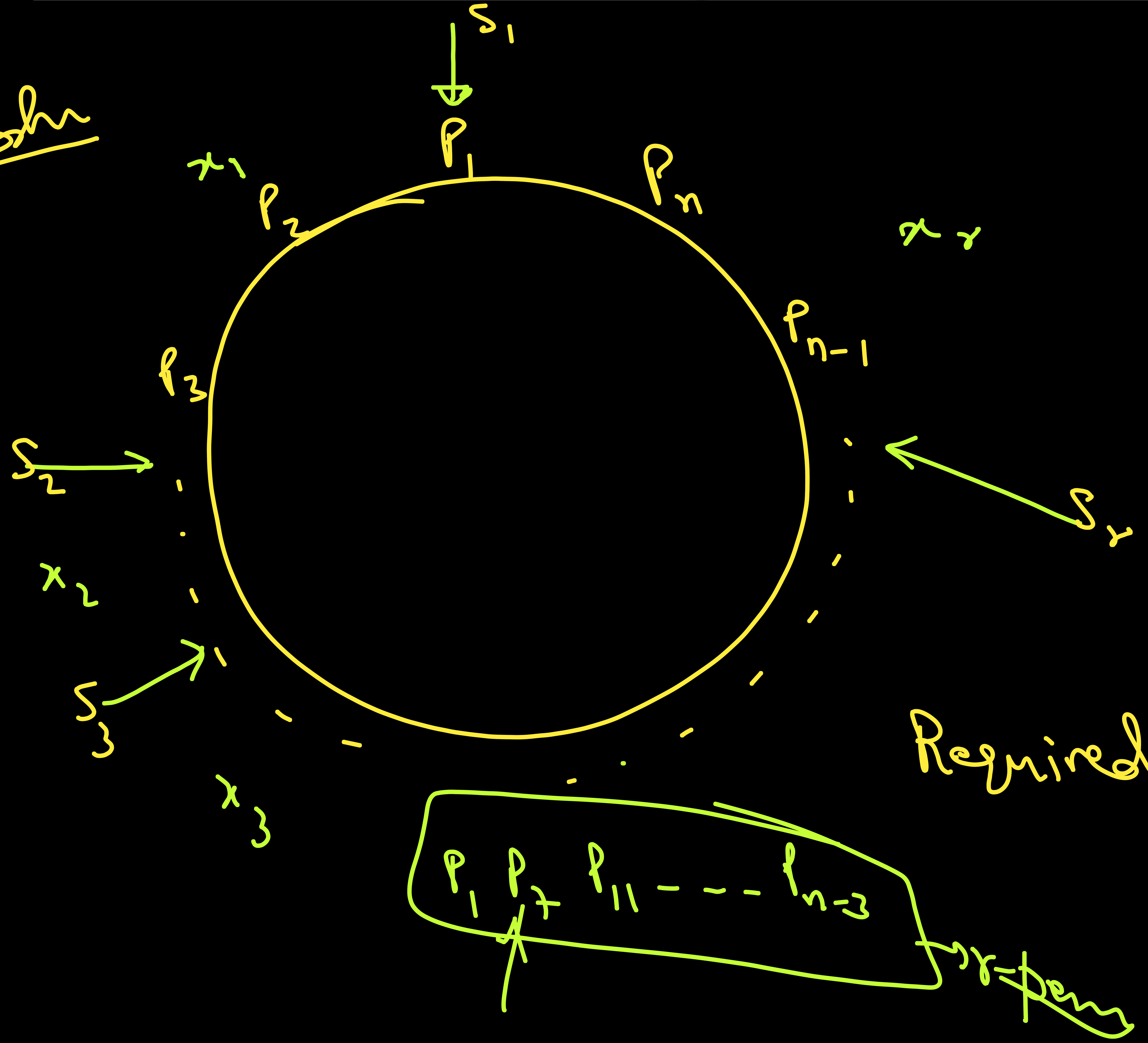
Que

In how many ways we can select r persons out of n persons sitting around a round table such that no two of them are neighbours.

Soln



$$x_1 + x_2 + x_3 + \cdots + x_r = n - r$$

$$x_1 \geq 1,\; x_2 \geq 1,\; \cdots,\; x_r \geq 1.$$

$$x_1' + x_2' + \cdots + x_r' = n - 2r$$

$$x_i' \geq 0.$$

$$\text{N.O.S.} = {}^{n-2r+r-1}C_{r-1} = {}^{n-r-1}C_{r-1}.$$

$$\text{Required Ans} = \left(\frac{n \cdot {}^{n-r-1}C_{r-1}}{r} \right) \text{Ans}$$

One

In how many ways we can distribute 999 identical balls in 3 identical boxes.

Sol

B B B

999-identical balls

same case

how many in a box.

What is the combination? X

B_1	B_2	B_3
x_1	x_2	x_3
99	450	450
450	99	450
450	450	99
999	0	0.
333	333	333
100	800	99
100	99	800
99	100	800
99	800	100

6 cases

$$x_1 + x_2 + x_3 = 999$$

$$\hookrightarrow \text{N.O.S.} = {}^{999+2}C_2 = {}^{1001}C_2$$

$$= \frac{1001 \times 1000}{2} = 1001 \times 500$$

$$= \underline{500500}$$

B_1 $\quad B_2$ $\quad \boxed{B_3}$

x_1 $\quad x_2$ $\quad x_3$

$x_1 + x_2 + x_3 = 999 \longrightarrow$ N.O.S. $= 500500$

$\alpha \neq \beta$ $\quad \alpha \neq \beta \neq \gamma$

α, α, α $\quad \alpha, \alpha, \beta$ $\quad \alpha, \beta, \gamma$

$\alpha = 333$ $\quad \Downarrow$

N.O.S. $= 1$ $\quad$ N.O.S. $= n_1$ $\quad$ N.O.S. $= n_2$

For n_1

$x_1 = x_2 \neq x_3$

$x_1 + x_1 + x_3 = 999$

$2x_1 + x_3 = 999.$

N.O.S. $= 500 - 1$

$= 499.$

$n_1 = 3 \cdot 499 = 1497$

$x_1 = x_2 \neq x_3$

$x_1 = x_3 \neq x_2$

$x_2 = x_3 \neq x_1$

$n_2 = 500500 - 1 - 1497$

$= 500500 - 1498$

$= 499002.$

Required Ans.

$= 1 + \dfrac{n_1}{3} + \dfrac{n_2}{3!}$

$= 1 + 499 + \dfrac{499002}{6}$

$= 500 + 83167$

$= \boxed{83667}$ Ans

Que

Find the total number of seven digit numbers $x_1x_2x_3x_4x_5x_6x_7$ having the property that $x_1 \le x_2 < x_3 < x_4 \le x_5 < x_6 < x_7$.

Soln

$x_1 \le x_2 < x_3 < x_4 \le x_5 < x_6 < x_7$

$x_1 < x_2 < x_3 < x_4 < x_5 < x_6 < x_7 \longrightarrow {}^9C_7$

$x_1 = x_2 < x_3 < x_4 < x_5 < x_6 < x_7 \longrightarrow {}^9C_6$

$x_1 < x_2 < x_3 < x_4 = x_5 < x_6 < x_7 \longrightarrow {}^9C_6$

$x_1 = x_2 < x_3 < x_4 = x_5 < x_6 < x_7 \longrightarrow {}^9C_5$

$+$

${}^9C_7 + {}^9C_6 + {}^9C_6 + {}^9C_5$

$= {}^{10}C_7 + {}^{10}C_6 = \boxed{{}^{11}C_7}$ Ans

Que

Find the possible number of ordered pairs (m, n, p) such that $1 \le m \le 100$, $1 \le n \le 50$, $1 \le p \le 25$ and $2^m + 2^n + 2^p$ is divisible by 3.

Sol

$$2^m + 2^n + 2^p = (3-1)^m + (3-1)^n + (3-1)^p$$

$$= (3k_1 + (-1)^m) + (3k_2 + (-1)^n) + (3k_3 + (-1)^p)$$

$$= 3(k_1 + k_2 + k_3) + (-1)^m + (-1)^n + (-1)^p$$

According to question either all 3 of m, n, p must be even or all 3 must be odd.

E E E or O O O

Hence, Required number of ordered pairs $= 50 \cdot 25 \cdot 12 + 50 \cdot 25 \cdot 13$

$= 50 \times 25 \times 25$

Ans

How many 5 letters word can be formed using the letters of word 'MANAGEMENT' such that if any two alike letters are there then they are always together.

Sol

M M
A A
N N
E E
G
T

5 - letter

All diff → ${}^{6}C_{5} \cdot \lfloor 5$

2-A + 3-diff → ${}^{4}C_{1} \cdot {}^{5}C_{3} \cdot \lfloor 4$

2-Alike + 2-Alike + 1-diff → ${}^{4}C_{2} \cdot {}^{4}C_{1} \cdot \lfloor 3$

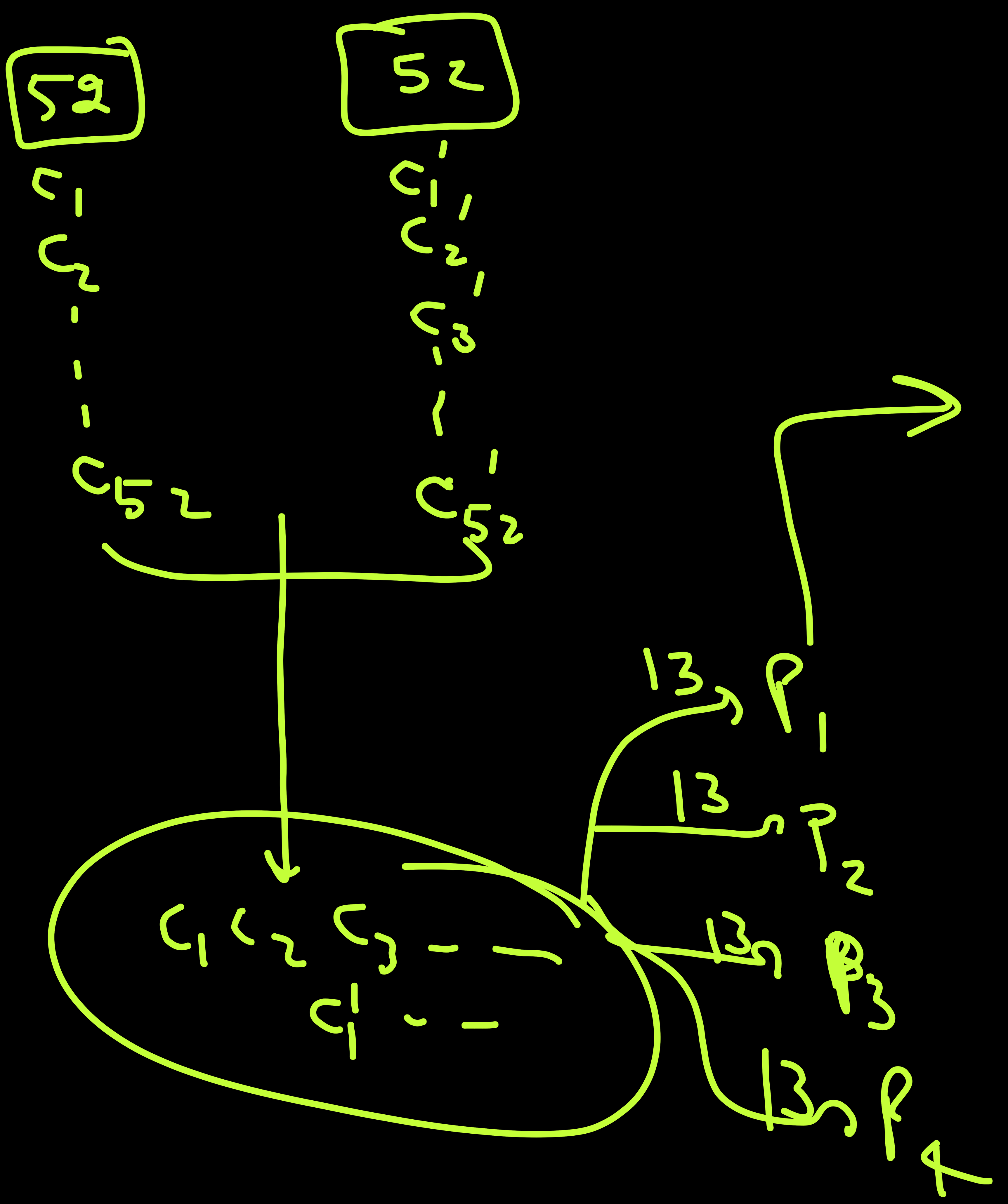
$6 \cdot \lfloor 5 + 4 \times 10 \cdot \lfloor 4 + 6 \times 4 \lfloor 3$

$= 14 \lfloor 5 + 6 \lfloor 4 = 76 \lfloor 4 = 76 \times 24$

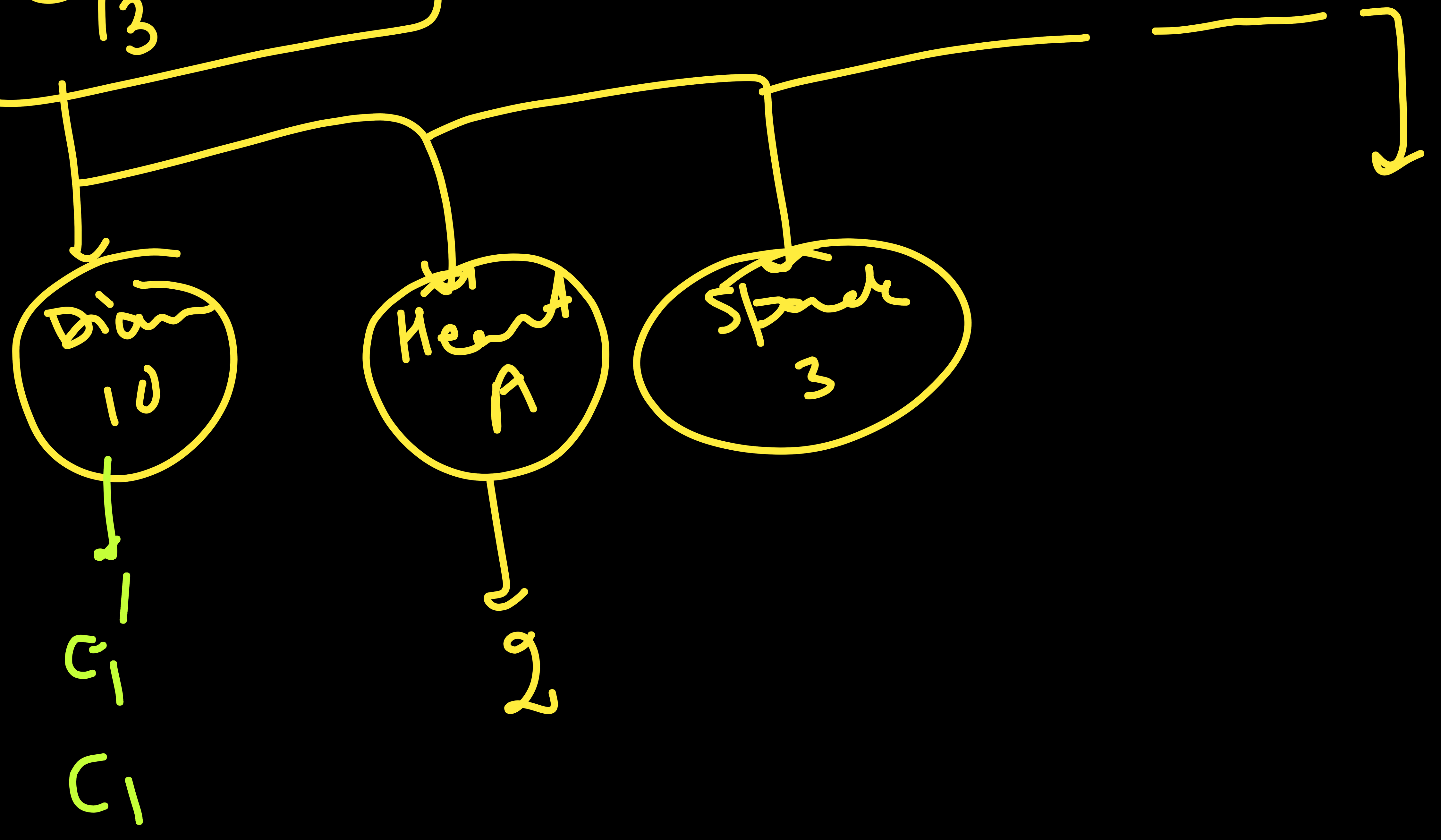
Que

Two different packs of card are shuffled together. Cards are distributed equally among 4 players, each getting 13 card. In how many ways can a player get his cards if no two cards are from the same suit and with the same denomination?

Sol



$$\boxed{{}^{52}C_{13} \cdot 2^{13}} \quad \text{Ans}$$



Que

There are mn balls, m each of n different colours (including Red, Blue, and Black). Find the number of ways in which they can be arranged in a row so that no black ball appears before any red ball and no red ball appears before any Blue ball. Discuss the cases when balls of same colour are identical and also when balls of same colour are distinct.

$C_1 \to B_1, B_2, B_3 - - -, B_m$

$C_2 \to B_1, B_2, - - \to B_m$

R

Blue

Black

$C_n \to B_1, B_2 - - -, B_m$

Blue | R | Black

$P_1\ P_2\ P_3 - - - - P_{mn}$

$B_{B_1}\ B_{B_2} \cdots B_{B_3} - - B_{B_m} - - R_{B_1} - R_{B_2} - - R_{B_m} \cdots B_{B_1} \cdots$

3m-position

n-3, m, m, m

C-1 Balls of same colours are identical

$$R.N.O.C. = {}^{mn}C_{3m} \cdot \frac{\lfloor (mn-3m)}{(m!)^{n-3}}$$

C-2 Balls of same colours are diff.

$$R.N.O.C. = {}^{mn}C_{3m} \cdot (\lfloor m)(\lfloor m)(\lfloor m) \cdot \lfloor mn-3m$$

Qw

In how many ways 2n identical black and 2n identical white balls can be arranged in a row such that on picking any n consecutive balls from the arrangement we get at least a black ball.

$2n \longrightarrow B$

$2n \longrightarrow W$

White balls $\to$ max $= n-1$

$\underbrace{\times \ \times \ \times \ - \ -}_{w_1\ w_2 \ldots}$ B_1 $\_\_\_\_$ B_2 $\_\_$ $\times \to 4n$-balls

n $\to$ atleast one black ball

No. of w-balls $\to$ x_1 $000\text{-}0B_1$ $00\text{-}0B_2$ x_2 x_3 $B_3 - - - - B_n$ x_{n+1}

R.N.O.W. = No. of Integral solns of $x_1+x_2+x_3+ - - +x_{n+1} = 2n$.

$0 \le x_i \le n-1$,

$= \text{coff of } x^{2n} \text{ in } \left((1+x+\ldots+x^{n-1})^{n+1}\right)$

$= \text{Coeff of } x^{2n} \text{ in } \left((1-x^n)^{n+1}(1-x)^{-(n+1)}\right)$

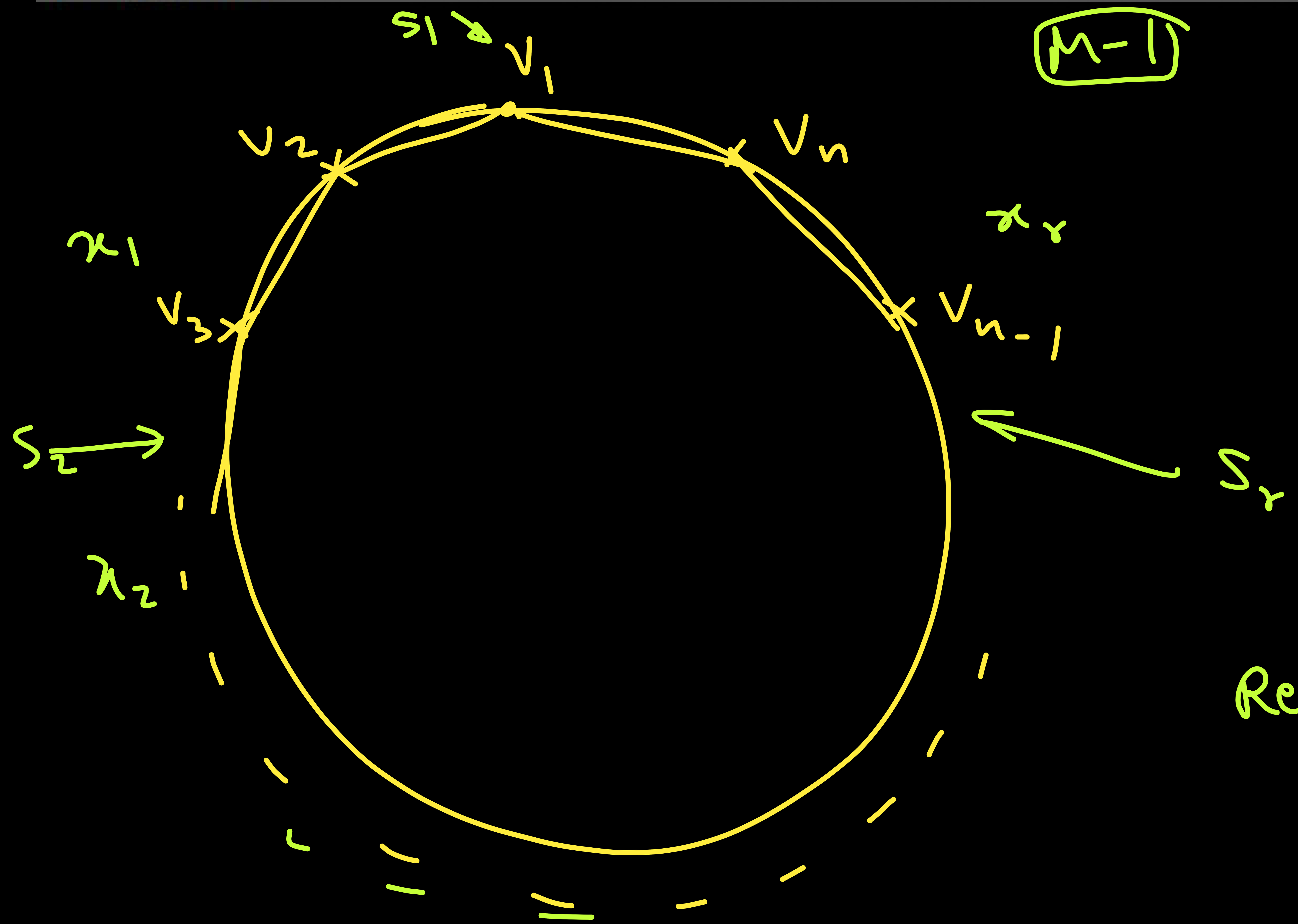
$= \text{" " " } \left(\left(1-(n+1)\cdot x^n + \frac{(n+1)n}{2}x^{2n} - - -\right)(1-x)^{-(n+1)}\right)$

$= {}^{2n+n}C_n - (n+1)\,{}^{n+n}C_n + \frac{(n+1)n}{2}\cdot 1$

$\text{Ans} = {}^{3n}C_n - (n+1)\cdot{}^{2n}C_n + \frac{n(n+1)}{2}$

One

r-sided polygons are formed by joining the vertices of an n–sided polygon. Find the number of polygons that can be formed, none of whose sides coincide with those of the n–sided polygon.



$\boxed{M-1}$

$$x_1 + x_2 + \text{---} + x_r = n - r$$

$$x_i \geq 1.$$

$$\Downarrow$$

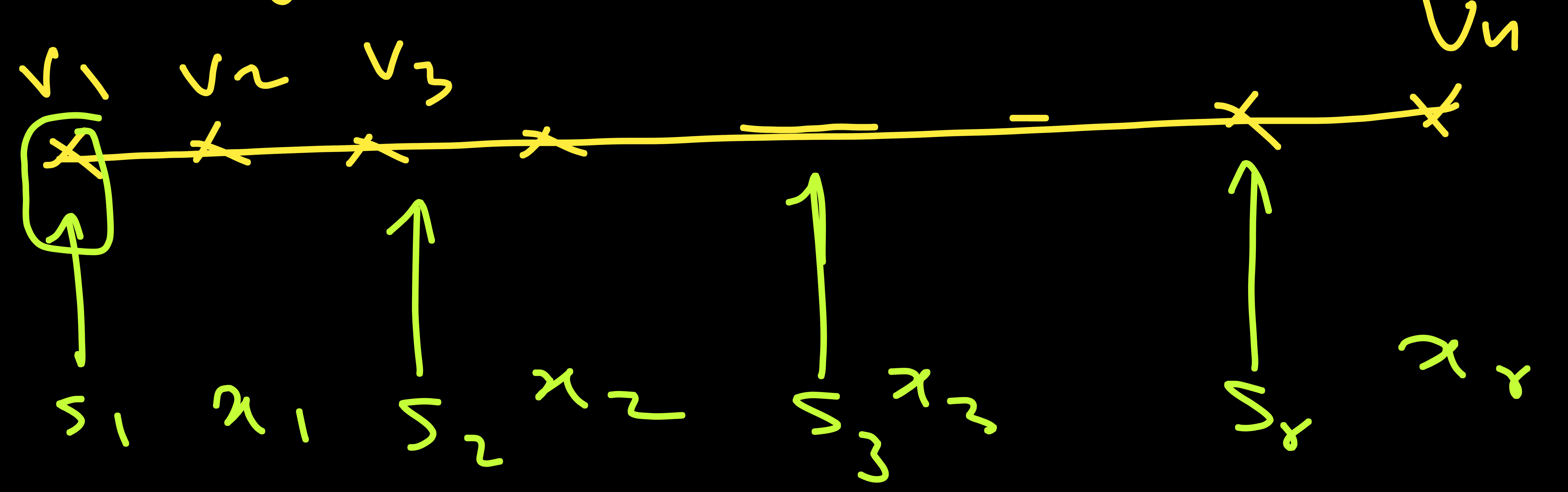
$$x_1' + x_2' + \text{---} + x_r' = n - 2r$$

$$x_i' \geq 0$$

$$N.O.S. = {}^{n-2r+r-1}C_{r-1} = {}^{n-r-1}C_{r-1}$$

$$\text{Required no. of polygons} = \frac{n \cdot {}^{n-r-1}C_{r-1}}{r}$$

C-2 Exactly one out of V_1 & V_n is selected.



$$x_1+x_2+ \cdots + x_r = n-r$$

$$x_i \geq 1$$

$$\rightarrow \text{N.O S.} = {}^{n-2r+r-1}C_{r-1} = {}^{n-r-1}C_{r-1}$$

$$\text{R.N.O.W}_1 = 2 \cdot {}^{n-r-1}C_{r-1}$$

$$\text{Final Ans} = {}^{n-r-1}C_r + 2 \cdot {}^{n-r-1}C_{r-1}.$$