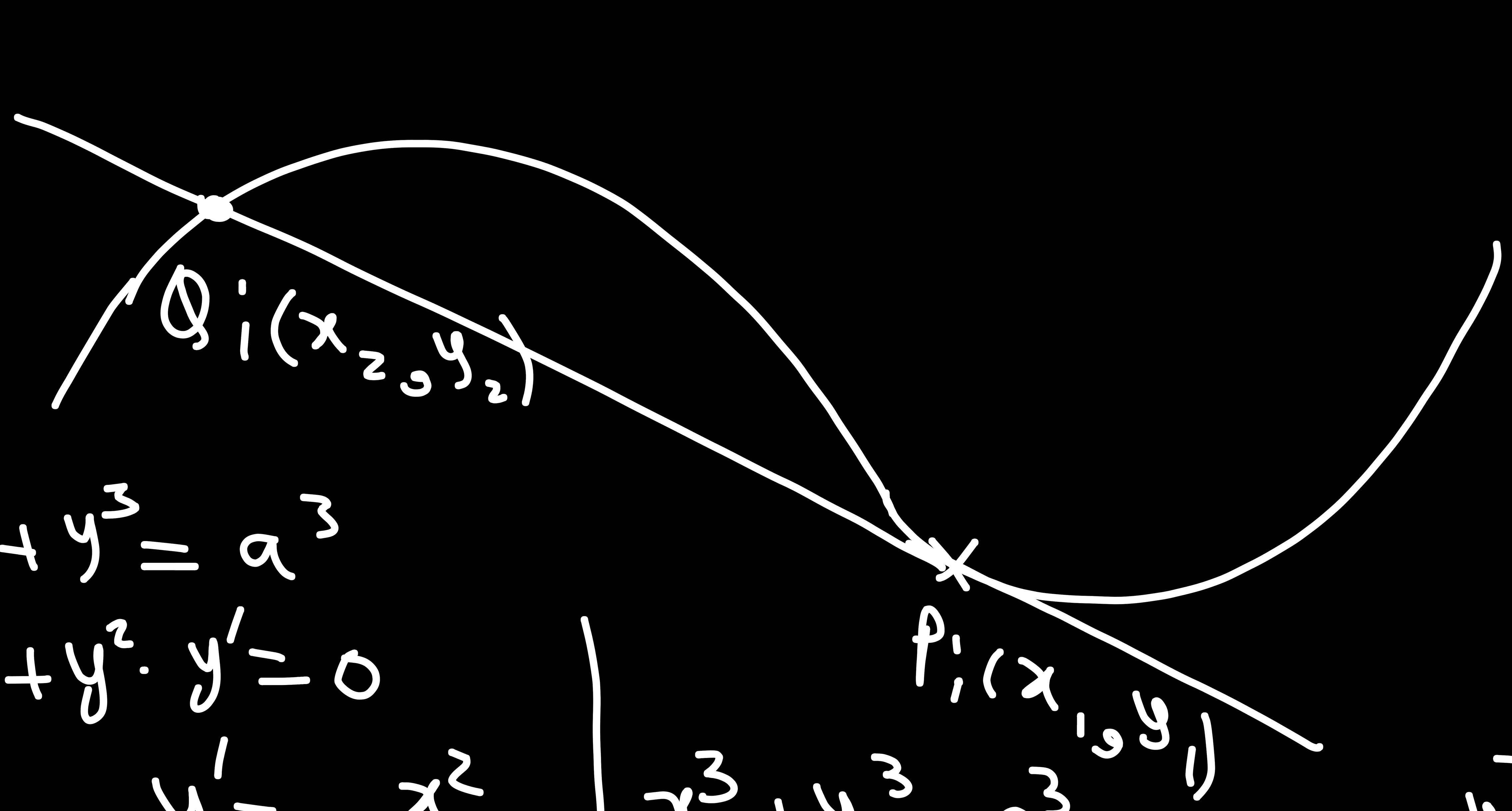
Que If the tangent at (x_1, y_1) to the curve $x^3+y^3=a^3$ meets the curve again at (x_2, y_2) then find $\frac{x_2}{x_1}+\frac{y_2}{y_1}$.

Sol



$x^3+y^3=a^3$

$x^2+y^2\cdot y'=0$

$y'=-\frac{x^2}{y^2}$

$x_1^3+y_1^3=a^3$

$-\,x_2^3+y_2^3=a^3$

$x_1^3-x_2^3+y_1^3-y_2^3=0$

$\Rightarrow y_2^3-y_1^3=-(x_2^3-x_1^3)$

$\frac{y_2-y_1}{x_2-x_1}=-\frac{x_1^2+x_2^2+x_1x_2}{y_1^2+y_2^2+y_1y_2}$

$m_{PQ}=m_{PQ}$

$-\frac{x_1^2}{y_1^2}=\frac{y_2-y_1}{x_2-x_1}$

$-\frac{x_1^2}{y_1^2}=-\frac{x_1^2+x_2^2+x_1x_2}{y_1^2+y_2^2+y_1y_2}.$

$1=\frac{1+\frac{x_2^2}{x_1^2}+\frac{x_2}{x_1}}{1+\frac{y_2^2}{y_1^2}+\frac{y_2}{y_1}}$

$\frac{x_2}{x_1}=\alpha$

& $\frac{y_2}{y_1}=\beta$

$$\frac{1+\alpha^2+\alpha}{1+\beta^2+\beta} = 1 \Rightarrow 1+\alpha^2+\alpha = 1+\beta^2+\beta$$

$$\alpha^2-\beta^2+\alpha-\beta = 0$$

$$(\alpha-\beta)\left(\alpha+\beta+1\right) = 0$$

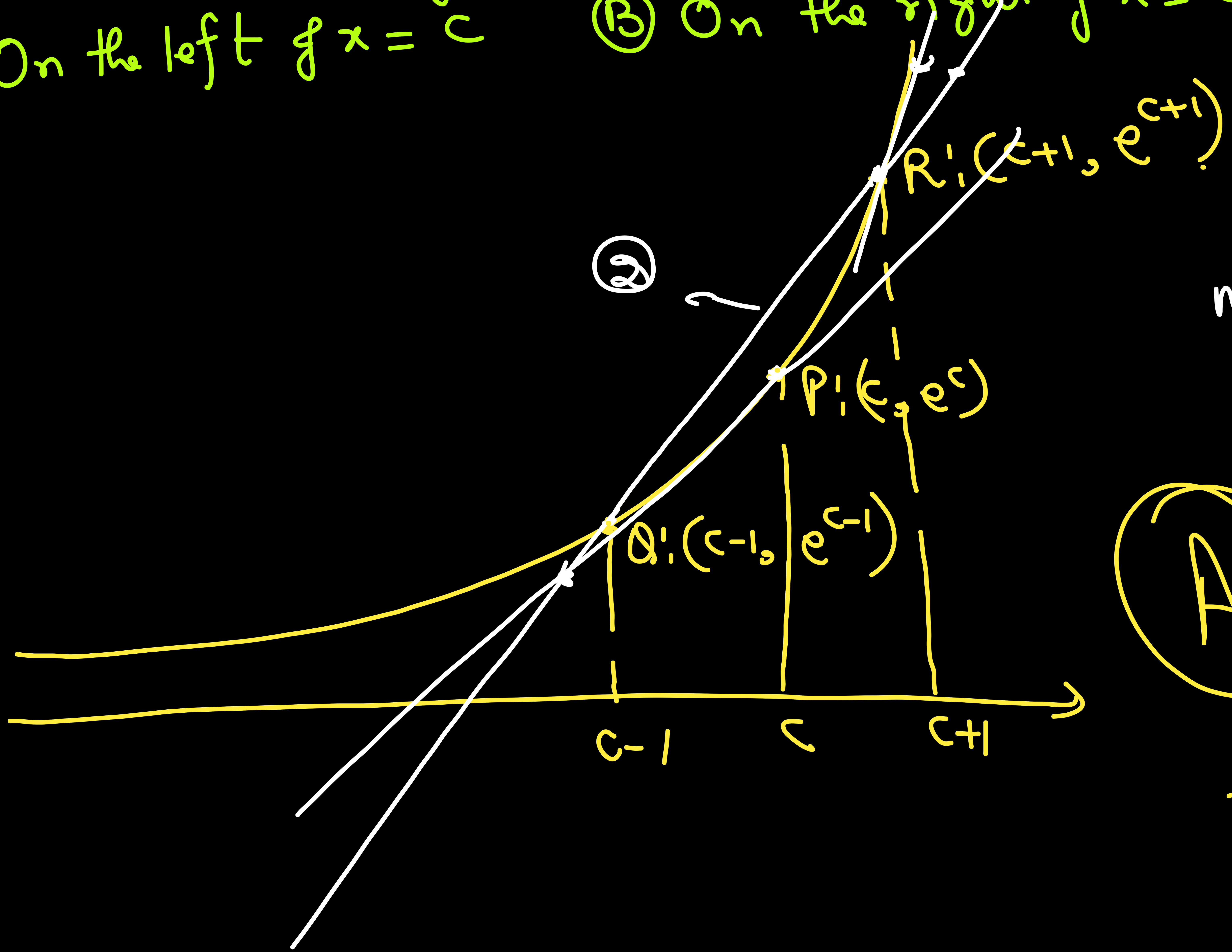
$$\alpha \neq \beta \Downarrow$$

$$\alpha+\beta+1 = 0$$

$$\frac{x_2}{x_1}+\frac{y_2}{y_1} = \boxed{-1}$$ Ans

Que. The tangent to the curve $y = e^x$ drawn at (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$, (A) On the left of $x = c$ (B) On the right of $x = c$ (C) at no point (D) both sides.

Sol



$$m_t^P = (e^x)'\Big]_{x=c} = e^c.$$

$$m_{RQ} = \frac{y_R - y_Q}{x_R - x_Q} = \frac{e^{c+1} - e^{c-1}}{c+1-(c-1)}$$

$$= e^c\left(\frac{e - e^{-1}}{2}\right) > e^c = m_t^P.$$

A

Ans

Que Find the interval of increase of $g(x)$ if $g(x) = f(x^2-2x+8) + f(14+2x-x^2)$

where $f''(x) \geq 0,\ \forall x \in R$.

Soln $f''(x) \geq 0 \Rightarrow f'$ $\uparrow$ing fn

$$g(x) = f(x^2-2x+8) + f(14+2x-x^2)$$

$$g'(x) = (2x-2)\cdot f'(x^2-2x+8) - (2x-2)\cdot f'(14+2x-x^2)$$

$$= \underbrace{2(x-1)}_{\beta} \cdot \left(\underbrace{f'(x^2-2x+8) - f'(14+2x-x^2)}_{= \alpha} \right)$$

If $\alpha > 0$ then $f'(x^2-2x+8) > f'(14+2x-x^2)$

$x^2-2x+8 > 14+2x-x^2 \Rightarrow x^2-2x-3 > 0$

$(x+1)(x-3) > 0$

$x \in (-\infty,-1) \cup (3,\infty)$

$\alpha > 0 \Rightarrow x \in (-\infty, -1) \cup (3, \infty)$

$\alpha < 0 \Rightarrow x \in (-1, 3)$

Sign of $g'(x)$

		-1		1		3	
$\alpha \rightarrow$	$+$		$-$		$-$		$+$
$\beta \rightarrow$	$-$		$-$		$+$		$+$
$g' \rightarrow$	$-$		$+$		$-$		$+$

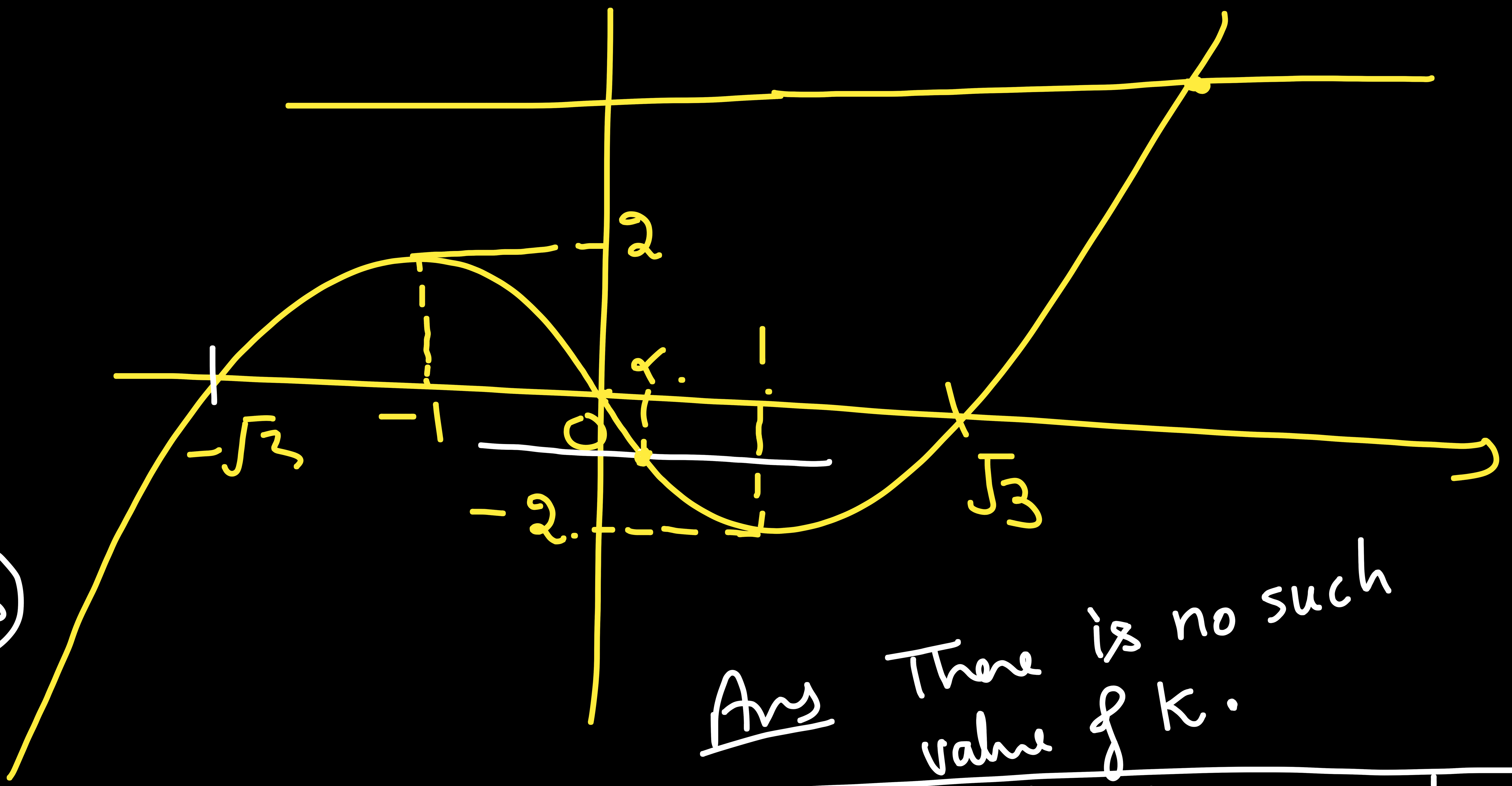
So $g(x)$ increases in $[-1, 1]$ and in $[3, \infty)$.

Que Find 'K' if the equation $x^3-3x+k=0$ has two distinct roots in $(0,1)$.

Soln

$$x^3-3x+k=0$$

$$\underbrace{x^3-3x}_{y} = \underbrace{-k}_{y}$$



$$y = x^3-3x = x(x-\sqrt{3})(x+\sqrt{3})$$

$$y' = 3x^2-3 = 3(x-1)(x+1)$$

+	−	+
	-1	1

Ans There is no such value of K.

S.Q. find 'k' if eqn. has atleast one root in $(0,1)$

$$-k\in(-2,0) \Rightarrow \boxed{k\in(0,2)}$$

Que Find the largest term of the sequence $a_n = \dfrac{n^2}{n^3+200}$, where $n \in \mathbb{N}$.

Soln

$$a_n = \frac{n^2}{n^3+200}$$

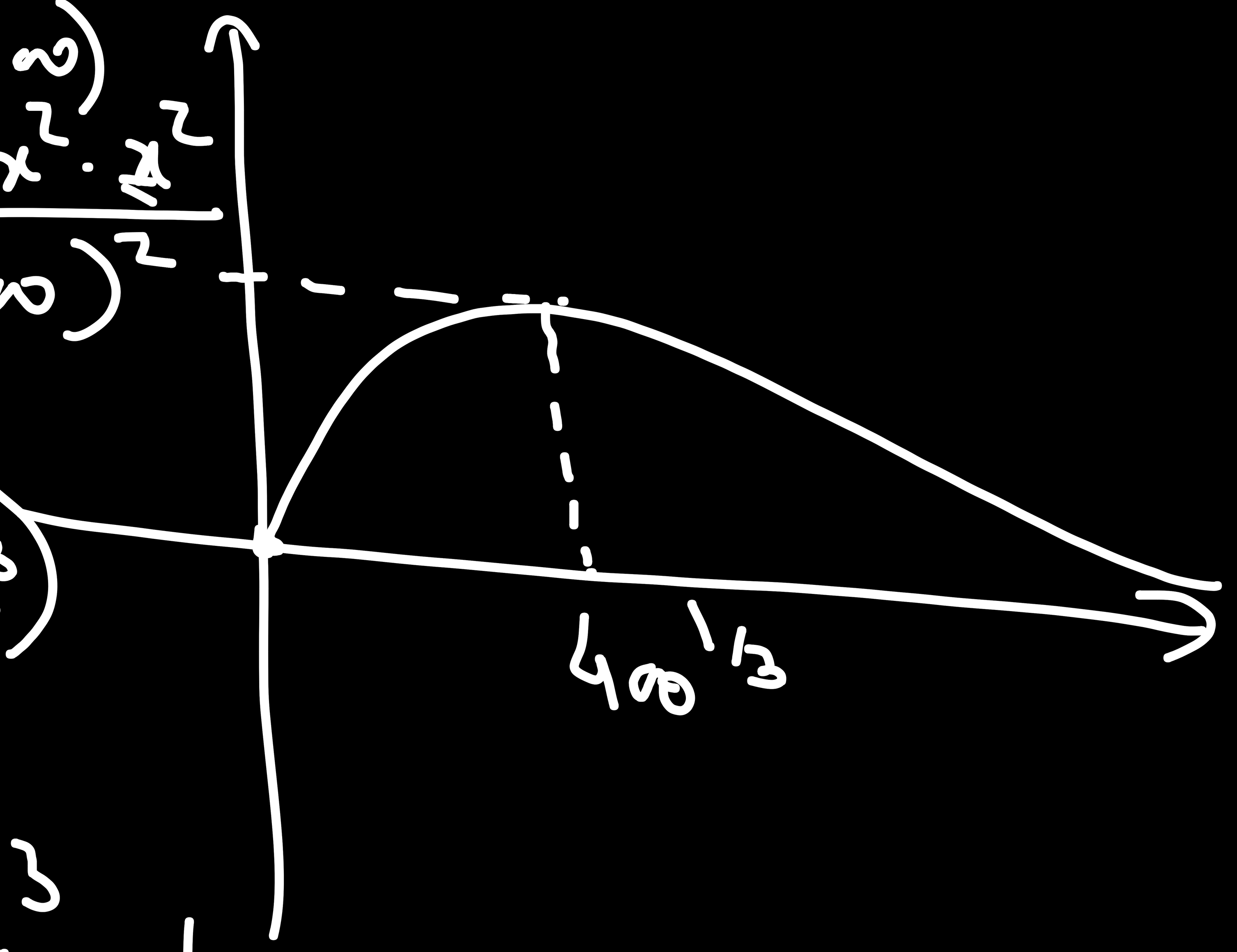
Let $f(x) = \dfrac{x^2}{x^3+200}$

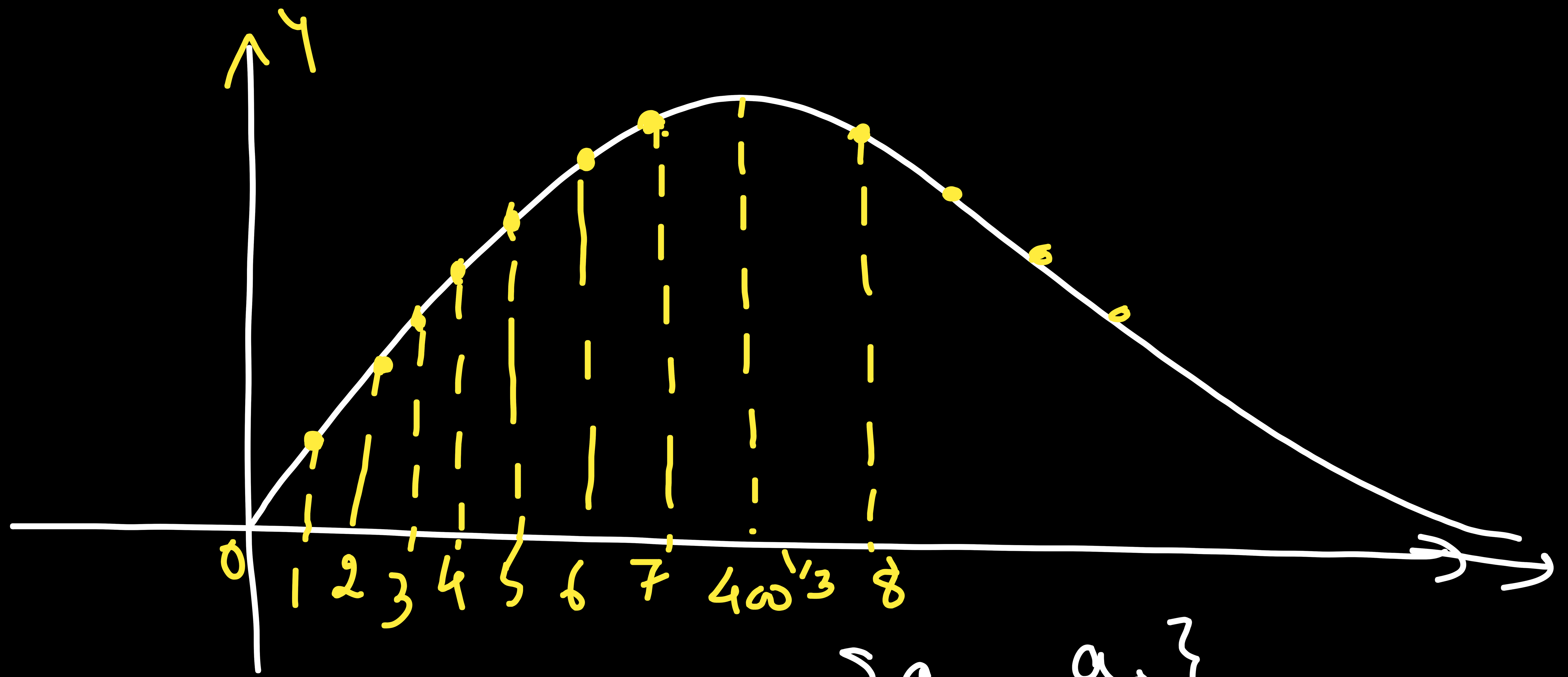
$$f'(x) = \frac{2x(x^3+200) - 3x^2 \cdot x^2}{(x^3+200)^2} = 0$$

$$x(2x^3+400-3x^3) = 0$$

$$\Rightarrow 400 = x^3$$

$$x = (400)^{1/3}$$





$$(a_n)_{max.} = \max.\{a_7, a_8\}.$$

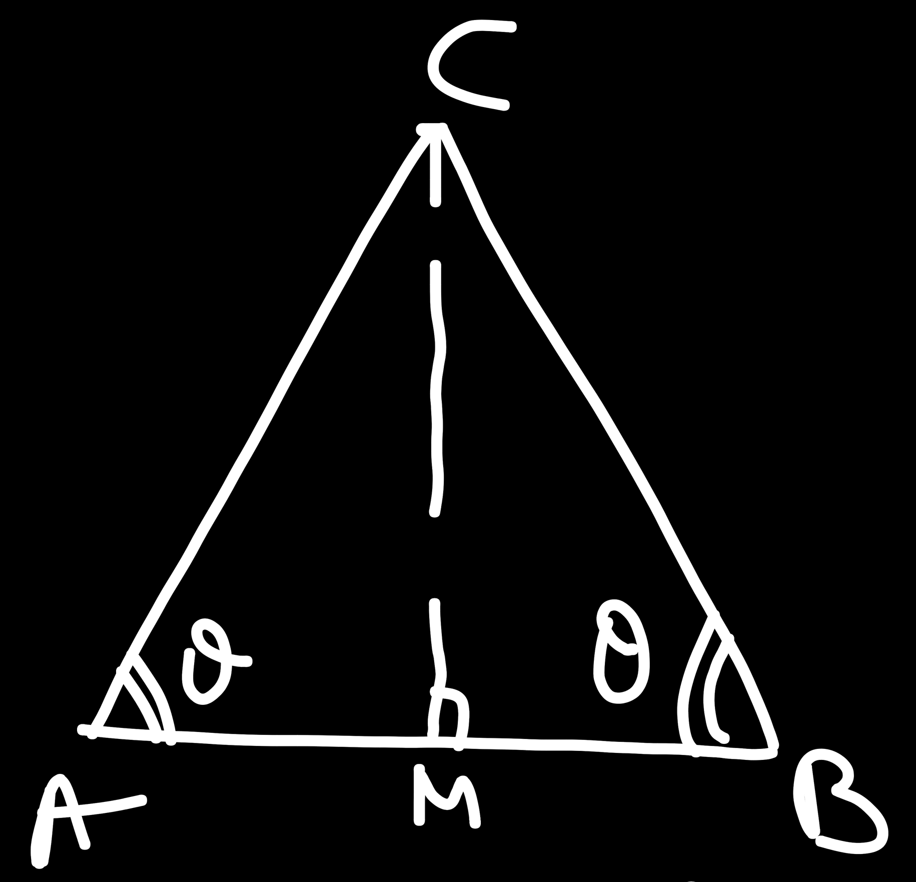
$$a_7 = \frac{49}{343+200} = \frac{49}{543} \approx \frac{1}{11}$$

$$a_8 = \frac{16}{512+200} = \frac{16}{712.} \approx \frac{1}{44.-}$$

$$a_7)_{max.} = \frac{49}{243}$$

Que Find number of non-similar isosceles triangles ABC, such that $\tan(A)+\tan(B)+\tan(C) = 100$.

Sol



Let $A = B = \theta$

$\theta \in (0, 90^\circ)$

$\tan(\theta) = \tan(A) = \tan(B) = t$ (say)

$\in (0, \infty)$

$t + t + \tan(C) = 100 \Rightarrow \tan(C) = 100 - 2t$.

Also in a Δ

$\sum \tan(A) = \prod \tan(A)$

$100 = t \cdot t \cdot (100 - 2t)$.

$\underbrace{100}_{y} = \underbrace{2t^2(50 - t)}_{y}$

$y' = 2\left(2t(50-t) - t^2\right)$

$= 2t(100 - 3t)$

$y\left(\frac{100}{3}\right) = 2\cdot\left(\frac{100}{3}\right)^2\cdot\left(50 - \frac{100}{3}\right) = \left(\frac{100}{3}\right)^3$

Required Δ's = 2 Ans

$9y^2 - 30y + 16 = 0$ and

Sol

Que Find the minimum and maximum value of $f(\alpha,\beta)$ where

(i) $f(\alpha,\beta) = (\alpha-\beta)^2 + (\alpha^2-4\beta-7)^2$ (ii) $f(\alpha,\beta) = (\alpha-\beta)^2 + (\alpha^2-4\beta+7)^2$.

$\alpha, \beta \in R$.

Sol (i) $f(\alpha,\beta) = (\alpha-\beta)^2 + (\alpha^2-4\beta-7)^2$.

If $(\alpha-\beta)=0$ & $\alpha^2-4\beta-7=0 \Rightarrow \alpha^2-4\alpha-7=0 \xrightarrow{\text{roots}} r_1, r_2$.

$\hookrightarrow \alpha=\beta$ $\hookrightarrow D>0 \Rightarrow \alpha \in R$.

$\Rightarrow [f(\alpha,\beta)]_{min} = 0$. for $\alpha=\beta=r_1$ & r_2

$(f(\alpha,\beta))_{max} = D.N.E$ $(\longrightarrow \infty)$.

(ii) $f(\alpha,\beta) = (\alpha-\beta)^2 + (\alpha^2-4\beta+7)^2$

Check $\left.\begin{array}{l}\alpha=\beta \\ \& \ \alpha^2-4\beta+7=0\end{array}\right\} \Rightarrow \alpha^2-4\alpha+7=0$

$\hookrightarrow D<0 \Rightarrow$ No real root.

$\Rightarrow f(\alpha,\beta)=0$ not possible.

$$f(\alpha,\beta) = (\alpha-\beta)^2 + (\alpha^2-4\beta+7)^2$$
$$= (x_1-x_2)^2 + (y_1-y_2)^2.$$

Assume

$P : (x_1, y_1) \equiv (\underset{x}{\overset{\shortparallel}{\alpha}}, \underset{y}{\overset{\shortparallel}{\alpha^2}}) \longrightarrow y = x^2$

$Q : (x_2, y_2) \equiv (\underset{x}{\overset{\shortparallel}{\beta}}, \underset{y}{\overset{\shortparallel}{4\beta-7}}) \longrightarrow y = 4x-7$

$4x - y - 7 = 0$

$\Rightarrow f(\alpha,\beta) = PQ^2$

for A $\quad y = x^2 \xrightarrow{\frac{d}{dx}} y' = 2x = 4.$

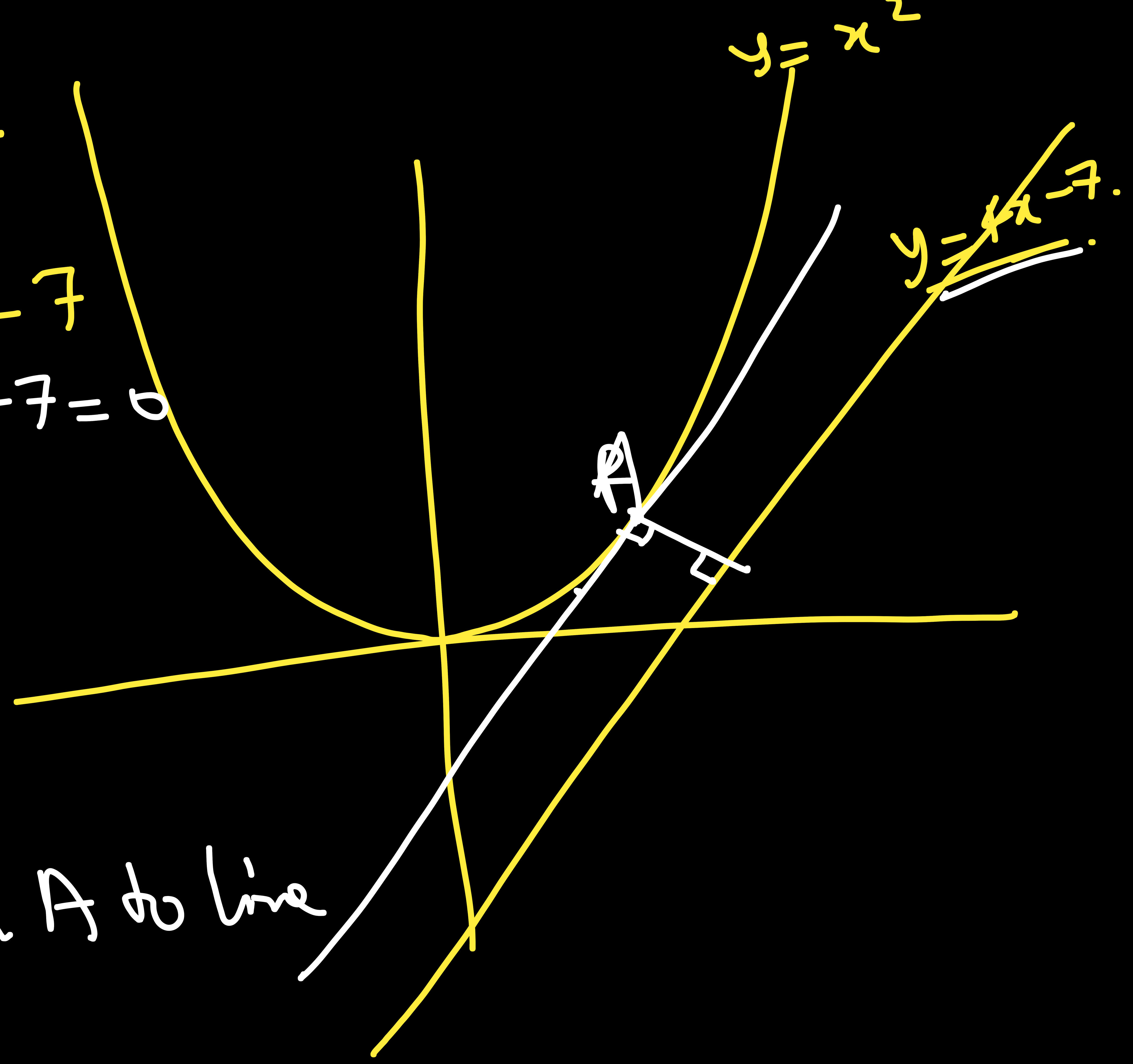
$\Rightarrow x = 2.$

$A : (2, 4)$

minimum distance PQ = length of $\perp$r from A to line

$$= \frac{|8-4-7|}{\sqrt{17}} = \frac{3}{\sqrt{17}}$$

$$\Rightarrow f(\alpha,\beta)_{min} = PQ^2_{min} = \boxed{\frac{9}{17}}\ \text{Ans}$$



$f(\alpha,\beta)_{max.} = \text{D.N.E.} \ (\rightarrow \infty \uparrow)$

Que: If $f'(x) > 0$, $f''(x) > 0$, $\forall x \in (0,1)$ and $f(0) = 0$ and $f(1) = 1$ then

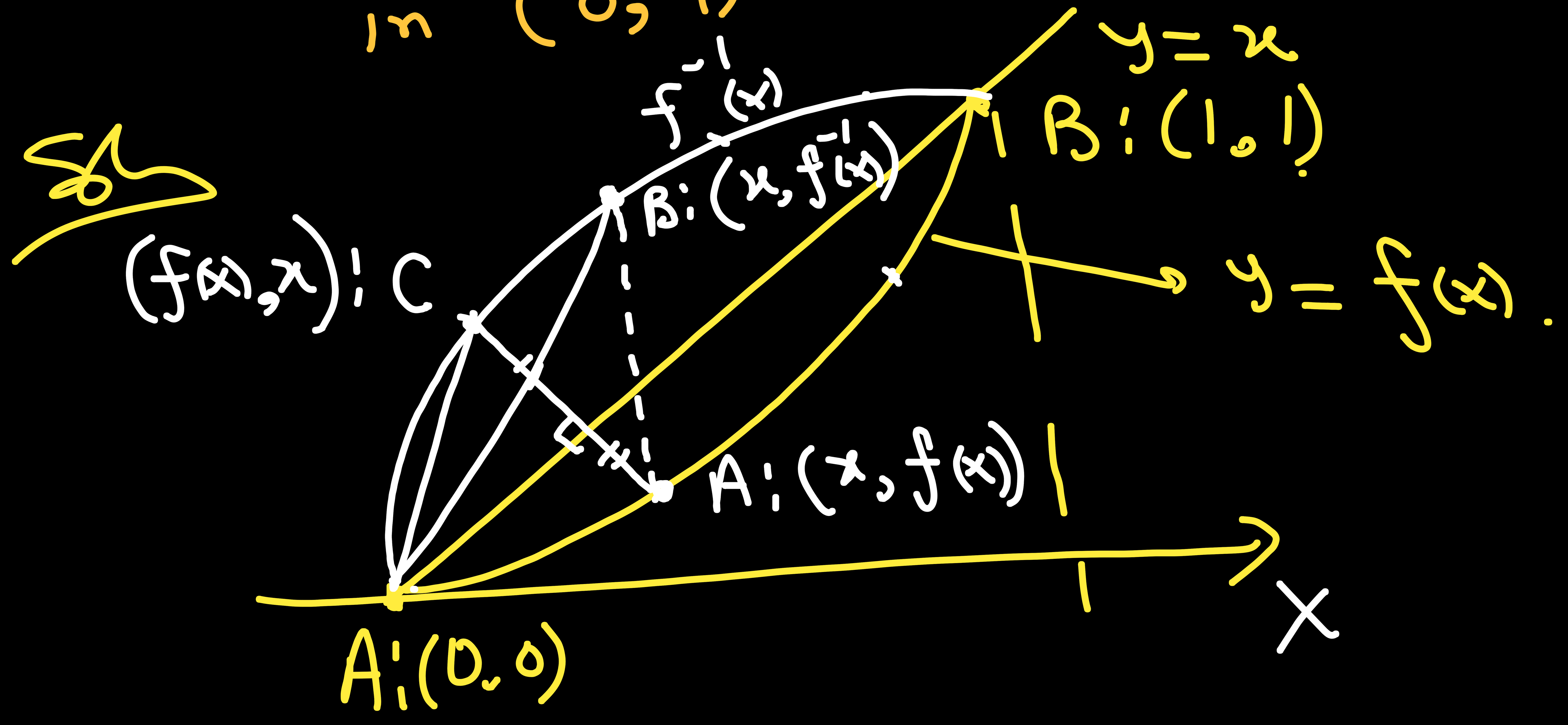
(A) $f(x) \cdot f^{-1}(x) < x^2$, $\forall x \in (0,1)$

(B) $f(x) \cdot f^{-1}(x) > x^2$, $\forall x \in (0,1)$

(C) $f(x) \cdot f^{-1}(x) = x^2$, has two roots in $(0,1)$

(D) $f(x) \cdot f^{-1}(x) = x^2$, has exactly one root in $(0,1)$.

Sol



$m_{AC} > m_{AB}$

$\Rightarrow \dfrac{x}{f(x)} > \dfrac{f^{-1}(x)}{x} \Rightarrow \boxed{f(x) \cdot f^{-1}(x) < x^2}$

A Correct

$\boxed{\text{Que}}$ If $\sin(\pi x)$

$f'(x) = \pi\cos(\pi x) + \underline{8x - 4}$

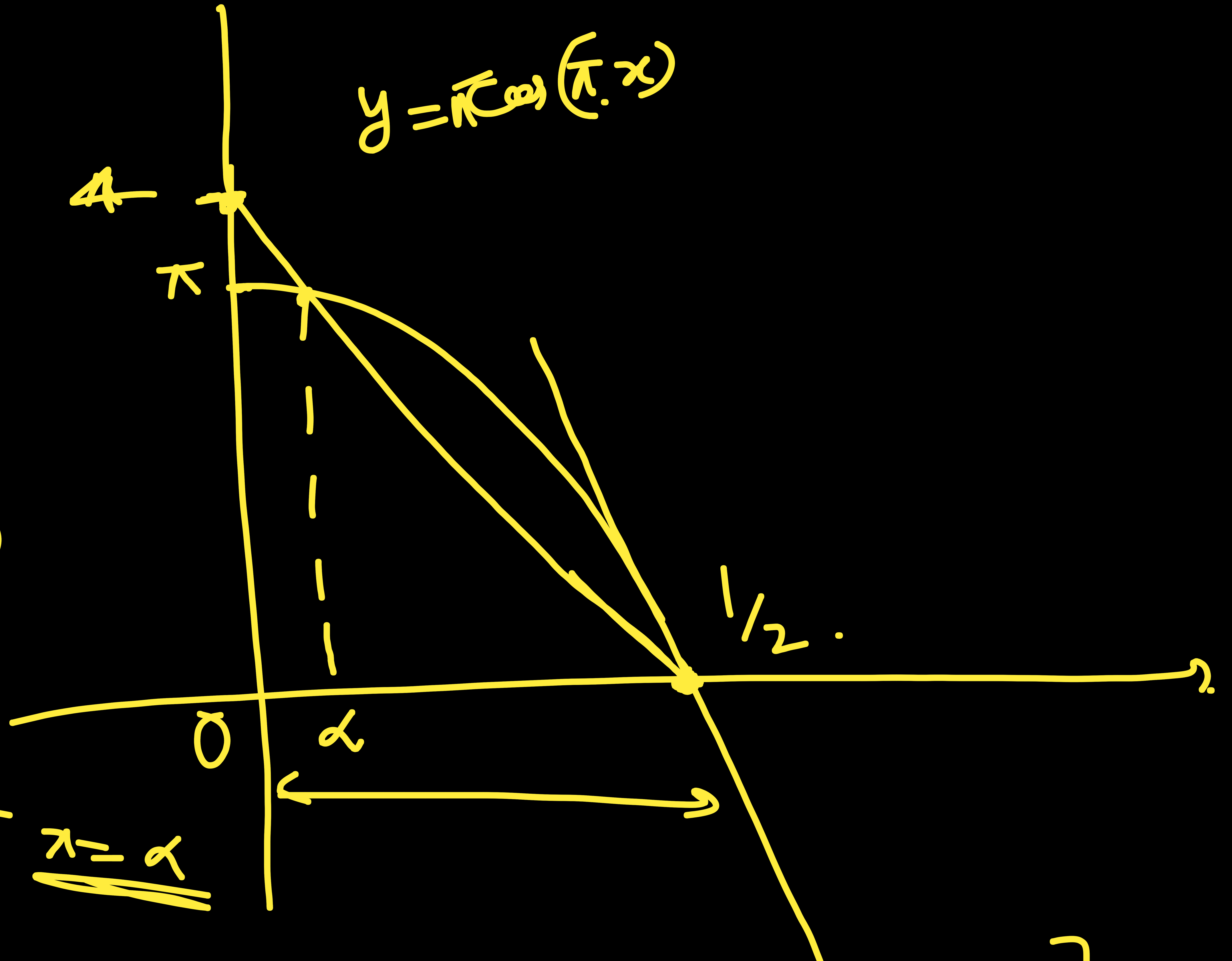
$\qquad\qquad\qquad\qquad\quad -(4 - 8x)$

$f'(0) = \pi - 4 < 0.$

$f'(1/2) = \pi\cos\left(\frac{\pi}{2}\right) + 4 - 4$

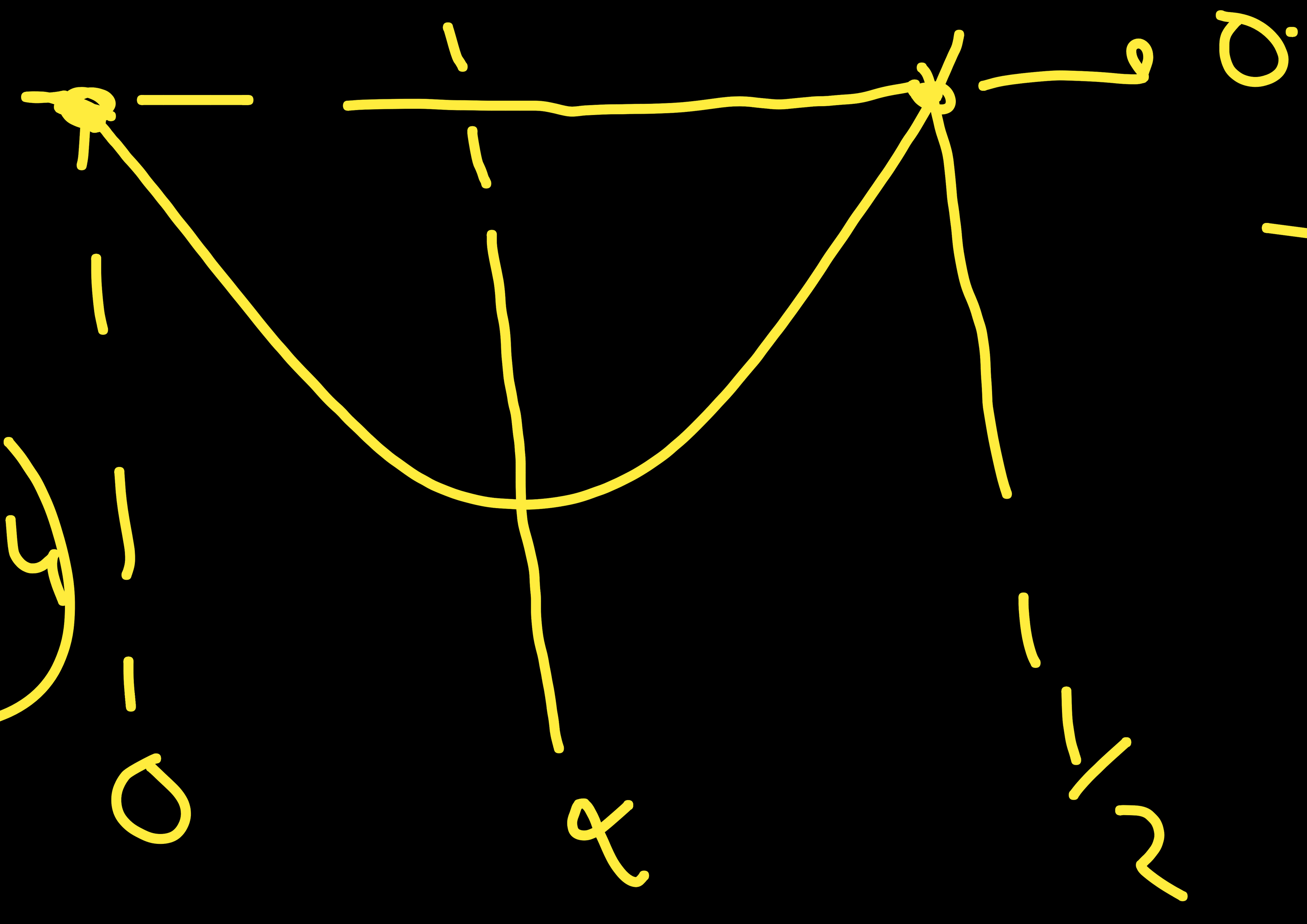
$= 0$

$\pi\cos(\pi x) = 4 - 8x$
$= 8\left(\frac{1}{2} - x\right)$



$\Rightarrow$ $f'(x)$ has point of local min. at $\underline{x = \alpha}$

$\Rightarrow$ $y = f(x)$



$f(0) = 0$

$f\left(\frac{1}{2}\right) = 0$ (for $\lambda = 4$)

$\Rightarrow$ $f(x) \le 0,\ \forall x \in \left[0, \frac{1}{2}\right]$

$\Downarrow$

$\sin(\pi x) \le 4x(1 - x),\ \forall x \in \left[0, \frac{1}{2}\right]$

$\Downarrow$

$\forall x \in [0, 1]$

$\boxed{\lambda_{min.} = 4}$