Que Let integral of $f(x) = \frac{x^{2009}}{(1+x^2)^{1006}}$ w.r.t. x is equal to $\frac{1}{n}\left(\frac{x^2}{1+x^2}\right)^m + C$, then find $m+n$.

Soln

$$\int \frac{x^{2009}}{(1+x^2)^{1006}} \cdot dx = \int \frac{x^{2009}}{\left(\frac{1}{x^2}+1\right)^{1006} \cdot x^{2012}} dx$$

$$= \int \frac{\frac{1}{x^3} \cdot dx}{\left(\frac{1}{x^2}+1\right)^{1006}} = \frac{1}{-2}\int \frac{\overbrace{\frac{-2}{x^3}}^{f'} dx}{(\underbrace{\tfrac{1}{x^2}+1}_{f})^{1006}} = -\frac{1}{2}\frac{\left(\frac{1}{x^2}+1\right)^{-1005}}{(-1005)} + C$$

$$= \frac{1}{2010}\left(\frac{x^2}{1+x^2}\right)^{1005} + C \Rightarrow \text{Company}$$

$\left.\begin{matrix} n = 2010 \\ m = 1005 \end{matrix}\right\} \Rightarrow m+n = \boxed{3015}$ Ans

Que Integrate $\int \frac{1+x\cos(x)}{x(1-x^2 e^{2\sin(x)})}\cdot dx$

Sol

$$\int \frac{1+x\cdot\cos(x)}{x\cdot(1-x^2\cdot e^{2\sin(x)})}\cdot dx = \int \frac{(1+x\cos(x))\cdot e^{\sin(x)}}{x\cdot e^{\sin(x)}\left(1-\left(xe^{\sin(x)}\right)^2\right)}\cdot dx$$

$$x\cdot e^{\sin(x)} = t$$

$$\left(e^{\sin(x)}+x\cdot\cos(x)\cdot e^{\sin(x)}\right)dx = dt$$

$$= \int \frac{dt}{t(1-t^2)} = \int \frac{dt}{t^3\left(\frac{1}{t^2}-1\right)}$$

$$= \frac{1}{-2}\int \frac{\overset{f'}{\left(-\frac{2}{t^3}\right)}\,dt}{\underset{f}{\frac{1}{t^2}-1}} = -\frac{1}{2}\ln\left|\frac{1}{t^2}-1\right| + C = -\frac{1}{2}\ln\left|\frac{1}{x^2 e^{2\sin(x)}}-1\right| + C.$$

Ans

Que Integrate $\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx$

Sol

$$\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx = \int \frac{(x^2-1)(x^2+1)}{x^2\sqrt{x^4+x^2+1}}\,dx$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)\cdot(x^2+1)}{x\cdot\sqrt{x^2+1+\frac{1}{x^2}}}\,dx = \int \frac{\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2-1}}\,dx$$

$$= \sqrt{\left(x+\frac{1}{x}\right)^2-1} + C \quad \text{Ans}$$

Rough

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

$$\left(\sqrt{x^2\pm1}\right)' = \frac{x}{\sqrt{x^2\pm1}}$$

Que Find $\int (x^6 + x^4 + x^2) \cdot (2x^4 + 3x^2 + 6)^{1/2} \cdot dx$

Sol

$$\int (x^6 + x^4 + x^2) \cdot (2x^4 + 3x^2 + 6)^{1/2} \cdot dx$$

x

$$= \int (x^5 + x^3 + x) \cdot (2x^6 + 3x^4 + 6x^2)^{1/2} \cdot dx$$

$$= \frac{1}{12} \int (\underbrace{12x^5 + 12x^3 + 12x}_{f'}) \cdot (\underbrace{2x^6 + 3x^4 + 6x^2}_{f})^{1/2} \cdot dx$$

$$= \frac{1}{12} \frac{(2x^6 + 3x^4 + 6x^2)^{3/2}}{\left(3/2\right)} + C \quad \text{Ans}$$

Que Evaluate $\displaystyle\int \frac{(x-\sin(x))^{3/2}}{\sqrt{x}} \cdot \left(\frac{6x^2\cdot \sin^2\left(\frac{x}{2}\right)}{x-\sin(x)} + 3x\right) \cdot dx$

Soln $\displaystyle\int \frac{(x-\sin(x))^{3/2}}{\sqrt{x}} \left(\frac{3x^2(1-\cos(x)) + 3x(x-\sin(x))}{x-\sin(x)}\right) \cdot dx$

$$= \cancel{3} \cdot \frac{\left(x^2 - x\sin(x)\right)^{3/2}}{\left(\cancel{3}/2\right)} + C$$

$$= 2\left(x^2 - x \cdot \sin(x)\right)^{3/2} + C$$

Que If $y(x-y)^2 = x$, and $\int \frac{dx}{x-3y} = \lambda \cdot \ln\left((x-y)^2 - 1\right) + C$ then find value of 'λ'.

Soln. $y(x-y)^2 = x \xrightarrow{\frac{d}{dx}} y'(x-y)^2 + 2y(x-y)(1-y') = 1.$

$$y'\left((x-y)^2 - 2y(x-y)\right) = 1 - 2y(x-y)$$

$$y' = \frac{1-2y(x-y)}{(x-y)(x-y-2y)} = \frac{1-2xy+2y^2}{(x-y)(x-3y)}$$

$$1-y' = 1 - \frac{1-2xy+2y^2}{(x-y)(x-3y)} = \frac{x^2-4xy+3y^2-1+2xy-2y^2}{(x-y)(x-3y)}$$

$$1-y' = \frac{x^2-2xy+y^2-1}{(x-y)(x-3y)} = \frac{(x-y)^2-1}{(x-y)(x-3y)}$$

$$\int \frac{dx}{x-3y} = \lambda \cdot \ln\left((x-y)^2 - 1\right)$$

$\frac{d}{dx}$:

$$\frac{1}{x-3y} = \lambda\left(\frac{2(x-y)\cdot(1-y')}{(x-y)^2-1}\right)$$

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$$1 - y' = \frac{(x-y)^2 - 1}{(x-y)(x-3y)}$$

$$\frac{(x-y)(1-y')}{(x-y)^2 - 1} = \frac{1}{x-3y}$$

from ①

$$\frac{1}{x-3y} = \lambda \frac{2 \cdot (x-y)(1-y')}{(x-y)^2 - 1}$$

$\Rightarrow$ Comparing $2\lambda = 1$

⇒

$$\boxed{\lambda = \frac{1}{2}}$$ Ans.

Que Evaluate $\int \frac{x^3+x^2+x-1}{(x^2-1)\cdot\sqrt{2-x^2}}\cdot dx$

Sol

$$\int \frac{x^3+x^2+x-1}{(x^2-1)\sqrt{2-x^2}}\cdot dx = \int \frac{x(x^2-1)+(x^2-1)+2x}{(x^2-1)\sqrt{2-x^2}}\,dx$$

$2x = (x-1)+(x+1)$

$$= \int \frac{x}{\sqrt{2-x^2}}\cdot dx + \int \frac{dx}{\sqrt{2-x^2}} + \int \frac{dx}{(x+1)\sqrt{2-x^2}} + \int \frac{dx}{(x-1)\sqrt{2-x^2}}.$$

$$= -\int \frac{\overset{f'}{-2x}}{2\underbrace{\sqrt{2-x^2}}_{f}}\cdot dx + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

$$= -\sqrt{2-x^2} + \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

$$+ f(x) + g(x) + c$$

$x+1 = \frac{1}{t} \Rightarrow x = \frac{1}{t} - 1 = \frac{1-t}{t}$

$dx = -\frac{dt}{t^2}$

$2-x^2 = 2-\left(\frac{1-t}{t}\right)^2$

$= \frac{t^2+2t-1}{t^2}$

$$= \int \frac{-\frac{dt}{\cancel{t^2}}}{\left(\frac{1}{\cancel{t}}\right)\frac{\sqrt{t^2+2t-1}}{\cancel{t}}}$$

$$f(x) = -\int \frac{dt}{\sqrt{t^2+2t-1}} = -\int \frac{dt}{\sqrt{(t+1)^2 - 2}}$$

$$= -\ln\left|t+1+\sqrt{(t+1)^2-2}\right| + C$$

Que Integrate $\int \frac{\sqrt{\cot(x)} - \sqrt{\tan(x)}}{1+3\sin(2x)} \cdot dx$

Soln

$$\int \frac{\sqrt{\cot(x)} - \sqrt{\tan(x)}}{1+3\sin(2x)} \cdot dx = \int \frac{\cos(x) - \sin(x)}{\sqrt{\sin(x)\cos(x)} \cdot (1+3\sin(2x))} \cdot dx$$

$$= \sqrt{2} \int \frac{c - s}{\sqrt{\sin(2x)}\,(1+3\sin(2x))} \cdot dx = \sqrt{2} \int \frac{c - s}{\sqrt{(s+c)^2 - 1} \cdot (1+3((s+c)^2-1))} \cdot dx$$

$$= \sqrt{2} \int \frac{c - s}{\sqrt{(s+c)^2 - 1}\,(3(s+c)^2 - 2)} dx$$

$s+c = t \Rightarrow (c-s)\,dx = dt$

$$= \sqrt{2} \int \frac{dt}{\sqrt{t^2-1}\,(3t^2-2)}$$

$$= \sqrt{2}\int \frac{dt}{(3t^2-2)\sqrt{t^2-1}}$$

$$1 - \frac{1}{t^2} = z^2$$

$$+\frac{\cancel{2}}{t^3}\,.dt = \cancel{2}z\,.dz$$

$$= \sqrt{2}\int \frac{\left(\frac{dt}{t^3}\right)}{\left(3-\frac{2}{t^2}\right)\sqrt{1-\frac{1}{t^2}}}$$

$$3 - \frac{2}{t^2} = 1 + 2\left(1-\frac{1}{t^2}\right)$$

$$= 1 + 2z^2$$

$$= \sqrt{2}\int \frac{\cancel{z}\,.\,dz}{(1+2z^2)\,.\,\cancel{z}} = \frac{1}{\sqrt{2}}\int \frac{dz}{z^2+\frac{1}{2}}$$

$$= \frac{1}{\cancel{\sqrt{2}}}\cdot\frac{1}{\left(\frac{1}{\cancel{\sqrt{2}}}\right)}\tan^{-1}\left(\frac{z}{\left(1/\sqrt{2}\right)}\right) + c = \tan^{-1}\left(\sqrt{2}\,z\right) + c$$

$$= \tan^{-1}\left(\sqrt{2}\sqrt{\frac{t^2-1}{t^2}}\right) + c = \tan^{-1}\left(\sqrt{2}\,\frac{\sqrt{(s+c)^2-1}}{s+c}\right) + c$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\,\sin(2x)}{\sin(x)+\cos(x)}\right) + c$$

Que Evaluate $\displaystyle\int \frac{x + e^{x}(\sin(x) + \cos(x)) + \sin(x)\cdot\cos(x)}{\left(x^{2} + 2e^{x}\cdot\sin(x) - \cos^{2}(x)\right)^{2}}\cdot dx$

$$(e^{x}\cdot f)' = e^{x}(f + f')$$

Sol

$$\int \frac{x+e^{x}(\sin(x)+\cos(x))+\sin(x)\cdot\cos(x)}{\left(x^{2}+2e^{x}(\sin(x)) - \cos^{2}(x)\right)^{2}}\cdot dx$$

$$= \frac{1}{2}\int \frac{\overbrace{2x+2e^{x}(\sin(x)+\cos(x)) + 2\sin(x)\cdot\cos(x)}^{f'}}{\Big(\underbrace{x^{2} + 2e^{x}\cdot\sin(x) - \cos^{2}(x)}_{f}\Big)^{2}}\cdot dx$$

$$= -\frac{1}{2}\cdot\frac{1}{x^{2}+2e^{x}\sin(x)-\cos^{2}(x)} + C$$

$\boxed{\text{Que}}$ Evaluate $\displaystyle\int x \cdot \sqrt{\frac{2\sin(x^2+1) - \sin(2(x^2+1))}{2\sin(x^2+1) + \sin(2(x^2+1))}} \cdot dx$

Let $x^2+1 = \theta$

$2x\,dx = d\theta$

Sol

$$\int x \cdot \sqrt{\frac{2\sin(x^2+1) - \sin(2(x^2+1))}{2\sin(x^2+1) + \sin(2(x^2+1))}} \cdot dx$$

$$= \frac{1}{2}\int \sqrt{\frac{2\sin(\theta) - \sin(2\theta)}{2\sin(\theta) + \sin(2\theta)}}\, d\theta = \frac{1}{2}\int \sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} \cdot d\theta = \frac{1}{2}\int \sqrt{\tan^2\left(\frac{\theta}{2}\right)} \cdot d\theta = \frac{1}{2}\int \tan\left(\frac{\theta}{2}\right) \cdot d\theta$$

$$= \cancel{\frac{1}{2}} \frac{\ln\left|\sec\left(\frac{\theta}{2}\right)\right|}{\cancel{\left(\frac{1}{2}\right)}} + c = \ln\left(\left|\sec\left(\frac{x^2+1}{2}\right)\right|\right) + c$$

Ans

Que Evaluate $\int e^x\left(\frac{x^4+2}{(1+x^2)^{5/2}}\right)dx$

Soln $\int e^x\left(\frac{x^4+2}{(1+x^2)^{5/2}}\right)dx$

$= \int e^x\left(\frac{x^4+2x^2+1+1-2x^2}{(1+x^2)^{5/2}}\right)dx$

$= \int e^x\left(\frac{(1+x^2)^2+1-2x^2}{(1+x^2)^{5/2}}\right)dx$

C-2

$\left(\frac{1}{f^3}\right)' = \frac{-3\cdot f'}{f^4}$

$\frac{\sim\sim}{(x+2)^7}$

$\frac{(x+2)\sim+\sim}{(x+2)^7}$

C-3

$\int e^x(f-f'')dx$

$\int e^x\left((f+f')-(f'+f'')\right)dx$

C-1

$\int e^x(f+f')\,dx$

$= \int(e^x\cdot f+e^x\cdot f')\,dx$

$= \int(e^x\cdot f)'\,dx$

$= e^x\cdot f(x).$

$\int \underset{II}{e^x}\cdot\underset{I}{f}\,dx + \cancel{\int e^x\cdot f'\,dx}$

$e^x\cdot f - \cancel{\int e^x\cdot f'\,dx}$

$$= \int e^x \left(\underbrace{\frac{1}{\sqrt{x^2+1}}}_{f} + \underbrace{\frac{1-2x^2}{(x^2+1)^{5/2}}}_{f''} \right) \cdot dx$$

$$= \int e^x \left(\underbrace{\frac{1}{\sqrt{x^2+1}}}_{f} \underbrace{- \frac{x}{(x^2+1)^{3/2}}}_{f'} + \underbrace{\frac{x}{(x^2+1)^{3/2}}}_{g} + \underbrace{\frac{1-2x^2}{(x^2+1)^{5/2}}}_{g'} \right) \cdot dx$$

$$= e^x \cdot \frac{1}{\sqrt{x^2+1}} + e^x \cdot \frac{x}{(x^2+1)^{3/2}}$$

$$+ C$$

Ans

Rough

$$\left((x^2+1)^{-1/2}\right)' = -\frac{1}{\cancel{2}}(x^2+1)^{-3/2} \cdot \cancel{2}x$$

$$= -\frac{x}{(x^2+1)^{3/2}}.$$

$$\left(\frac{x}{(x^2+1)^{3/2}}\right)' = \frac{(x^2+1)^{3/2} - x \cdot \frac{3}{\cancel{2}} \cdot (x^2+1)^{1/2} \cdot \cancel{2}x}{(x^2+1)^3}$$

$$= \frac{x^2+1-3x^2}{(x^2+1)^{5/2}} = \frac{1-2x^2}{(x^2+1)^{5/2}}$$

$$= \int e^{\overbrace{x\sin(x)+\cos(x)}^{g}} \cdot \left(\cos^2(x) - \left(\frac{x\sin(x)+\cos(x)}{x^2} \right) \right) dx$$

$$= \int e^{\overbrace{x\sin(x)+\cos(x)}^{g}} \left(\overset{g'}{(x \cdot \cos(x))} \underbrace{\left(\frac{\cos(x)}{x} \right)}_{f} + \underbrace{\left(\frac{-x\sin(x)-\cos(x)}{x^2} \right)}_{f'} \right) \cdot dx$$

$$= e^{x\sin(x)+\cos(x)} \cdot \frac{\cos(x)}{x} + C$$

$$= \frac{\cos(x) \cdot e^{x\sin(x)+\cos(x)}}{x} + C$$

Rough

$$\int e^x(\underbrace{\sin(x)}_{f'} - \underbrace{\cos(x)}_{f})\,dx$$

$$= -\cos(x)\cdot e^x + C$$