Que

A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. Find the probability of each pair consisting of balls of different colours.

Sol

$n - W$

$n - R$

M-1

Required Probability $= \dfrac{2^n \cdot \lfloor n \, \lfloor n}{\lfloor 2n}$ Ans

Alt.

T.N.O.C. $= {}^{2n}C_2 \cdot {}^{2n-2}C_2 \cdot {}^{2n-4}C_2 \cdots {}^{2}C_2 = \left(\dfrac{(2n)(2n-1)}{\lfloor 2}\right)\left(\dfrac{(2n-2)(2n-3)}{\lfloor 2}\right)\cdots\left(\dfrac{2\cdot 1}{\lfloor 2}\right) = \dfrac{\lfloor 2n}{2^n}$

Favourable no. of C. $= (n \cdot n)\big((n-1)(n-1)\big)\big((n-2)(n-2)\big)\cdots(1\cdot 1)$

$= (\lfloor n)^2$

$\Rightarrow$ Req. P. $= \dfrac{(\lfloor n)^2}{\left(\dfrac{\lfloor 2n}{2^n}\right)} = \dfrac{2^n \cdot (\lfloor n)^2}{\lfloor 2n}$ Ans

One

A bag contains 'W' white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. What is the probability that this procedure for drawing the balls will come to an end at the r^{th} draw.

Sol

$W \rightarrow$ White

$3 \rightarrow$ Black balls

$B_1 \,|\, B_2 \,|\, B_3 \,|\, B_4 \,|\, - \; - \;$ Black. $\; - \; B_{w+3}$

2 – Black-balls

$r-3 \rightarrow$ white.

$r-1$ places

r^{th} draw

$$T.N.O.C. = |\underline{w+3}$$

$$T.N.O.W. = {}^{r-1}C_2 \cdot |\underline{3} \cdot |\underline{w}$$

$$= (r-1)(r-2)\, 3 \cdot |\underline{w}.$$

$$\text{Required Probability} = \frac{3(r-1)(r-2)\,|\underline{w}}{|\underline{w+3}}$$

$$= \boxed{\frac{3(r-1)(r-2)}{(w+3)(w+2)(w+1)}}$$

Ans

Alt

W - White
3 - Black

$D_1 \to B$
$D_2 \to B$
$B_3 \to W$
$D_4 \to W$
⋮
$D_{r-1} — W$
$D_r \to B$

W B W B W . . W $\to r-3$ W

$$R.P. = {}^{r-1}C_2 \left(\left(\frac{3}{w+3}\right) \cdot \left(\frac{2}{w+2}\right) \cancel{\left(\frac{w}{w+1}\right)} \cancel{\left(\frac{w-1}{w}\right)} \cancel{\left(\frac{w-2}{w-1}\right)} \cdots \cancel{\left(\frac{w-r+4}{w-r+5}\right)} \cdot \frac{1}{\cancel{w-r+4}} \right)$$

$$= \frac{(r-1)(r-2)}{\cancel{2}} \left(\frac{3 \cdot \cancel{2}}{(w+3)(w+2)(w+1)} \right)$$

$$= \frac{3(r-1)(r-2)}{(w+3)(w+2)(w+1)}.$$

Que

If 6n tickets numbered 0, 1, 2, ⋯, 6n −1 are placed in a bag, and three are drawn without replacement. Find the probability that the sum of the numbers of tickets is 6n.

Sol

0, 1, 2, - - -, 6n-1

T.N.O.C. = $^{6n}C_3$.

M-1 $a < b < c$.

If $a = 0$ then No. of cases $= 3n-1$

(for $b = 1, 2, --, 3n-1$)

If $a = 1$ then No. of cases $= 3n-2$.

for $b = 2, 3, ---, 3n-1$

$c \to 6n-3, 6n-4, ---, 3n$

If $a = 2$ then No. of cases $= 3n-4$

for $b = 3, 4, ---, 3n-2$.

$c = 6n-5, 6n-6, \quad, 3n$.

If $a = 3$ then

no. of cases $= 3n-5$

$b = 4, 5, -----, 3n-2$.

$c = 6n-7, 6n-8, ----, 3n-1$

If $a = 4$ then

no. of cases $= 3n-7$

$b = 5, 6, -----, 3n-3$

$c = 6n-9, ------, 3n-1$

$\frac{6n-4}{2} \to 3n-2$

$$\text{Fav. no. of cases} = (3n-1)+(3n-2)+(3n-4)+(3n-5)+(3n-7)+(3n-8)+\cdots$$

$$\cdots\cdots+4+2+1$$

$$=(1+2+3+4+\cdots+3n)-(3+6+9+\cdots+3n)$$

$$=\frac{3n(3n+1)}{2}-\frac{3.\,n(n+1)}{2}=\frac{3n}{2}\left(3n+\cancel{1}-n-\cancel{1}\right)$$

$$=\frac{3n}{\cancel{2}}.\cancel{2}n = 3n^2.$$

$$\text{R. Prob.} = \boxed{\frac{3n^2}{{}^{6n}C_3}} \text{ Ans}$$

Alt

M-2

$\boxed{0}$ $\boxed{1}$ $\boxed{2}$ ⋮ $\boxed{6n-1}$ → 3 nos.

Unordered

& Unequal

& Sum = 6n

Let

$x_1 + x_2 + x_3 = 6n.$

$0 \le x_i \le 6n-1.$

→ N.O.S. $= {}^{6n+2}C_2 = \dfrac{(6n+2)(6n+1)}{2} = (3n+1)(6n+1)$

Problem

Ordered

$6n, 0, 0$ ✗

2-equal and 3-equal

$x_1 = x_2 = x_3 = 2n \longrightarrow$ N.O.C. = 1.

Exactly two equal (eg. $x_1 = x_2 \ne x_3$.)

$x_1 + x_1 + x_3 = 6n$

$2x_1 + x_3 = 6n$

$0, 1, \ldots, 3n.$ — $3n+1$ — $2n, 2n, 2n.$

$-3n.$

$\llcorner 3 \cdot (3n)$

$\to x_1 = x_2 \ne x_3$

$\to x_1 = x_3 \ne x_2$

$\to x_2 = x_3 \ne x_1$

all unequal

$\llcorner (3n+1)(6n+1) - 9n - 1$

all unequal & Unordered $= \dfrac{(3n+1)(6n+1) - (9n+1)}{\lfloor 3}$

$= \dfrac{18n^2 + \cancel{9n} + \cancel{1} - \cancel{9n} - \cancel{1}}{6} = 3n^2.$

R.P. $= \boxed{\dfrac{3n^2}{{}^{6n}C_3}}$ Ans

Que

There are two bags, each containing 5 red and 3 black balls. Two persons, A and B are given one bag each. Each of them is to draw one ball at random from the bag till both of them get a black ball (not necessarily in the same draw). The balls are to be replaced after each draw. Find the probability that the number of trials required is n.

5 - RE
3 - BL
↓
A

5 - RE
3 - BL
↓
B

If $P(r)$ denotes the probability that process of drawing ball for a person ends at r^{th} draw then

$$P(r) = \underbrace{\frac{5}{8}\cdot\frac{5}{8}\cdot\,\text{----}\,\frac{5}{8}}_{\text{'}r-1\text{' times}}\cdot\frac{3}{8} = \frac{3}{8}\cdot\left(\frac{5}{8}\right)^{r-1}.$$

The probability that the process for a person end in r^{th} draw or any previous drawn

$$= P(1) + P(2) + P(3) + \text{——} + P(r)$$

$$= \frac{3}{8} + \frac{3}{8}\left(\frac{5}{8}\right) + \frac{3}{8}\left(\frac{5}{8}\right)^2 + \text{--} + \frac{3}{8}\left(\frac{5}{8}\right)^{r-1} = \frac{\cancel{3/8}\left(1-\left(\frac{5}{8}\right)^r\right)}{\cancel{1-5/8}} = 1 - \left(\frac{5}{8}\right)^r.$$

$$R.P. = P(A_1 \cap B_n) + P(A_2 \cap B_n) + P(A_3 \cap B_n) + \cdots + P(A_n \cap B_n)$$

$$+ P(A_n \cap B_1) + P(A_n \cap B_2) + \cdots + P(A_n \cap B_{n-1})$$

$$= P_A(r \le n) \cdot P_B(r \le n) - P_A(r \le n-1) \cdot P_B(r \le n-1)$$

$$= \left(1 - \left(\frac{5}{8}\right)^n\right) \cdot \left(1 - \left(\frac{5}{8}\right)^n\right) - \left(1 - \left(\frac{5}{8}\right)^{n-1}\right) \cdot \left(1 - \left(\frac{5}{8}\right)^{n-1}\right)$$

$$= 1 - 2\left(\frac{5}{8}\right)^n + \left(\frac{5}{8}\right)^{2n} - \left(1 - 2 \cdot \left(\frac{5}{8}\right)^{n-1} + \left(\frac{5}{8}\right)^{2n-2}\right)$$

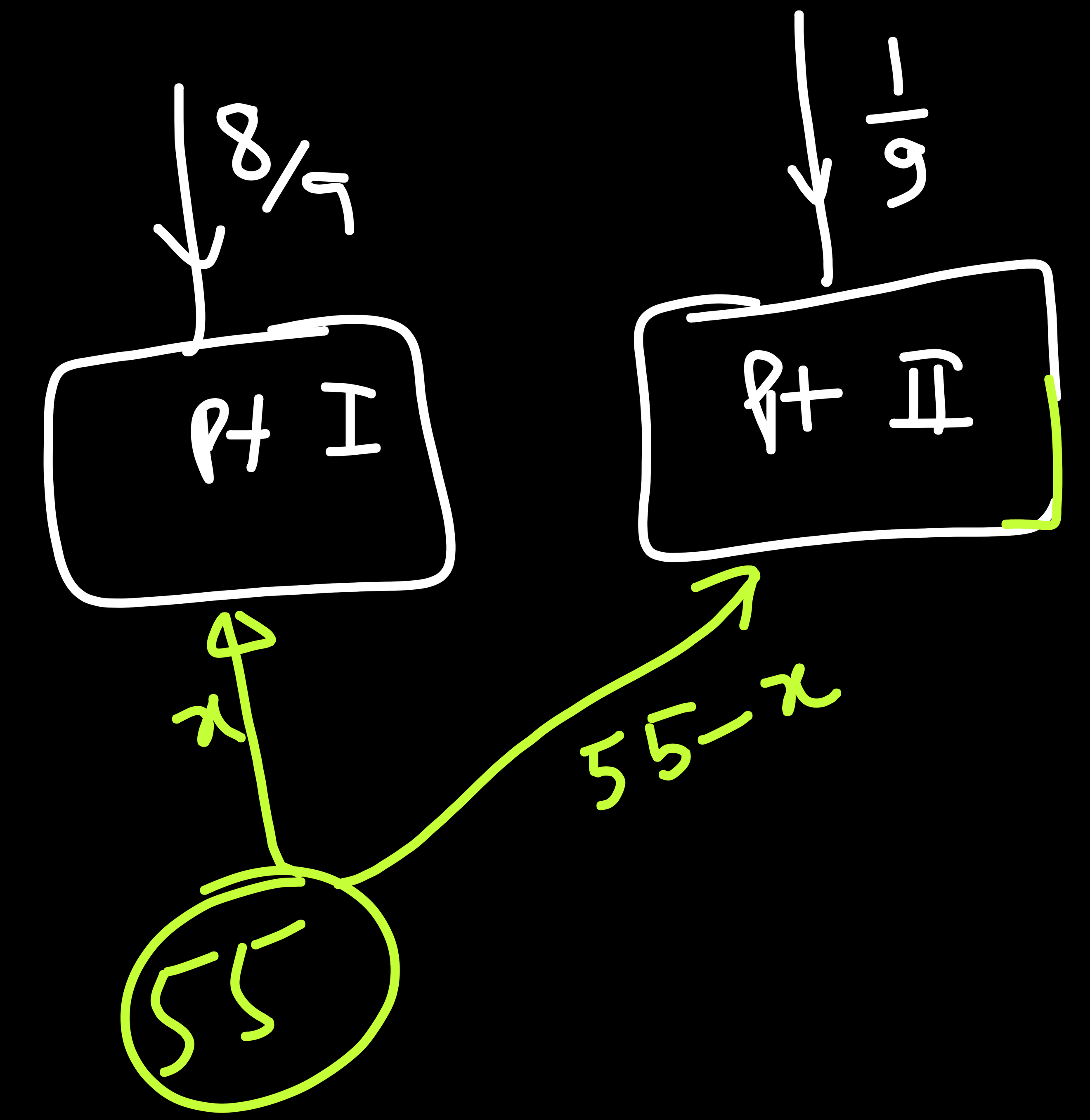
$$= \left(\frac{5}{8}\right)^{2n-2}\left(\left(\frac{5}{8}\right)^2 - 1\right) + 2\left(\frac{5}{8}\right)^{n-1}\left(1 - \frac{5}{8}\right)$$

$$= \frac{3}{4}\left(\frac{5}{8}\right)^{n-1} - \frac{39}{64} \cdot \left(\frac{5}{8}\right)^{2n-2}$$

Que

An artillery target may be either at point I, with probability 8/9 or at point II with probability 1/9. We have 55 shells, each of which can be fired either at point I or II. Each shell may hit the target, independent of the other shells, with probability ½. How many shells must be fired at point I to hit the target with maximum probability?

Sol



Probability that target is hit

$$P(x) = \frac{8}{9}\cdot\Big(1-\underbrace{\left(\tfrac{1}{2}\right)^{x}}_{\text{No-hit}}\Big) + \frac{1}{9}\left(1-\left(\tfrac{1}{2}\right)^{55-x}\right),\quad x = 0, 1, \ldots, 55$$

$$P'(x) = \frac{8}{9}\left(-\left(\tfrac{1}{2}\right)^{x}\cdot \ln\left(\tfrac{1}{2}\right)\right) + \frac{1}{9}\left(+\left(\tfrac{1}{2}\right)^{55-x}\cdot \ln\left(\tfrac{1}{2}\right)\right)$$

$$= \frac{\ln\left(\frac{1}{2}\right)}{9}\cdot\left(-\left(\tfrac{1}{2}\right)^{x-3} + \left(\tfrac{1}{2}\right)^{55-x}\right)$$

$$= \frac{\ln(2)}{9}\cdot\left(\tfrac{1}{2}\right)^{x-3}\cdot\left(1-\left(\tfrac{1}{2}\right)^{58-2x}\right) = \frac{\ln(2)}{9}\left(\tfrac{1}{2}\right)^{x-3}\left(1-2^{2x-58}\right)$$

$$= 0$$

$$1 = \left(\tfrac{1}{2}\right)^{58-2x}$$

$$x = 29$$

One

Three dice are rolled together till a sum of either 4 or 5 or 6 is obtained. Find the probability that 4 come before 5 or 6.

$D_1 \quad D_2 \quad D_3$

$x_1 + x_2 + x_3 = n.$

$x_1 \to 1, \dots 6.$

$(x + x^2 + \dots + x^6)^3$

$x^3(1 + x + \dots + x^5)^3$

$= x^3(1 - x^6)^3 (1 - x)^{-3}$

$= x^3(1 - 3x^6 + 3x^{12} - x^{18}).(1 - x)^{-3}$

Coff of x^4 = N.O.W. of obtaining sum of $4 = {}^{1+2}C_2 = 3$

" " x^5 = ——— $5 = {}^{2+2}C_2 = 6$

" " " = ——— $6 = {}^{3+2}C_2 = 10$

A throw will result in sum 4 with prob $= \frac{3}{216}$.

——— 5 ——— $= \frac{6}{216}$.

——— 6 ——— $= \frac{10}{216}$.

N ——— $\neq 4,5,6$ ——— $= 1 - \frac{19}{216}$.

$= \frac{197}{216}$.

4 or N4 or NN4 or NNN4

$R.P. = \frac{3}{216} + \left(\frac{197}{216}\right)\left(\frac{3}{216}\right) + \left(\frac{197}{216}\right)^2\left(\frac{3}{216}\right) + \dots \infty$

$= \frac{3/216}{1 - \frac{197}{216}}$

$= \frac{\left(\frac{3}{\cancel{216}}\right)}{\left(\frac{19}{\cancel{216}}\right)} = \boxed{\frac{3}{19}}$

2^n players of equal strength are playing a knock out tournament. If they are paired at randomly in all rounds, find out the probability that out of two particular players S_1 and S_2 exactly one will reach in semi final. ($n \in N, n \geq 2$).

Ans

2^n

R-1 (2), (2), (2) - - - (2). $\to 2^{n-1}$

Winner 1 1 1 $\to 2^{n-1}$

R-2 $\to$ (2) (2) (2) - - - (2) $\to 2^{n-2}$

1 1 1 1 Winner 2^{n-2}.

Semifinal P_1, P_2, P_3, P_4 $\to S_1, S_2$ - - - -

S_1, S_2 - - -

Final F_1, F_2

T.N.O.C. for S.F. line-up $= {}^{2^n}C_4$.

F.N.O.C. for S.F. line-up $= 2 \cdot {}^{(2^n-2)}C_3$

$$R.P. = \frac{2 \cdot {}^{(2^n-2)}C_3}{{}^{2^n}C_4} = \frac{2 \cdot (2^n-2)(2^n-3)(2^n-4) \cdot 4}{6 \cdot 2^n (2^n-1)(2^n-2)(\;)(\;)}$$

$$= \frac{8}{2^n \cdot (2^n-1)}$$

Ans

One

The probability that a positive two digit number selected at random has its tens digit at least three more than its unit digit is

(A) 14/45 (B) 7/45 (C) 36/45 (D) 1/6

$\boxed{T}\ \boxed{U}$

T.N.O.C. = $9 \times 10 = 90$.

× × × × – – ——× → 10

× · × × × × × × – – × → $^8C_2$

$^8C_2$ = F.N.O.C.

$$R.P. = \frac{^8C_2}{90} = \frac{\cancel{8}^{2} \times 7}{\cancel{2} \times \cancel{90}_{45}}$$

$$= \frac{14}{45}$$

A

Que

A 5 digit number is formed by using the digits 0, 1, 2, 3, 4 & 5 without repetition. The probability that the number is divisible by 6 is :

(A) 8 % (B) 17 % (C) 18 % (D) 36 %

Sol

0, 1, 2, 3, 4, 5

Sum → $\frac{5\times 6}{2} = 15$

div. by 6 ⇓ div by 3 & div. by 2.

Total no. of no. $= 5\times5\times4\times3\times2$.

$\bar{0}$ → 1, 2, 3, 4, 5 → Even ⇒ $2 \cdot \underline{|4} = 48$

$\bar{3}$ → 0, 1, 2, 4, 5 → 0 → $\underline{|4} = 24$

→ $^3C_1 \cdot 2 \cdot \underline{|3} = 36$

→ 108

$$R.P. = \frac{5\times5\times\cancel{4}\times\cancel{3}\times2}{\cancel{108}\ \cancel{36}\ 9}^{-1} = \frac{9}{50} = 18\%$$

(C)