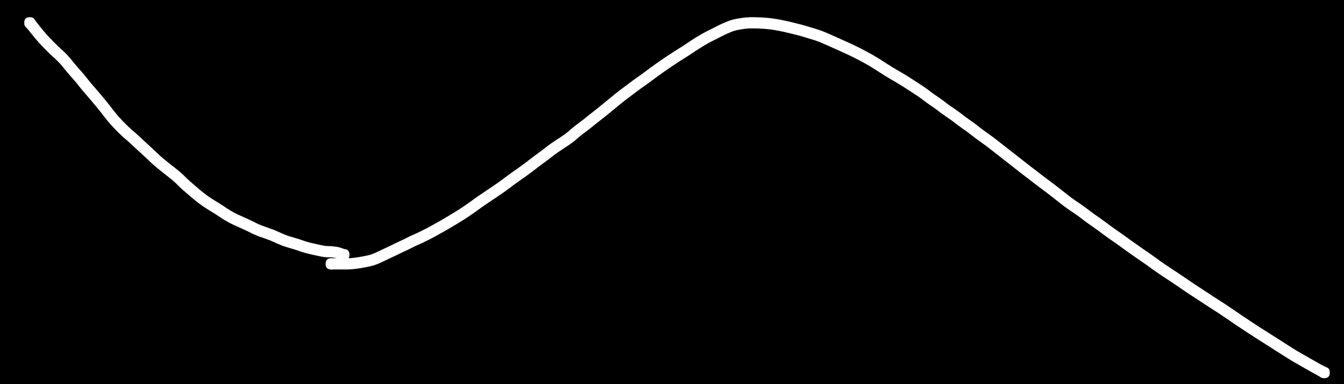
$$f(x) = \frac{x}{x^2+1}$$

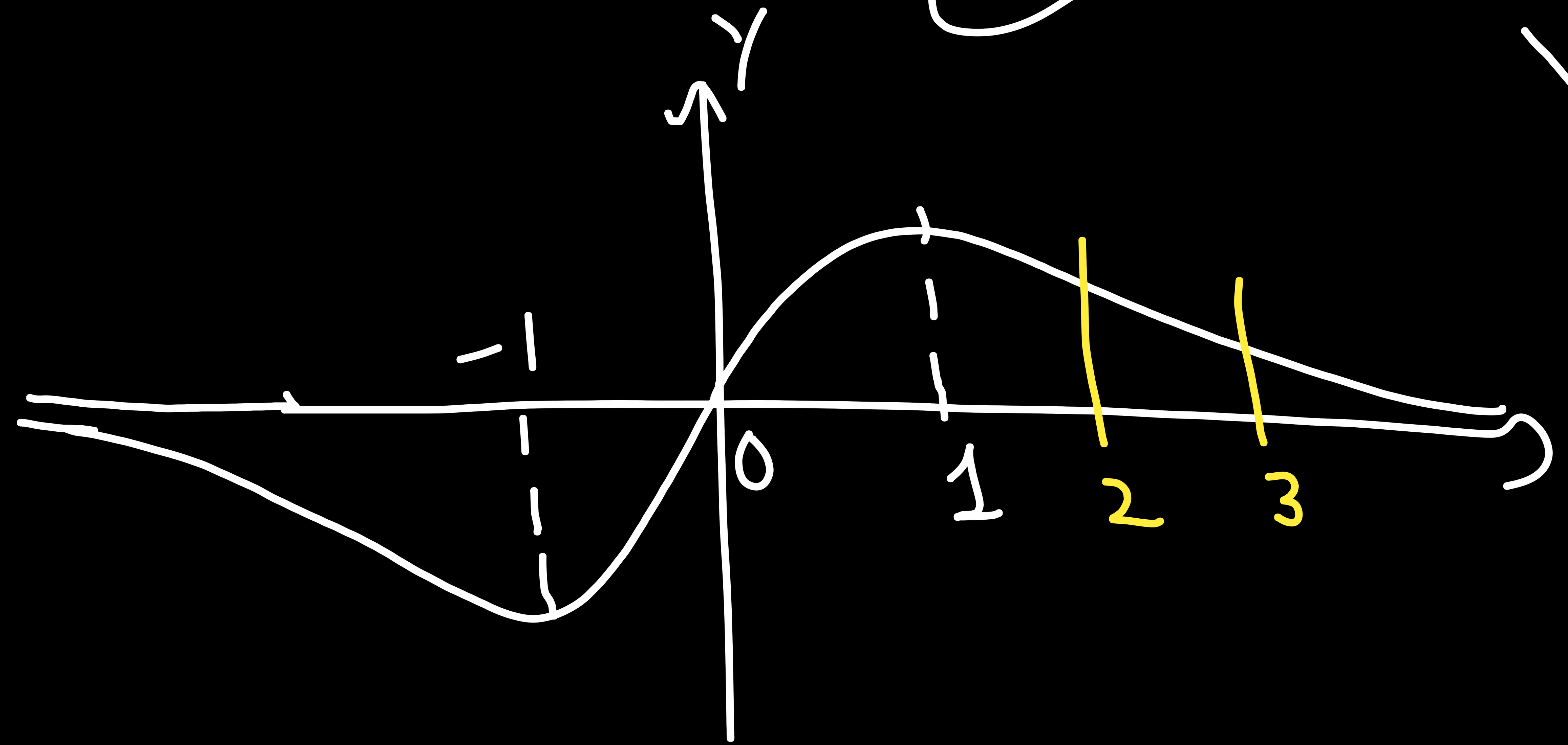
$$f'(x) = \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}.$$

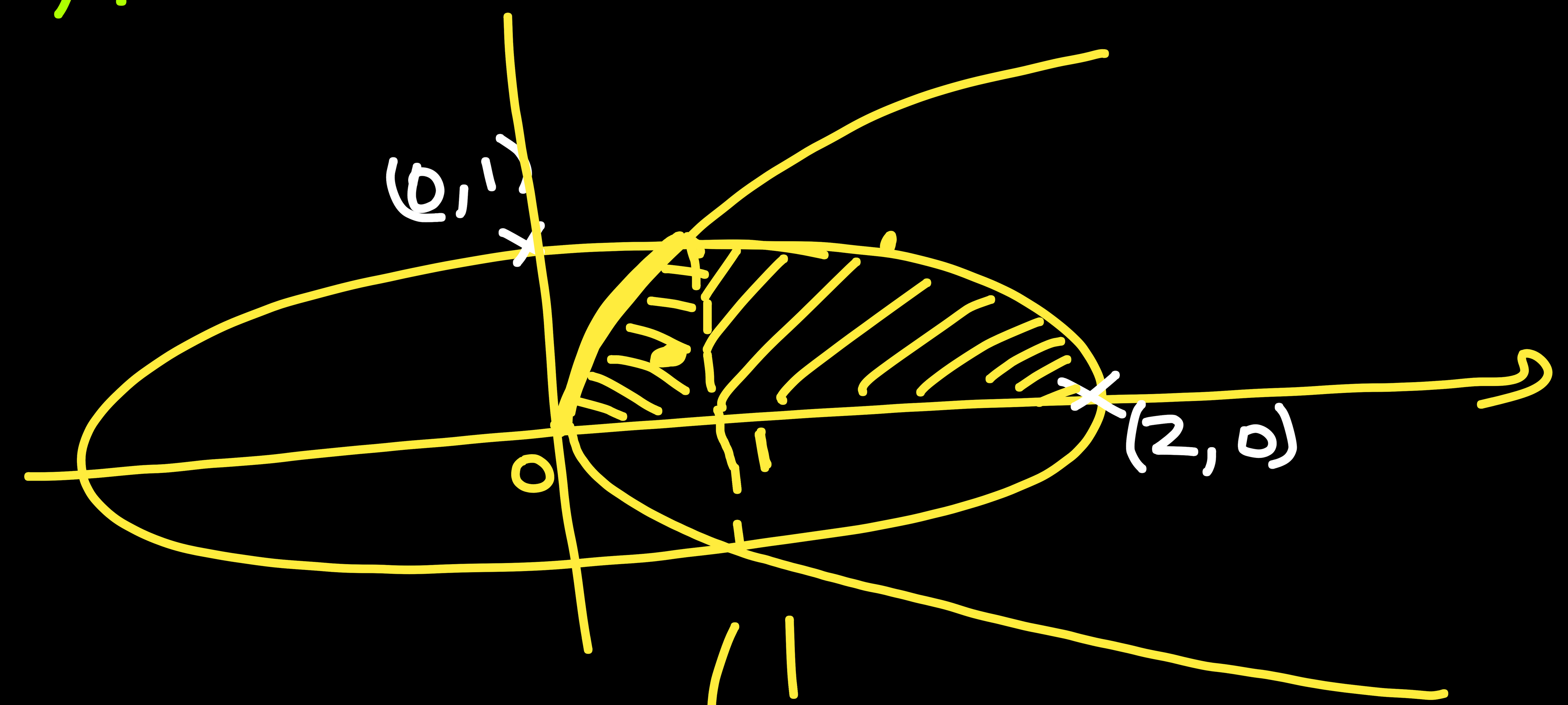
sign of f'

$-$	-1	$+$	1	$-$





Que Find area of the smaller region bounded by $\frac{(x-1)^2}{4}+(y-1)^2=1$ and $(y-1)^2=\frac{3}{4}(x-1)$.



Sol $\frac{(x-1)^2}{4}+(y-1)^2=1$

$(y-1)^2=\frac{3}{4}(x-1)$

$\Downarrow$

$\frac{x^2}{4}+y^2=1$

$y^2=\frac{3}{4}x$

$\Rightarrow \frac{x^2}{4}+\frac{3}{4}x=1$

$x=1$

$$R.A.=2\left(\int_0^1 \frac{\sqrt{3}}{2}\sqrt{x}\,dx+\int_1^2\sqrt{1-\frac{x^2}{4}}\,dx\right)$$

Ans

Que Let S be the area bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$, then identify correct statement(s).

(A) $S \geq \frac{1}{e}$ (B) $S \geq 1 - \frac{1}{e}$ (C) $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

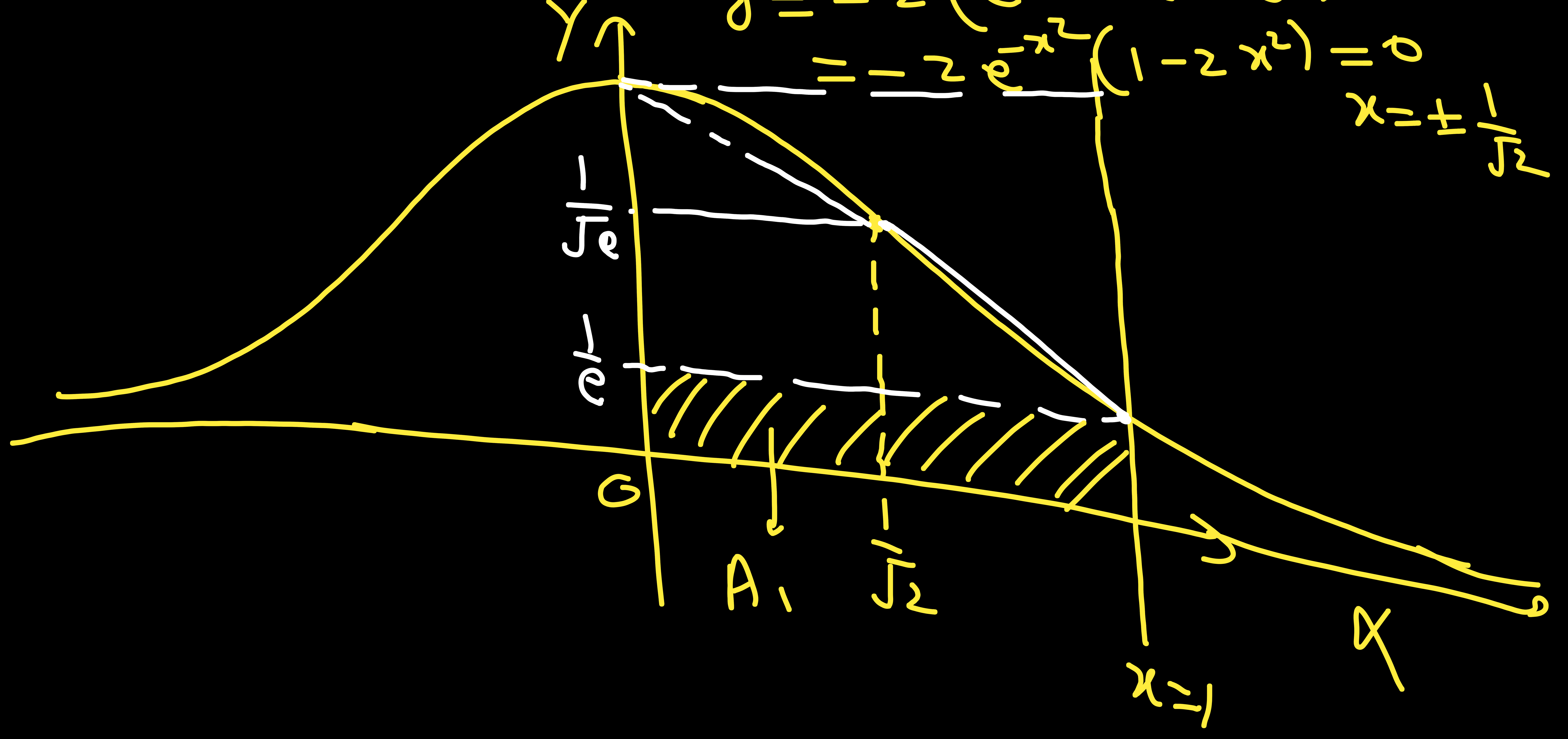
Sol

$$y = e^{-x^2} \Rightarrow y' = -2x e^{-x^2}$$

$$y'' = -2\left(e^{-x^2} - 2x^2 e^{-x^2}\right)$$

$$= -2e^{-x^2}(1 - 2x^2) = 0$$

$$x = \pm\frac{1}{\sqrt{2}}$$



Clearly $S > A_1 = 1 \cdot \frac{1}{e}$

$S > \frac{1}{e}$. (A) ✓

In $(0,1)$ $e^x > e^{x^2}$

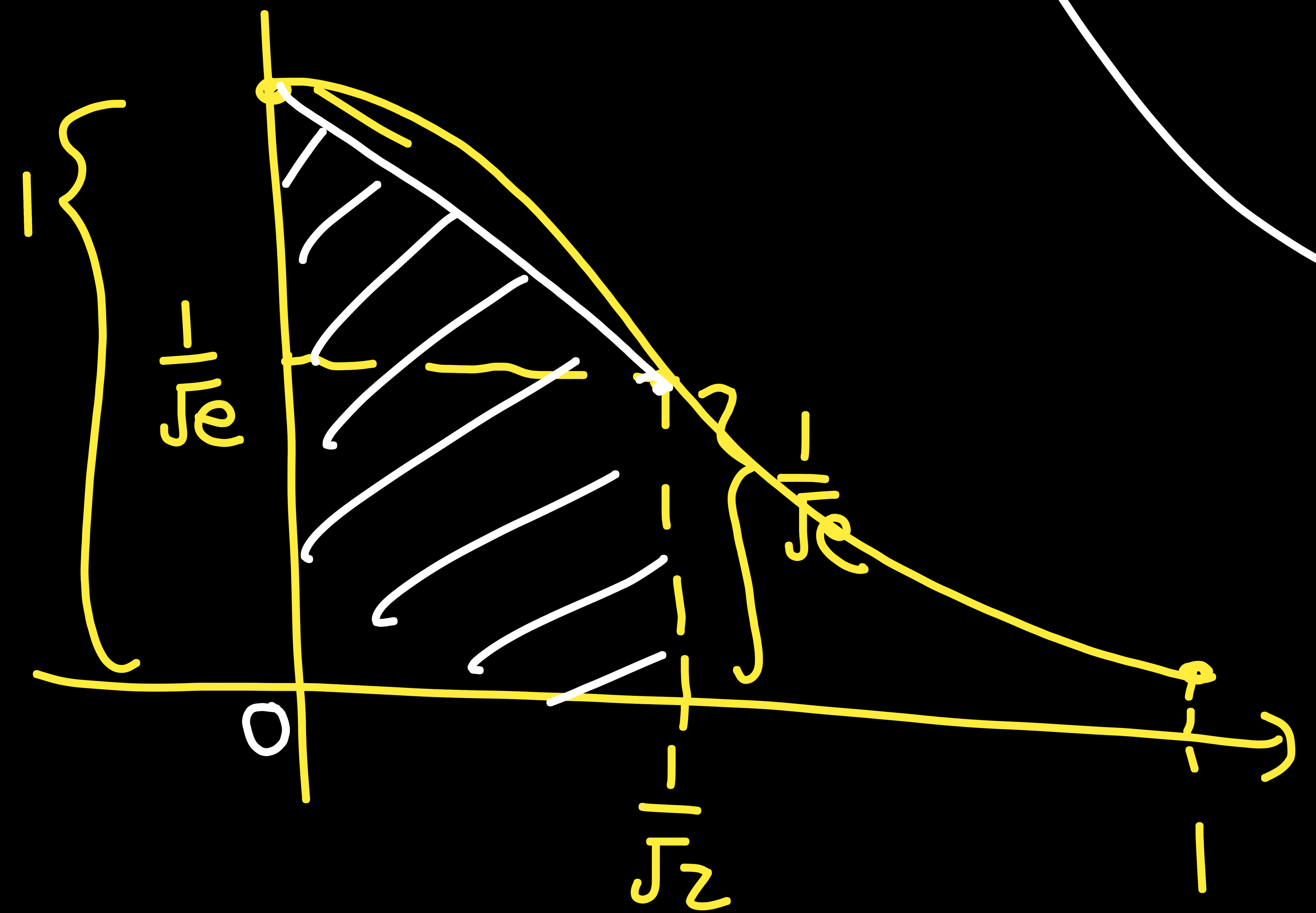
$$e^{-x} < e^{-x^2}$$

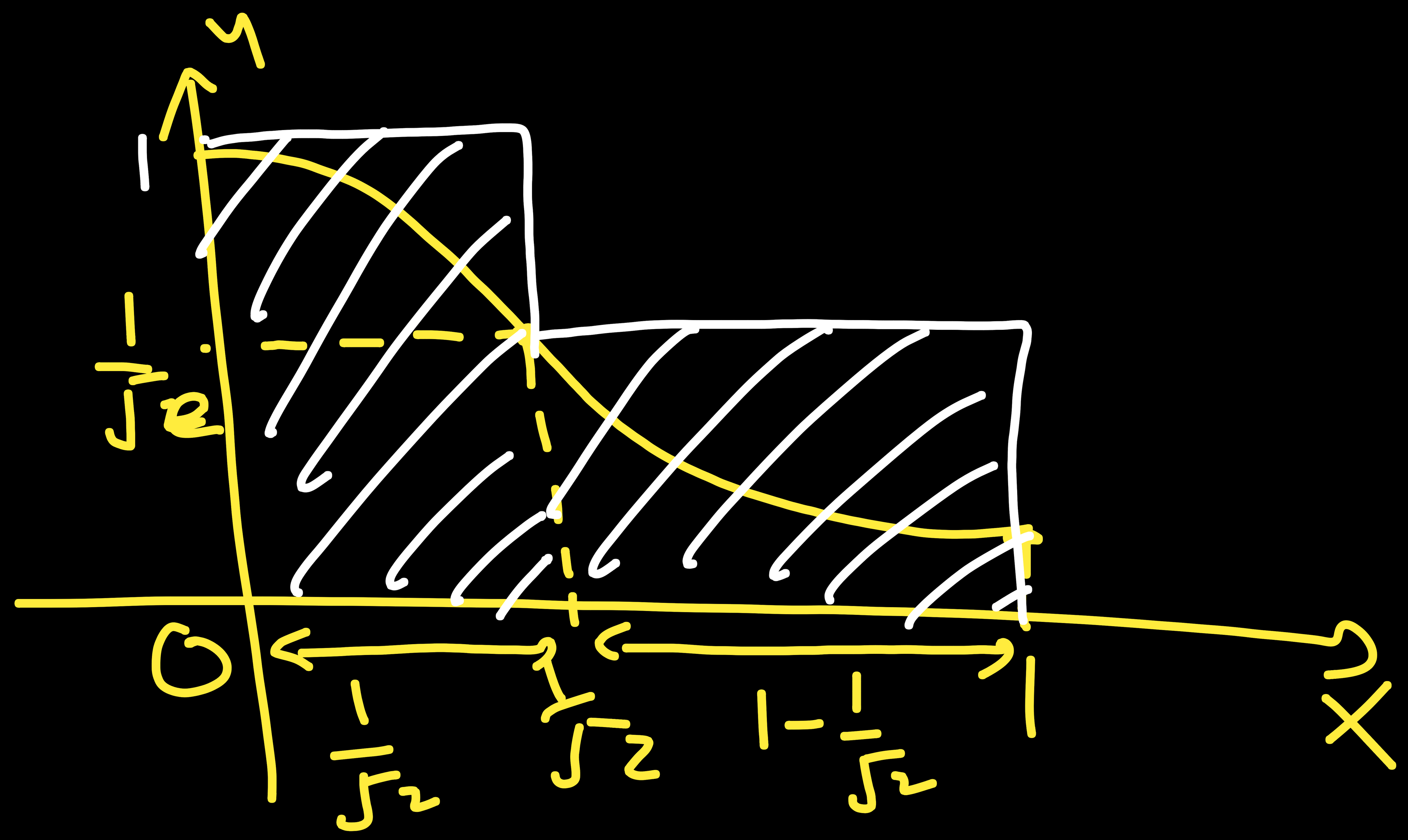
$$\int_0^1 e^{-x}\,dx < \int_0^1 e^{-x^2}\,dx$$

$$\left[e^{-x}\right]_1^0 = 1 - \frac{1}{e}$$

$$\Rightarrow S > 1 - \frac{1}{e}$$ (B) ✓

C $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ D $S \le \frac{1}{\sqrt{2}}\cdot 1 + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$





A B D

$$S > \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{e}}+1\right) > \frac{1}{2}\cdot\frac{1}{2}\left(\frac{1}{\sqrt{e}}+1\right)$$
$$= \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$$

$\cancel{C}$.

$$S < 1\cdot\frac{1}{\sqrt{2}} + \left(1-\frac{1}{\sqrt{2}}\right)\cdot\frac{1}{\sqrt{e}}.$$

D.

Que Find area of the region in which $P:(x, y)$ can move while satisfying $x^2+y^2 \le 100$ and $2n \le x+y \le 2n+1$, $\forall n \in I$.

Sol

$x^2+y^2 \le 100$

$2n \le x+y \le 2n+1$

$\boxed{0 \le x+y \le 1}$

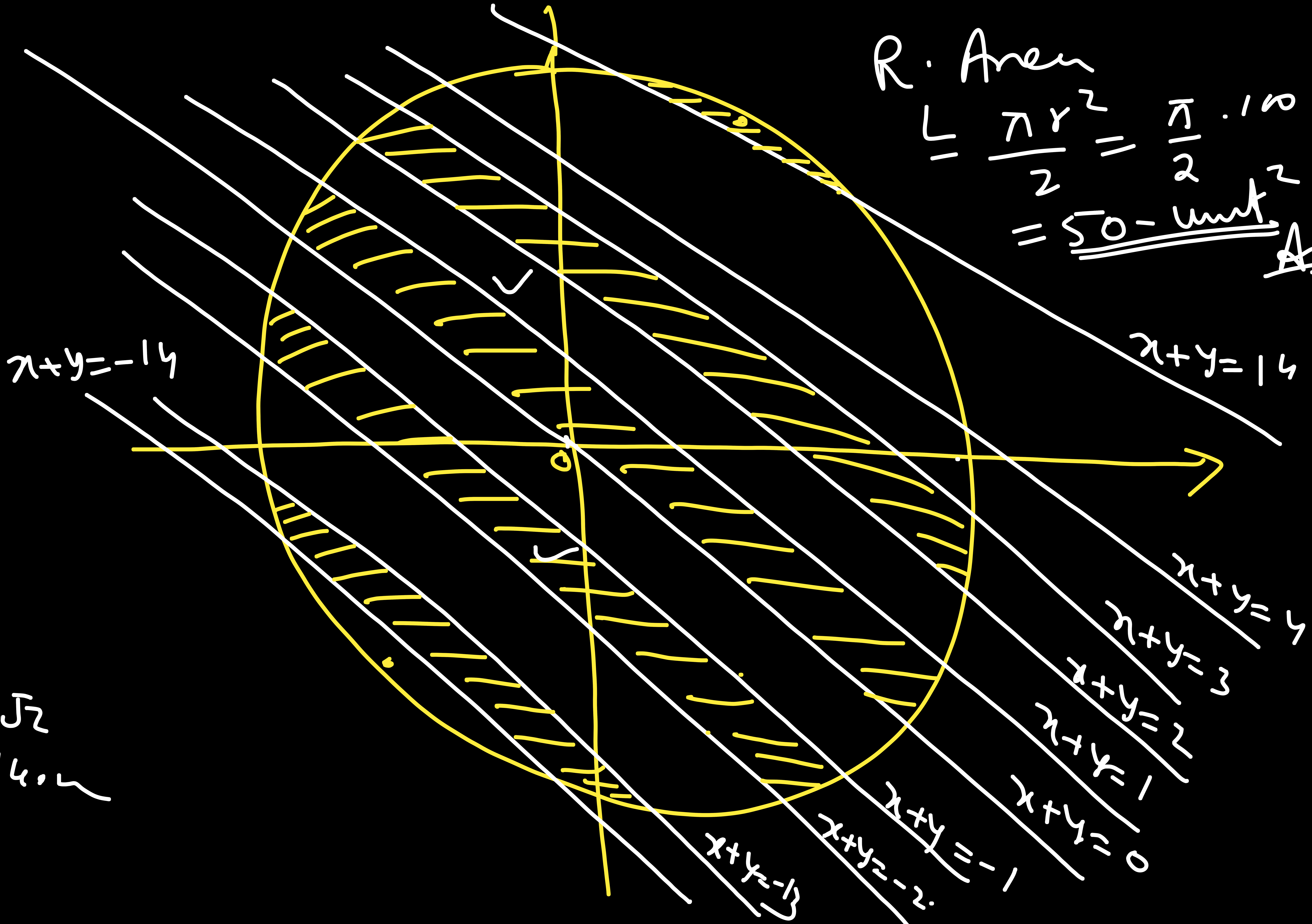
$2 \le x+y \le 3$

$-2 \le x+y \le -1$

$x+y = n$

$\frac{n}{\sqrt{2}} = 10 \Rightarrow n = 10\sqrt{2}$

$= 14.\ldots$



R. Area

$= \frac{\pi r^2}{2} = \frac{\pi}{2} \cdot 100$

$= \underline{\underline{50\text{ - unit}^2}}$

Ans.

Que. Find $b\ (\in R^+)$ if the area bounded by $y = x - bx^2$ & $y = \frac{x^2}{b}$ is maximum.

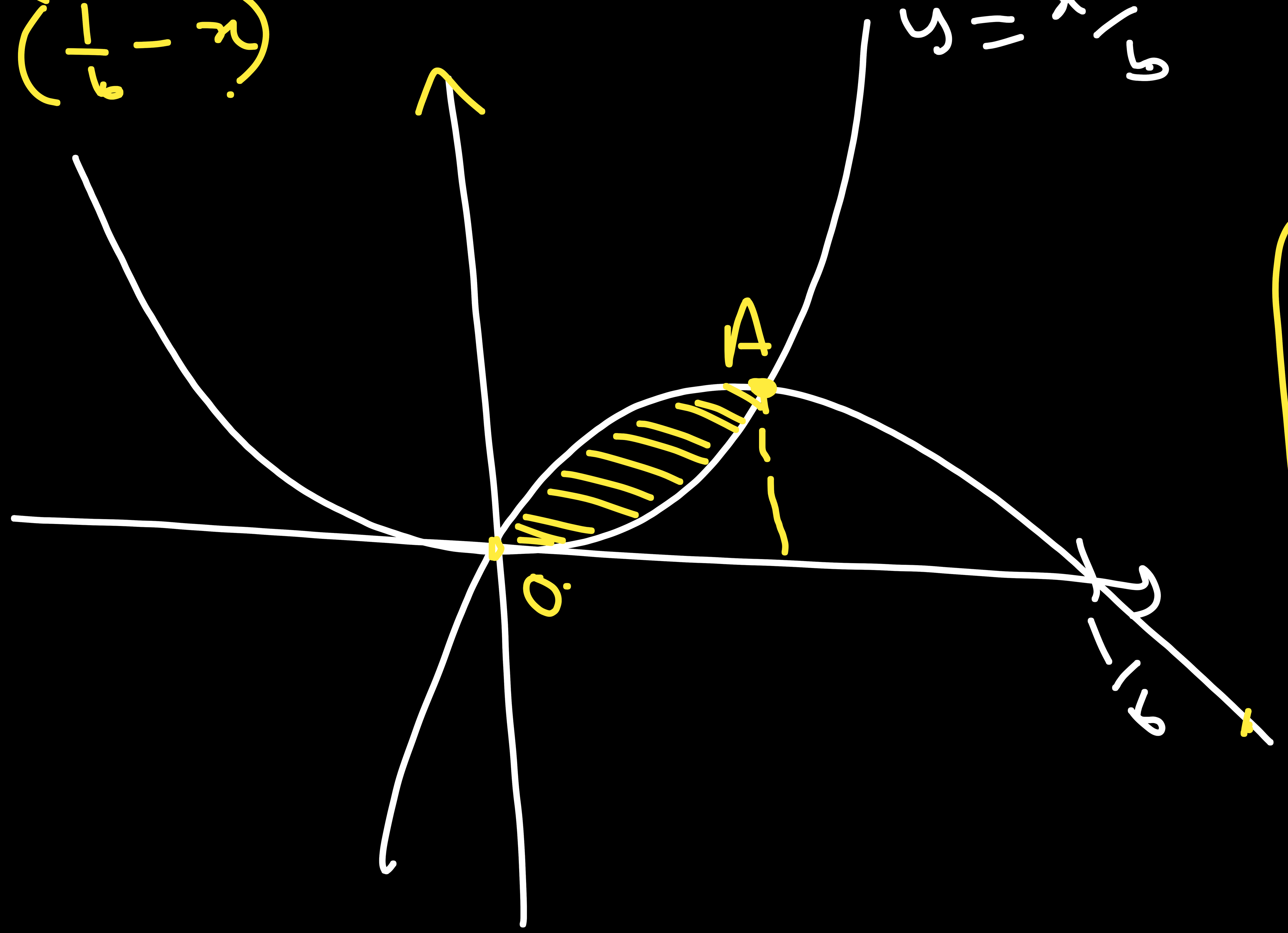
Sol.

$$y = x - bx^2$$
$$= x(1 - bx)$$
$$= bx\left(\frac{1}{b} - x\right)$$

$$y = \frac{x^2}{b}$$

for A

$$x - bx^2 = \frac{x^2}{b}$$
$$1 - bx = \frac{x}{b} \Rightarrow x\left(b + \frac{1}{b}\right) = 1$$
$$x = \frac{b}{b^2+1}$$
$$= b_1$$



$$\text{Area} = A(b) = \int_0^{b_1}\left((x - bx^2) - \frac{x^2}{b}\right)dx.$$

$$A'(b) = 0$$

$$b_1'\left(b_1 - \cancel{b b_1^2 - \frac{b_1^2}{b}}\right) + \int_0^{b_1}\left(-x^2 + \frac{x^2}{b^2}\right)dx = 0.$$

$$\int_0^{b_1} x^2\left(\frac{1}{b^2} - 1\right).dx = 0.$$

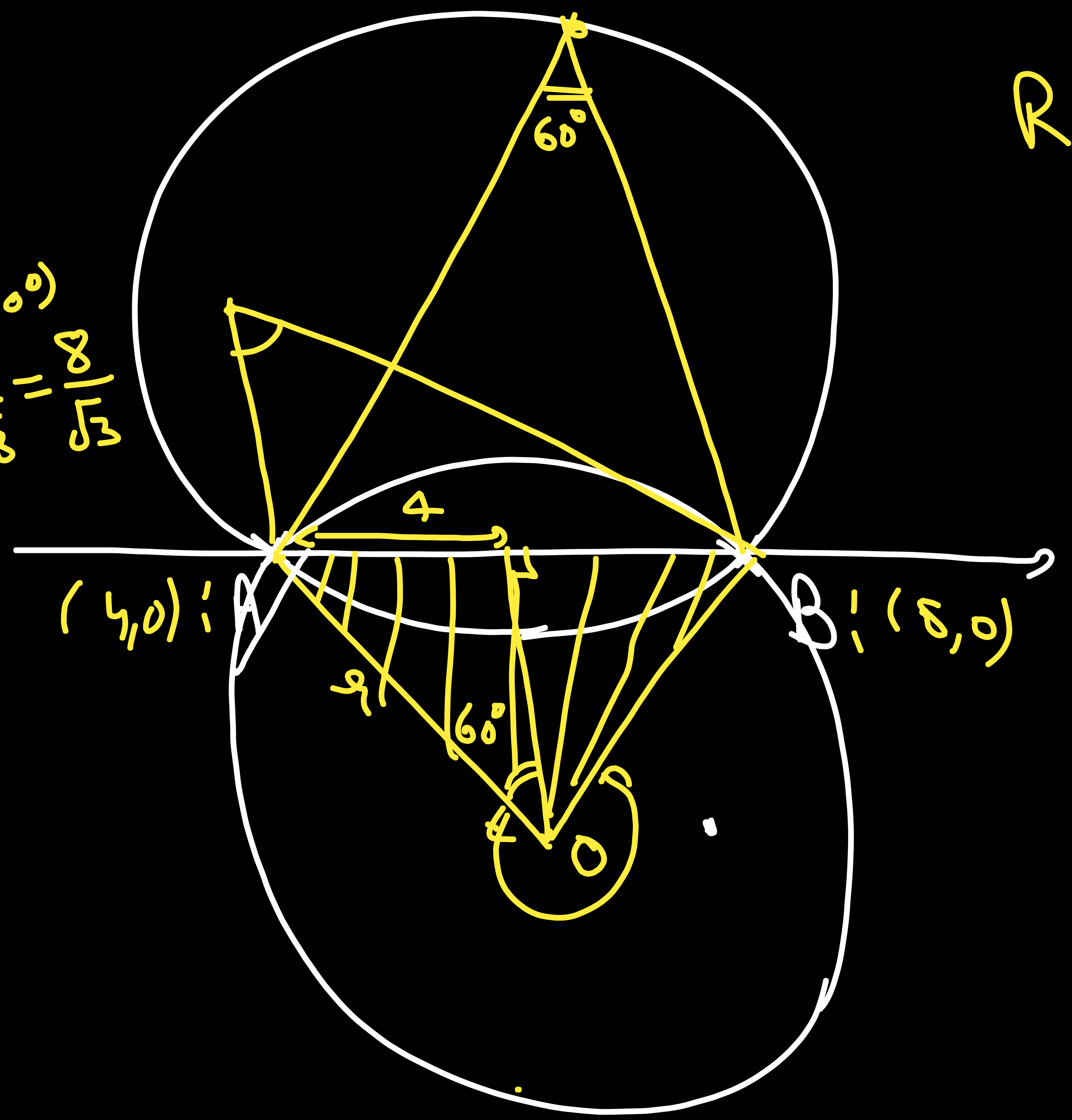
$$\frac{x^3}{3}\left(\frac{1}{b^2} - 1\right) = 0 \Rightarrow \boxed{b = 1}$$

$$\Rightarrow \frac{b_1^3}{3}\left(\frac{1}{b^2} - 1\right) = 0$$

A

Que Let A : (4, 0) and B : (8, 0), if P is a variable point such that $\angle APB > 60^\circ$ then find area of the region in which P can move.

Sol



$r = 4\csc(60^\circ)$

$= 4 \cdot \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$

$$R.A. = \cancel{2} \cdot \left(\frac{1}{\cancel{2}} \cdot r^2 \cdot \sin(120^\circ) + \frac{1}{\cancel{2}} \cdot r^2 \cdot \frac{4\pi}{3} \right)$$

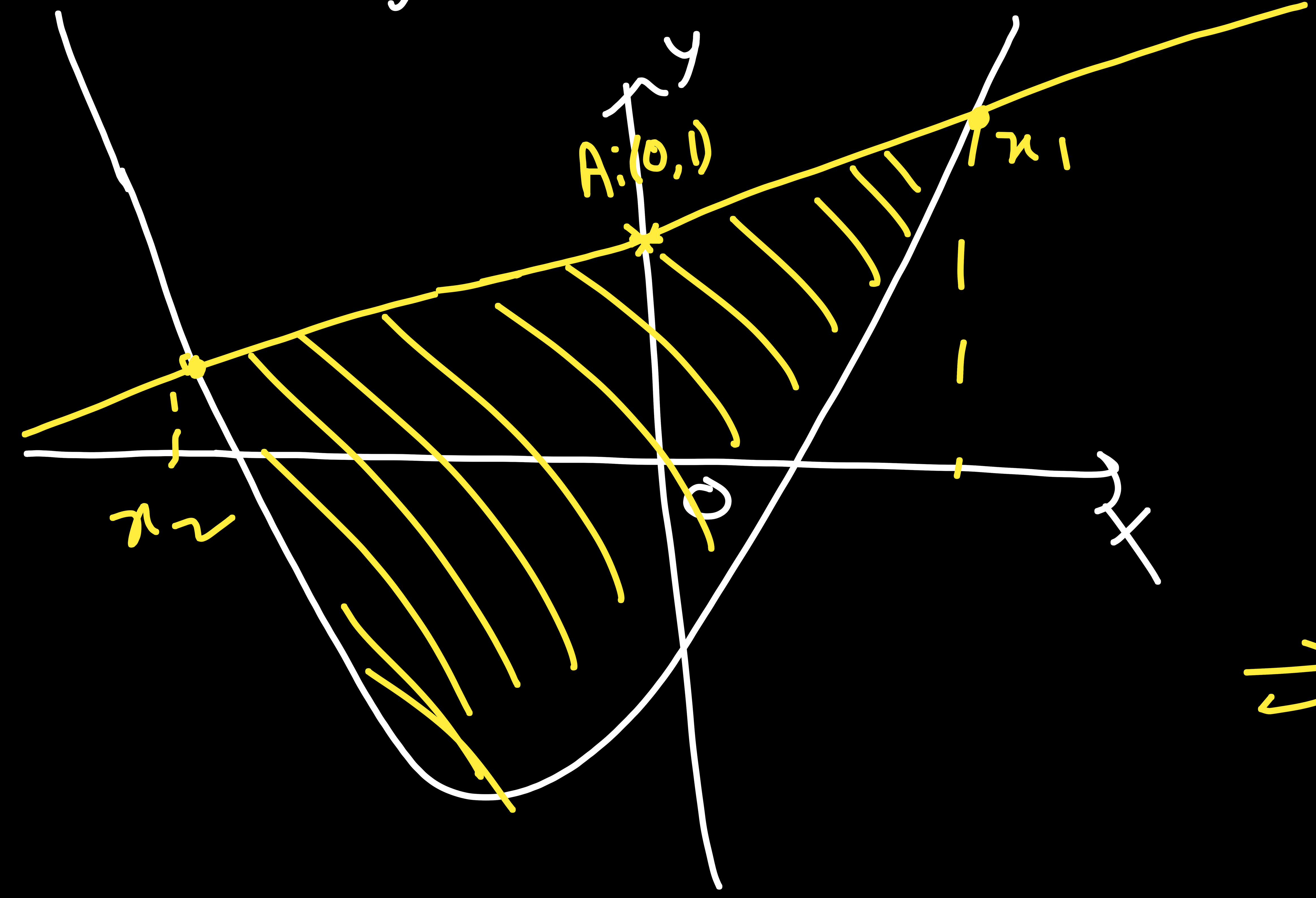
$$= \frac{64}{3} \left(\frac{\sqrt{3}}{2} + \frac{4\pi}{3} \right) \text{ unit}^2$$

Que If the area bounded by $y = x^2+2x-3$ and the line $y = kx+1$ is least, then find 'k' and the corresponding area.

Sol

$y = x^2+2x-3$

$y = kx+1$



for x_1, x_2

$x^2+2x-3 = kx+1 \longrightarrow x_1, x_2.$

$\hookrightarrow k-2 = x_1+x_2$

$$A(k) = \int_{x_2}^{x_1} \left(kx+1-(x^2+2x-3)\right)dx$$

$$A'(k) = 0 = x_1' \cdot \cancel{\left(kx_1+1-(x_1^2+2x_1-3)\right)} - x_2'\cancel{\left(kx_2+1-(x_2^2+2x_2-3)\right)} \;\; 0$$

$$+ \int_{x_2}^{x_1} x\,dx$$

'+'

$$\Rightarrow \frac{x_1^2-x_2^2}{2} = 0 \Rightarrow \frac{(x_1-x_2)(x_1+x_2)}{2} = 0$$

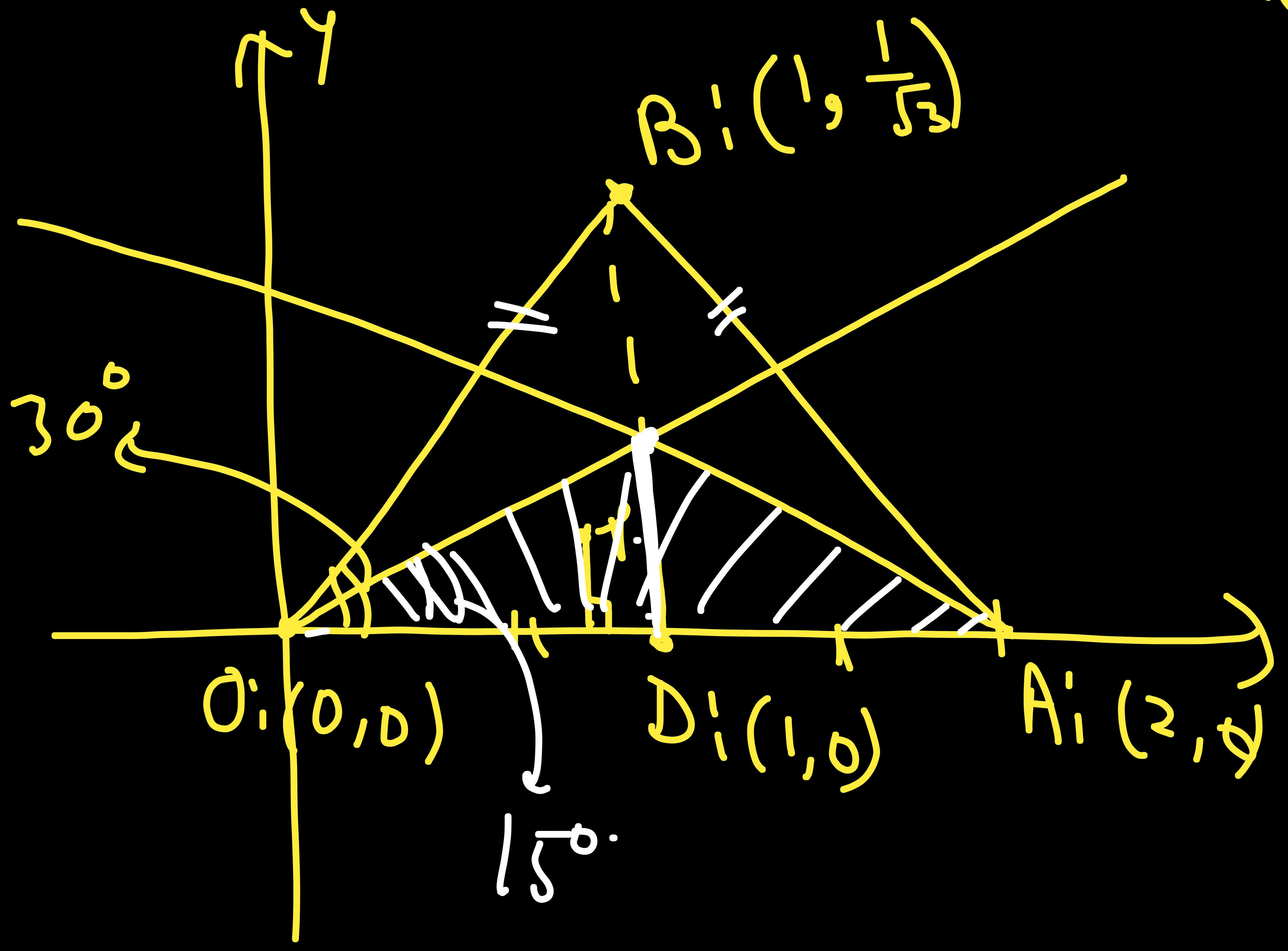
$x_1 + x_2 = 0.$

$\hookrightarrow k - 2 = 0 \Rightarrow \boxed{K = 2}$

Corresponding Area $=$ D.I.Y.

Que Let $O:(0,0)$, $A:(2,0)$ & $B:(1, \frac{1}{\sqrt{3}})$ be the vertices of a triangle and let R be the region consisting of all those points P inside the ΔOAB such that $d(P, OA) \leq \min\{d(P,OB), d(P, AB)\}$ where $d(P, L)$ denotes distance of point P from line 'L'. Find area of region 'R'.

Sol



✓ $d(P, OA) \leq d(P, OB)$

$d(P, OA) \leq d(P, AB)$.

$$R.\text{Area} = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \left(1 \cdot \tan(15^\circ)\right)$$

$$= \tan(15^\circ)$$

Ans