

# MAGNETISM

## GEO-MAGNETISM (BY DR. WILLIAM GILBERT)

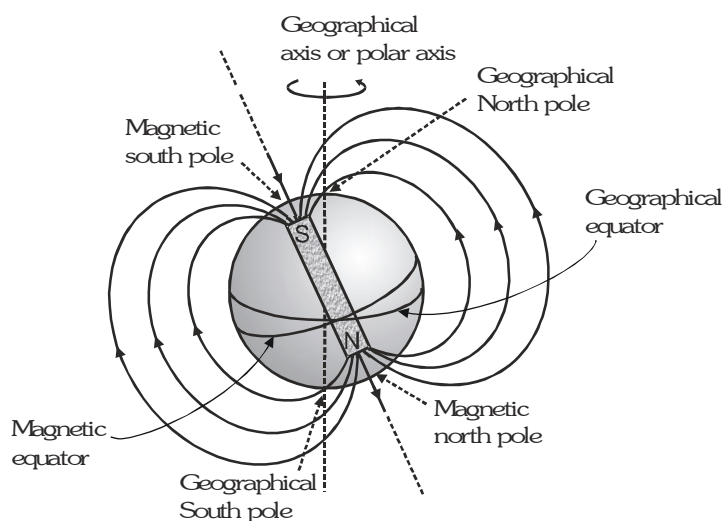
The branch of physics which deals with the study of earth's magnetic field is called geomagnetism.

**Important definitions :**

(a) **Geographic axis :** It is a straight line passing through the geographical poles of the earth. It is also called axis of rotation or polar axis of the earth.

(b) **Geographic Meridian (GM) :** It is a vertical plane at any place which passing through geographical axis of the earth.

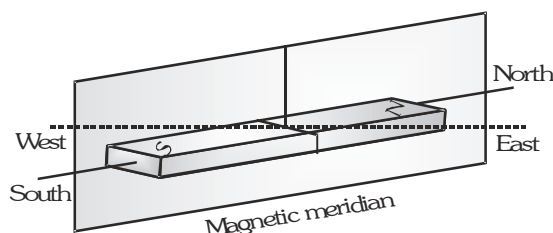
(c) **Geographic equator :** It is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distance from the geographic poles.



A great plane which passes through geographic equator and perpendicular to the geographic axis called **geographic equatorial plane**. This plane cuts the earth in two equal parts, a part has geographic north called **northern hemisphere (NHS)** and another part has geographic south called **southern hemisphere (SHS)**.

(d) **Magnetic axis :** It is a straight line passing through magnetic poles of the earth. It is inclined to the geographic axis at nearly  $17^\circ$ .

(e) **Magnetic Meridian (MM) :** (i) It is a vertical plane at any place which passing through magnetic axis of the earth.  
 (ii) It is a vertical plane at any place which passing through axis of free suspended bar magnet or magnetic needle.  
 (iii) It is a vertical plane at any place which contains all the magnetic field lines of earth of that place.

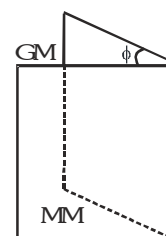


(f) **Magnetic equator :** It is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distance from the magnetic poles.

## MAIN ELEMENTS OF EARTH'S MAGNETIC FIELD

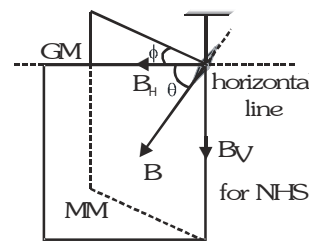
### Angle of declination ( $\phi$ )

At a given place the acute angle between geographic meridian and the magnetic meridian is called angle of declination, i.e. at a given place it is the angle between the geographical north south direction and the direction indicated by a magnetic compass needle in its equilibrium.



### Angle of dip ( $\theta$ )

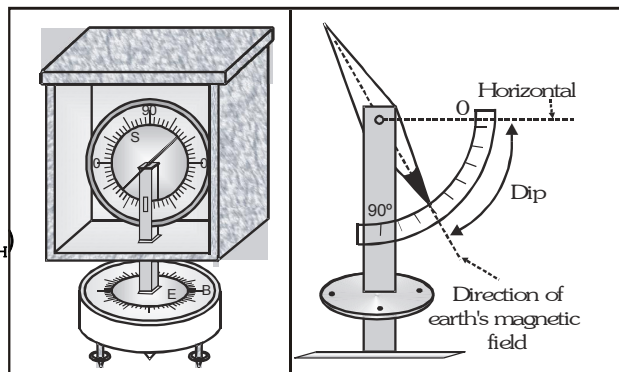
- It is an angle which the direction of resultant magnetic field of the earth subtends with the horizontal line in magnetic meridian at the given place.
- It is an angle which the axis of freely suspended magnetic needle (up or down) subtends with the horizontal line in magnetic meridian at a given place.



In northern hemisphere, north pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface. In southern hemisphere, south pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface.

**Dip circle** : Angle of dip at a place is measured by the instrument called 'Dip-circle' in which a magnetic needle is free to rotate in vertical plane. About its horizontal axis. The ends of the needle move over a vertical scale graduated in degree.

**Horizontal component of earth magnetic field ( $B_H$ )**  
Horizontal component of earth magnetic field at a given place is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.



$$B_H = B \cos \theta \quad \text{and} \quad B_V = B \sin \theta \quad \dots\dots(1)$$

$$\text{so that} \quad \tan \theta = \frac{B_V}{B_H} \quad \text{and} \quad B = \sqrt{B_H^2 + B_V^2} \quad \dots\dots(2)$$

At magnetic poles  $\theta = 90^\circ$

$B_H = 0$  and only  $B_V$  exist

❖ At magnetic equator  $\theta = 0^\circ$

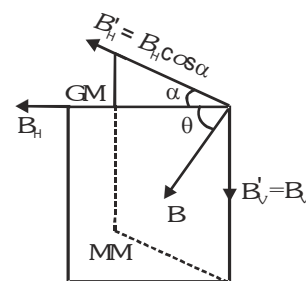
$B_V = 0$  and only  $B_H$  exist

$\phi$  decides the plane in which magnetic field lies at any place, ( $\phi$ ) and ( $\theta$ ) decides the direction of magnetic field and ( $B_H$ ) decides the magnitude of the field.

### Apparent angle of dip ( $\theta'$ ) :

When the plane of vertical scale of dip circle is in the magnetic meridian, the needle rests in the direction of earth's magnetic field. The angle made by the needle with the horizontal is called true dip or actual dip. If the plane of vertical scale of dip circle is not kept in magnetic meridian, then the needle will not indicate the correct direction of earth magnetic field.

In this situation the angle made by the needle with the horizontal is called the apparent angle of dip. Suppose the dip circle is set at an angle  $\alpha$  to the magnetic meridian. Effective horizontal component in this plane will be  $B_H \cos \alpha$  and no effect on vertical component  $B_V$ .



$$\text{Apparent angle of dip} \quad \tan \theta' = \frac{B'_V}{B'_H} \Rightarrow \tan \theta' = \frac{B_V}{B_H \cos \alpha} \Rightarrow \tan \theta' = \frac{\tan \theta}{\cos \alpha}$$

❖ For a vertical plane other than magnetic meridian

$$\alpha > 0 \Rightarrow \cos \alpha < 1 \Rightarrow \tan \theta' > \tan \theta \Rightarrow \theta' > \theta,$$

so apparent angle of dip is always more than actual angle of dip at any place.

❖ For a vertical plane perpendicular to magnetic meridian  $\alpha = 90^\circ \Rightarrow \tan \theta' = \frac{\tan \theta}{\cos 90^\circ} = \infty$

$\theta' = 90^\circ$ , so in a plane perpendicular to magnetic meridian dip needle becomes just vertical.

**Example**

At a certain place, the horizontal component of earth's magnetic field is  $\sqrt{3}$  times of the vertical component.

What the angle of dip at that place.

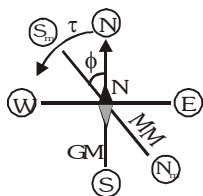
**Solution**

$$B_H = \sqrt{3} B_V, \quad \tan\theta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3}B_V} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

**Example**

A compass needle of magnetic moment is  $60 \text{ A-m}^2$  pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is  $40 \mu\text{T}$ , experiences a torque  $1.2 \times 10^{-3} \text{ N-m}$ .

What is the declination of that place.

**Solution**

$$\tau = MB \sin\phi \Rightarrow \sin\phi = \frac{\tau}{MB} = \frac{1.2 \times 10^{-3}}{24 \times 10^{-4}} = \frac{1}{2} \Rightarrow \phi = 30^\circ$$

**Example**

If the dip circle is set at  $45^\circ$  to the magnetic meridian, then the apparent dip is  $30^\circ$ . Calculate the true dip.

**Solution**

$$\tan\theta' = \frac{\tan\theta}{\cos\alpha}; \quad \tan\theta = \tan\theta' \cos\alpha = \tan 30^\circ \cos 45^\circ = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}}; \quad \tan\theta = \frac{1}{\sqrt{6}} \Rightarrow \theta = \tan^{-1} \left( \frac{1}{\sqrt{6}} \right)$$

**Example**

A magnetic needle is free to rotate in a vertical plane and that plane makes an angle of  $60^\circ$  with magnetic meridian. If the needle stays in a direction making an angle of  $\tan^{-1} \left( \frac{2}{\sqrt{3}} \right)$  with the horizontal direction, what would be the actual dip at that place?

**Solution**

$$\tan\theta = \tan\theta' \cos\alpha \quad \left( \because \theta' = \tan^{-1} \left( \frac{2}{\sqrt{3}} \right), \alpha = 60^\circ \right)$$

$$\therefore \tan\theta = \tan \left( \tan^{-1} \frac{2}{\sqrt{3}} \right) \cos 60^\circ \quad \tan\theta = \frac{2}{\sqrt{3}} \cdot \frac{1}{2} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

**Example**

A 1-meter long narrow solenoid having 1000 turns is placed in magnetic meridian. Find the current in the solenoid which neutralises the earth's horizontal field of 0.36 oersted at the centre of the solenoid.

**Solution**

The magnetic field intensity at the centre of solenoid is  $H = ni = 1000 i \text{ A/m} = 4\pi i \text{ oersted}$   
 $(\because 1 \text{ amp/meter} = 4\pi \times 10^{-3} \text{ oersted})$

Since it neutralises the earth's field of 0.36 oersted, it is equal and opposite to the earth's field.

$$\therefore 4\pi i = 0.36$$

$$\Rightarrow i = \frac{0.36}{4 \times 3.14} = 0.0286 \text{ ampere} = 28.6 \text{ milli-ampere or } 28.6 \text{ mA}$$

### Example

If  $\theta_1$  and  $\theta_2$  are angles of dip in two vertical planes at right angle to each other and  $\theta$  is true dip then prove  $\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$ .

### Solution

If the vertical plane in which dip is  $\theta_1$  subtends an angle  $\alpha$  with meridian than other vertical plane in which dip is  $\theta_2$  and is perpendicular to first will make an angle of  $90 - \alpha$  with magnetic meridian. If  $\theta_1$  and  $\theta_2$  are apparent dips then

$$\tan \theta_1 = \frac{B_V}{B_H \cos \alpha} \quad \tan \theta_2 = \frac{B_V}{B_H \cos(90 - \alpha)} = \frac{B_V}{B_H \sin \alpha}$$

$$\cot^2 \theta_1 + \cot^2 \theta_2 = \frac{1}{(\tan \theta_1)^2} + \frac{1}{(\tan \theta_2)^2} = \frac{B_H^2 \cos^2 \alpha + B_H^2 \sin^2 \alpha}{B_V^2} = \frac{B_H^2}{B_V^2} = \left( \frac{B \cos \theta}{B \sin \theta} \right)^2 = \cot^2 \theta$$

$$\text{So } \cot^2 \theta_1 + \cot^2 \theta_2 = \cot^2 \theta$$

### Example

Considering earth as a short bar magnet show that the angle of dip  $\theta$  is related to magnetic latitude  $\lambda$  as  $\tan \theta = 2 \tan \lambda$ .

### Solution

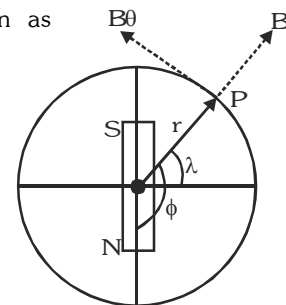
For a magnetic dipole the field components at point P ( $r, \phi$ ) are given as

$$B_r = \frac{\mu_0}{4\pi} \frac{2M \cos \phi}{r^3} \quad B_\theta = \frac{\mu_0}{4\pi} \frac{M \sin \phi}{r^3}$$

$$\tan \theta = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta} = -\frac{\mu_0}{4\pi} \frac{2M \cos \phi}{r^3} \frac{4\pi r^3}{\mu_0 M \sin \phi}$$

$$\text{or } \tan \theta = -2 \cot \phi$$

$$\text{From figure } \phi = \frac{\pi}{2} + \lambda \quad \text{So } \tan \theta = -2 \cot \left( \frac{\pi}{2} + \lambda \right) \quad \text{or } \tan \theta = 2 \tan \lambda$$



### Example

At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of the magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

### Solution

$$\text{From formula, } B_H = B \cos \theta$$

$$B = \frac{B_H}{\cos \theta} = B_H \sec \theta = 0.16 \times 2 = 0.32 \text{ G.}$$

The earth's field is 0.32 G, in direction 60° upwards from horizontal, in a plane (magnetic meridian) 12° West of geographical meridian.

### Example

A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated through 90° what apparent dip will it show?

### Solution

Let  $\theta_1$  and  $\theta_2$  be apparent dip shown by dip circle in two perpendicular positions then true dip  $\theta$  is given by

$$\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2 \quad \text{or } \cot^2 45 = \cot^2 60 + \cot^2 \theta_2$$

$$\text{or } \cot^2 \theta_2 = \frac{2}{3} \quad \text{or } \cot \theta_2 = 0.816 \text{ giving } \theta_2 = 51^\circ$$

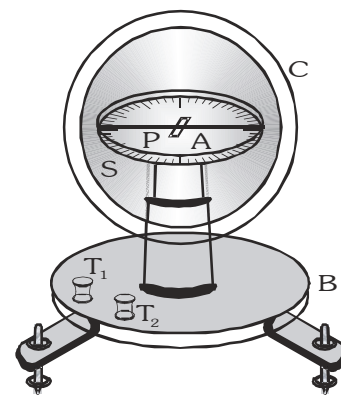
**Applications of Geo-magnetism :-** (Based on  $B_H$ )

**Tangent galvanometer :-**

It is an instrument which can detect/measures electric currents. It is also called moving magnet galvanometer.

**Principle :-** It is based on 'tangent law'

**Construction :-** (i) It consists of a circular coil of a large number of turns of insulated copper wire wound over a vertical circular frame.  
 (ii) A small magnetic compass needle is pivoted at the centre of vertical circular coil. This needle can rotate freely in a horizontal plane.



**Tangent law :-** If a current is passed through the vertical coil, then magnetic field produced at its centre is perpendicular to the horizontal component of earth's magnetic field since coil is in magnetic meridian. So in the effect of two crossed fields ( $B_H \perp B_0$ ) compass needle comes in equilibrium according to tangent law.

Torque on needle due to ( $B_0$ ) -  $\tau_1 = MB_0 \sin(90-\theta)$

Torque on needle due to ( $B_H$ ) -  $\tau_2 = MB_H \sin\theta$

At equilibrium condition of needle net torque on it is zero

$$\vec{\tau}_1 + \vec{\tau}_2 = 0 \quad ; \quad |\vec{\tau}_1| = |\vec{\tau}_2|$$

$$MB_0 \sin(90-\theta) = MB_H \sin\theta$$

$$B_0 \cos\theta = B_H \sin\theta$$

$$B_0 = B_H \frac{\sin\theta}{\cos\theta} \quad ; \quad B_0 = B_H \tan\theta \quad \text{where } B_H = B \cos\theta, \quad \theta \rightarrow \text{angle of dip.}$$

$$\therefore \frac{\mu_0 NI}{2R} = B_H \tan\theta \quad \text{so } I = \left( \frac{2B_H R}{\mu_0 N} \right) \tan\theta$$

$$I = K \tan\theta, \text{ so for this galvanometer } I \propto \tan\theta$$

The electric current is proportional to the tangent of the angle of deflection

**Reduction factor :** is a constant for the given galvanometer at given place.

The reduction factor of a tangent galvanometer is numerically equal to the current required to produce a deflection of  $45^\circ$  in it.

$$\theta = 45^\circ \Rightarrow I = K \tan(45^\circ) \quad ; \quad I = K$$

SI unit of 'K'  $\Rightarrow$  ampere

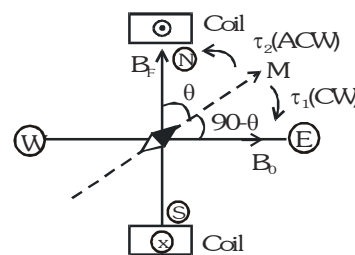
- ☉ **Sensitivity :** A tangent galvanometer is both sensitive and accurate if the change in its deflection is large for a given fractional change in current.

$$I = K \tan\theta \quad \text{or} \quad dI = K \sec^2\theta d\theta$$

$$\frac{dI}{I} = \frac{d\theta}{\sin\theta \cos\theta} = \frac{2d\theta}{\sin 2\theta} \quad \text{or} \quad d\theta = \frac{\sin 2\theta}{2} \frac{dI}{I}$$

$$d\theta = (d\theta)_{\max} \quad \text{if } \sin 2\theta = 1 = \sin \frac{\pi}{2} \quad \text{so} \quad \theta = \frac{\pi}{4}$$

The tangent galvanometer has maximum sensitivity when  $\theta = 45^\circ$ .



### Example

Why is a short magnetic needle used in tangent galvanometer ?

### Solution

It is kept short so that the field around it is uniform.

### Example

Why does a tangent galvanometer fail to work at magnetic poles of the earth?

### Solution

Because horizontal component of earth's field ( $B_H$ ) is zero at the magnetic poles.

### Example

A cell of an emf of 2V and internal resistance of  $0.5\ \Omega$  is sending current through a tangent galvanometer of resistance  $4.5\Omega$ . If another external resistance of  $95\ \Omega$  is introduced, the deflection of galvanometer is 45 . Calculate the raduction factor of galvanometer.

### Solution

$$I = \frac{E}{r + R + R'} = \frac{2}{0.5 + 4.5 + 95} = \frac{2}{100} \text{ ampere}$$

From  $I = K \tan \theta$  reduction factor  $K = \frac{I}{\tan \theta} = \frac{2}{100 \times \tan 45^\circ} = 0.02$  ampere

### Example

Two tangent galvanometers A and B have their number of turns in the ratio 1 : 3 and diameters in the ratio 1 : 2

- (a) Which galvanometer has greater reduction factor
- (b) Which galvanometer shown greater deflection, when both are connected in series to a d.c. source.

### Solution

(a)  $\frac{N_A}{N_B} = \frac{1}{3}, \frac{R_A}{R_B} = \frac{1}{2}$ ; Reduction factor  $K = \Rightarrow \frac{K_A}{K_B} = \frac{R_A}{R_B} \cdot \frac{N_B}{N_A} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2} \Rightarrow K_A > K_B$

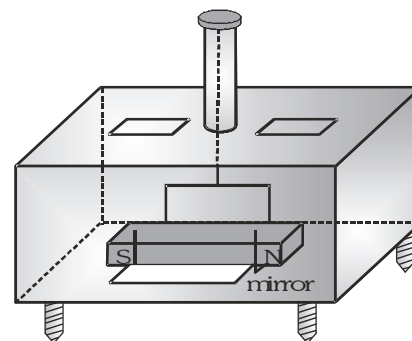
- (b) From  $I = K \tan \theta$ , ( $I$  = same in series combination)  $\therefore K_A > K_B \Rightarrow \tan \theta_A < \tan \theta_B \Rightarrow \theta_A < \theta_B$

### VIBRATION MAGNETOMETER :

It is an instrument used to compare the horizontal components of magnetic field of earth of two different places, to compare magnetic fields and magnetic moments of two bar magnets. It is also called oscillation magnetometer.

**Principle :** This device works on the principle, that whenever a freely suspended bar magnet horizontal component in earth magnetic field ( $B_H$ ) is slightly disturbed from its equilibrium position then, it will experience a torque and executes angular S.H.M.

\*Rotation is possible only in horizontal plane.



**Angular S.H.M of magnetic dipole :-** When a dipole is suspended in a uniform magnetic field it will align itself parallel to field. Now if it is given a small angular displacement  $\theta$  about its equilibrium position. The restoring torque acts on it :

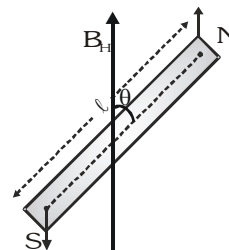
$$\tau = -M B_H \sin \theta \Rightarrow I \alpha = -M B_H \sin \theta = -M B_H \theta, \quad (\because \sin \theta \simeq \theta)$$

$$\Rightarrow \alpha = \frac{M B_H}{I} (-\theta) \Rightarrow \alpha = \omega^2 (-\theta) \Rightarrow \omega^2 = \frac{M B_H}{I}$$

The time period of angular S.H.M.  $\Rightarrow T = 2\pi \sqrt{\frac{I}{M B_H}}$

$M$  = magnetic moment of bar magnet

$I$  = moment of inertia of bar magnet about its geometric axis



### Comparison of magnetic moments of magnets of the same size

Let the two magnets of same size have moment of inertia  $I$  and magnetic moments  $M_1$  and  $M_2$ . Suspend the two given magnets turn by turn in the metal stirrup of the vibration magnetometer and note the time period in each case.

Then  $T_1 = 2\pi \sqrt{\frac{I}{M_1 B}}$  and  $T_2 = 2\pi \sqrt{\frac{I}{M_2 B}}$

Dividing,  $\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$  or  $\frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$

Since  $T_1$  and  $T_2$  are known therefore the ratio  $\frac{M_2}{M_1}$  can be determined.

### Comparison of magnetic moments of magnets of different sizes

Let the two magnets have moments of inertia  $I_1$  and  $I_2$  and magnetic moments  $M_1$  and  $M_2$  respectively. Place the two given magnets one upon the other as shown in Fig. (a). This combination is called 'sum combination'. It has moment of inertia  $(I_1 + I_2)$  and magnetic moment  $(M_1 + M_2)$ . Put this combination in the magnetometer and

set it into oscillations. The time period  $T_1$  is determined.

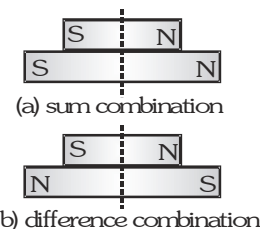
$$T_1 = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B}}$$

Now, the two magnets are placed as shown in Fig. (b). This combination is called 'difference combination'. It has moment of inertia  $(I_1 + I_2)$  and magnetic moment  $(M_1 - M_2)$ . This combination is put in the magnetometer and its time period  $T_2$  is determined.

$$T_2 = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}} \dots\dots(2)$$

Dividing,  $\frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$  [from equation (1) and (2)]

knowing  $T_1$  and  $T_2$ , we can determine  $\frac{M_1}{M_2}$ .



$$\dots\dots\dots(1)$$

☉ **Comparison of earth's magnetic field at two different places**

Let the vibrating magnet have moment of inertia  $I$  and magnetic moment  $M$ . Let it be vibrated in places where earth's magnetic field is  $B_{H_1}$  and  $B_{H_2}$

$$\text{Then, } T_1 = 2\pi\sqrt{\frac{I}{MB_{H_1}}} \text{ and } T_2 = 2\pi\sqrt{\frac{I}{MB_{H_2}}}$$

$T_1$  and  $T_2$  are determined by placing magnetometer at two different places, turn by turn.

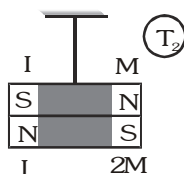
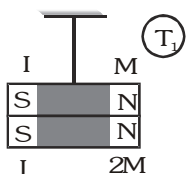
$$\text{Dividing, } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \text{ or } \frac{T_1^2}{T_2^2} = \frac{B_{H_2}}{B_{H_1}} = \frac{B_2 \cos \theta_2}{B_1 \cos \theta_1} \Rightarrow \frac{B_1}{B_2} = \frac{T_2^2 \cos \theta_2}{T_1^2 \cos \theta_1}$$

Knowing  $T_1$ ,  $T_2$  and  $\theta_1$ ,  $\theta_2$  the ratio  $\frac{B_1}{B_2}$  can be determined.

**Example**

Magnetic moments of two identical magnets are  $M$  and  $2M$  respectively. Both are combined in such a way that their similar poles are same side. The time period in this is case ' $T_1$ '. If polarity of one of the magnets is reversed its period becomes ' $T_2$ ' then find out ratio of their time periods respectively.

**Solution**



$$M_{\text{system}} = 2M + M = 3M \quad M_{\text{system}} = 2M - M = M$$

$$I_{\text{system}} = 2I \quad I_{\text{system}} = 2I$$

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \quad (I_{\text{system}} \rightarrow \text{same, } B_H \rightarrow \text{same}) \quad \boxed{T \propto \frac{1}{\sqrt{M}}};$$

$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{M}{3M}} = \frac{1}{\sqrt{3}}$$

**Example**

A magnet is suspended in such a way when it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is  $30^\circ$  and 15 oscillations per min at a place where dip angle is  $60^\circ$ . Find the ratio of the total earth's magnetic field at the two places.

**Solution**

$$f = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}} \Rightarrow f^2 = \frac{1}{4\pi^2} \cdot \frac{MB \cos \theta}{I} \quad \text{I and M are same in given cases}$$

$$\frac{B_1}{B_2} = \frac{f_1^2}{f_2^2} \cdot \frac{\cos \theta_2}{\cos \theta_1} = \frac{20 \times 20}{15 \times 15} \cdot \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{16}{9\sqrt{3}}$$



**Example**

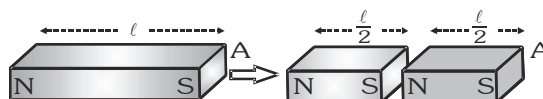
Time period of thin rectangular bar magnet of vibration magnetometer is 'T'. If it is broken into two equal parts then find out time period of each part at the same place.

- (a) Along its length (b) Perpendicular to its length

**Solution**

$$\text{Time period of thin rectangular bar magnet } T = 2\pi \sqrt{\frac{I}{MB_H}} \quad (\text{here } I = \frac{ml^2}{12})$$

**Case I** perpendicular to its axis :



for each part mass and length both become half

$$\text{moment of inertia } I' = \frac{I}{8} \quad \text{magnetic moment } M' = \frac{M}{8}$$

$$\text{Time period of each part } T' = \sqrt{\frac{I/8}{M/2}} = \frac{T}{2} \quad [B_H \rightarrow \text{same}]$$

If bar magnet broken into 'n' equal parts perpendicular to its axis then time period of each part becomes

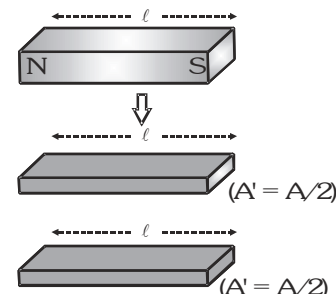
$$T' = \frac{T}{n}$$

**Case - II** parallel to its axis :-

for each part: mass become half and length remain same

$$\text{moment of inertia } I' = \frac{I}{2} \quad \text{magnetic moment } M' = \frac{M}{2}$$

$$\text{Time period of each part } T' = \sqrt{\frac{I/2}{M/2}}$$



If bar magnet cuts into 'n' equal parts parallel to its axis then time period of each part remain equal to time period original magnet

**Example**

A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is  $2^{5/4}$  sec. One of the magnets is removed and if the other magnet oscillates in the same field, calculate the time period.

**Solution**

Magnetic moment is a vector quantity. If the magnetic moments of the two magnets are M each then, the net magnetic moment when the magnets are placed perpendicular to each other, is

$$M_{\text{eff.}} = \sqrt{M^2 + M^2} = M\sqrt{2} \quad \text{and the moment of inertia is } 2I. \quad \text{so } T = 2\pi \sqrt{\frac{2I}{M\sqrt{2}H}}$$

$$\text{When one of the magnets is withdrawn, the time period is } T' = 2\pi \sqrt{\frac{I}{MH}}$$

$$\therefore \frac{T'}{T} = \sqrt{\frac{1}{2^{1/2}}} \quad \text{or } T' = \frac{T}{2^{1/4}} = 2^{5/4 - 1/4} = 2 \text{ sec}$$

## NEUTRAL POINT

It is a point where net magnetic field is zero.

At this point magnetic field of bar magnet or current carrying coil or current carrying wire is just neutralised by magnetic field of earth. ( $B_H$ )

A compass needle placed at this neutral point can set itself in any direction.

### Location of Neutral Points :

- (a) **When N-pole of magnet directed towards North :-** Two neutral points symmetrically located on equatorial line of magnet. Let distance of each neutral point from centre of magnet is 'y' then

$$B_{eq} = B_H$$

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{M}{(y^2 + \ell^2)^{3/2}}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{y^3} = B_H \quad (\text{If } y \gg \ell)$$

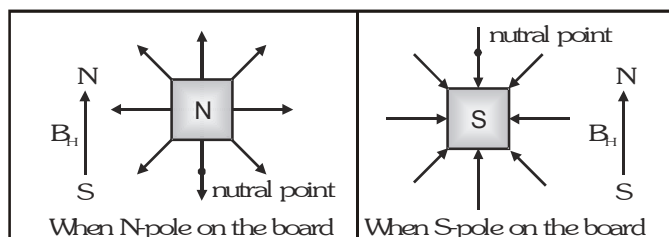
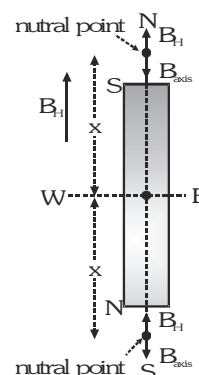
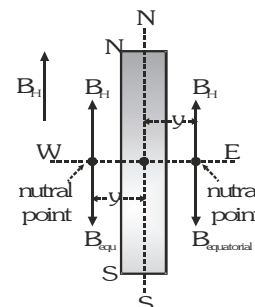
- (b) **When S-pole of magnet directed towards North :-**

Two neutral points symmetrically located on the axial line of magnet. Let distance of each neutral points from centre of the magnet is x, then

$$B_{axis} = B_H \Rightarrow B_H = \frac{\mu_0}{4\pi} \cdot \frac{2Mx}{(x^2 - \ell^2)^2}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3} = B_H \quad (\text{If } x \gg \ell)$$

- (c) If magnet is held vertically on the board, then only one neutral point is obtained on the horizontal board.



### Example

The magnetic field at a point x on the axis of a small bar magnet is equal to the field at a point y on the equator of the same magnet. Find the ratio of the distances of x and y from the centre of the magnet.

### Solution

$$B_{axis} = B_{equatorial} \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{M}{y^3} \Rightarrow \frac{2}{x^3} = \frac{1}{y^3} \Rightarrow \frac{x^3}{y^3} = \frac{2}{1} \Rightarrow \frac{x}{y} = 2^{1/3}$$

### Example

A coil of 0.1 m radius and 100 turns placed perpendicular magnetic meridian. When current of 2 ampere is flow through the coil then the neutral point is obtained at the centre. Find out magnetising field of earth.

### Solution

$$\text{Magnetic field at centre of coil } B = \frac{\mu_0 NI}{2R} = \mu_0 H_c \quad (\because H_c = H_e)$$

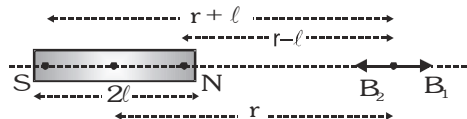
$$\text{Magnetising field of earth } H_e = \frac{NI}{2R} = \frac{100 \times 2}{2 \times 0.1} = 1000 \text{ A/m}$$

**Example**

A short magnet of moment  $6.75 \text{ A-m}^2$  produces a neutral point on its axis. If the horizontal component of earth's magnetic field  $5 \times 10^{-5} \text{ Wb/m}^2$ , Calculate the distance of the neutral point from the centre.

**Solution**

$$B_H = \frac{2KM}{d^3} \Rightarrow d = \left( \frac{2KM}{B_H} \right)^{1/3} = \left( \frac{2 \times 10^{-7} \times 6.75}{5 \times 10^{-5}} \right)^{1/3} = 0.3 \text{ m} = 30 \text{ cm}$$

**MISCELLANEOUS****Magnetic field of long Bar magnet****(i) At Axial position :-**

Magnetic field at point 'P' due to north pole  $B_1 = \frac{\mu_0}{4\pi} \frac{m}{(r-l)^2}$  (away from north pole)

Magnetic field at point 'P' due to south pole  $B_2 = \frac{\mu_0}{4\pi} \frac{m}{(r+l)^2}$  (towards north pole)

Net magnetic field at point 'P'

$$B_{\text{axis}} = B_1 - B_2, (\because B_1 > B_2) = \frac{\mu_0 m}{4\pi} \left[ \frac{1}{(r-l)^2} - \frac{1}{(r+l)^2} \right] = \frac{\mu_0 m}{4\pi} \left[ \frac{4rl}{(r^2 - l^2)^2} \right]$$

$$B_{\text{axis}} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}, \text{ where } M = m(2l)$$

If magnet is short  $r \gg l$ , then  $B_{\text{axis}} \sim \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

**(ii) At equatorial position :-**

Magnetic field at point 'P' due to north pole :-

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{(\sqrt{r^2 + l^2})^2} \dots (1) \text{ (along NP line)}$$

Magnetic field at point 'P' due to south pole :-

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{(\sqrt{r^2 + l^2})^2} \dots (2) \text{ (along PS line)}$$

From equation (1) & (2)  $B_1 = B_2 = \frac{\mu_0}{4\pi} \frac{m}{r^2 + l^2} = B$  (Let)

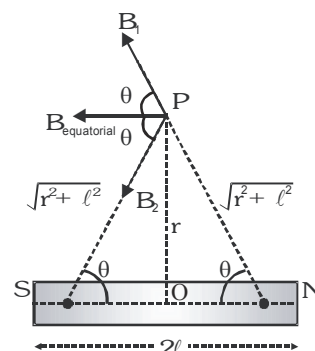
Net magnetic field at point 'P'

$$B_{\text{eq}} = 2B \cos\theta = 2 \frac{\mu_0}{4\pi} \frac{m}{(r^2 + l^2)} \cos\theta, \quad [\text{where } \cos\theta = \frac{l}{\sqrt{r^2 + l^2}}]$$

$$= 2 \frac{\mu_0}{4\pi} \frac{m}{(r^2 + l^2)} \frac{l}{\sqrt{r^2 + l^2}}$$

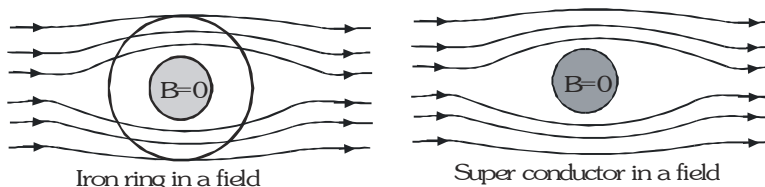
$$B_{\text{eq.}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}}, \quad \text{where } M = m(2l)$$

If magnet is short  $r \gg l$ , then  $B_{\text{eq.}} \sim \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$



## Magnetic shielding

If a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding and is used to protect costly wrist-watches and other instruments from external magnetic fields by enclosing them in a soft-iron case or box.



- (i) Super conductors also provides perfect magnetic screening due to exclusion of lines of force. This effect is called '**Meissner effect**'
- (ii) Relative magnetic permeability of super conductor is zero. So we can say that super conductors behaves like perfect diamagnetic.

## Magnetic map

This map represents basic three elements of earth magnetic field at all position of earth, and following conclusions are normally used for magnetic survey of earth's magnetic field.

### Lines joining

- ❖ Position of same  $\phi \rightarrow$  Isogonic lines,
- ❖ Positions of zero  $\phi \rightarrow$  Agonic lines
- ❖ Positions of same  $\theta \rightarrow$  Isoclinic lines ,
- ❖ Positions of zero  $\theta \rightarrow$  Aclinic lines, mag. equator
- ❖ Positions of same  $B_H \rightarrow$  Isodynamic lines
- ❖ Positions of zero  $B_H \rightarrow$  Adynamic line or magnetic axis of earth

## Dipole - Dipole Interactions :

S.No.	Relative position of dipoles	Magnetic force ( $F_m$ )
(a)		$\frac{\mu_0}{4\pi} \cdot \frac{6M_1M_2}{r^4} \text{ (along } r\text{)}$
(b)		$\frac{\mu_0}{4\pi} \cdot \frac{3M_1M_2}{r^4} \text{ (along } r\text{)}$
(c)		$\frac{\mu_0}{4\pi} \cdot \frac{3M_1M_2}{r^4} \text{ (perpendicular to } r\text{)}$

# MAGNETIC MATERIALS

## IMPORTANT DEFINITIONS AND RELATIONS

**Magnetising field or Magnetic intensity ( $\vec{H}$ )**

Field in which a material is placed for magnetisation, called as magnetising field.

$$\text{Magnetising field } \vec{H} = \frac{\vec{B}_0}{\mu_0} = \frac{\text{magnetic field}}{\text{permeability of free space}}$$

SI Unit  $\vec{H}$  : ampere/meter

**Intensity of magnetisation ( $\vec{I}$ )**

When a magnetic material is placed in magnetising field then induced dipole moment per unit volume of

that material is known as intensity of magnetisation  $\vec{I} = \frac{\vec{M}}{V}$

$$\text{SI Unit : ampere/meter} \quad \left[ \because \frac{\vec{M}}{V} = \frac{IA}{V} = \frac{\text{ampere} \times \text{meter}^2}{\text{meter}^3} \right]$$

**Magnetic susceptibility ( $\chi_m$ )**

$$\chi_m = \frac{I}{H} \quad [\text{It is a scalar with no units and dimensions}]$$

Physically it represent the ease with which a magnetic material can be magnetised

A material with more  $\chi_m$ , can be change into magnet easily.

**Magnetic permeability  $\mu$**

$$\mu = \frac{B_m}{H} = \frac{\text{total magnetic field inside the material}}{\text{magnetising field}}$$

It measures the degree to which a magnetic material can be penetrated (or permeated) by the magnetic field lines

$$\text{SI Unit of } \mu : \mu = \frac{B_m}{H} \equiv \frac{\text{Wb} / \text{m}^2}{\text{A} / \text{m}} \equiv \frac{\text{Wb}}{\text{A} - \text{m}} \equiv \frac{\text{H} - \text{A}}{\text{A} - \text{m}} = \frac{\text{H}}{\text{m}}$$

$$[\because \phi = LI \therefore \text{weber} \equiv \text{henry} - \text{ampere}]$$

**Relative permeability  $\mu_r = \frac{\mu}{\mu_0}$**

It has no units and dimensions.

**Relation between permeability and susceptibility**

When a magnetic material is placed in magnetic field  $\vec{B}_0$  for magnetisation then total magnetic field in material

$$\vec{B}_m = \vec{B}_0 + \vec{B}_i, \text{ where } \vec{B}_i = \text{induced field. } \therefore \vec{B}_0 = \mu_0 \vec{H}; \quad \vec{B}_i = \mu_0 \vec{I}$$

$$\therefore \vec{B}_m = \mu_0 \vec{H} + \mu_0 \vec{I} \Rightarrow \vec{B}_m = \mu_0 (\vec{H} + \vec{I}) = \mu_0 \vec{H} \left( 1 + \frac{I}{H} \right)$$

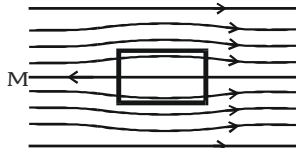
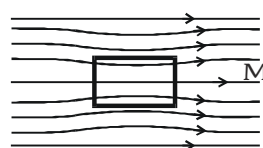
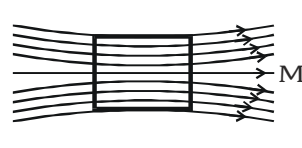
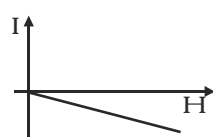
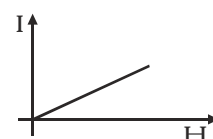


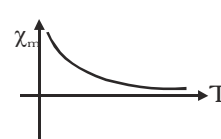
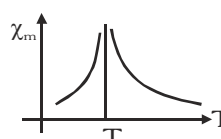
$$\Rightarrow \frac{B}{H} = \mu_0 \left( 1 + \frac{I}{H} \right) \Rightarrow \mu = \mu_0 (1 + \chi_m) \Rightarrow \mu_r = 1 + \chi_m$$



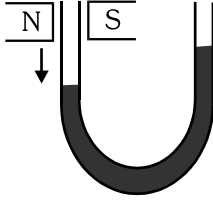


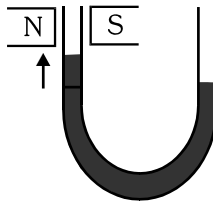


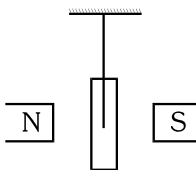
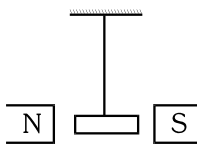
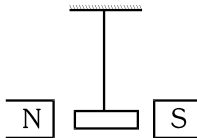
for vacuum  $\chi_m = 0$ , ( $\because \mu_r = 1$ )

at STP for air  $\chi_m = 0.04$  ( $\because$  at S.T.P. for air  $\mu_r = 1.04$ )

## CLASSIFICATION OF MAGNETIC MATERIALS

On the basis of magnetic properties of the materials [as magnetisation intensity ( $I$ ), Susceptibility ( $\chi$ ) and relative permeability ( $\mu_r$ )] Faraday divide these materials in three classes –

PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in uniform magnetic field.	Poor magnetisation in opposite direction. Here $B_m < B_0$ 	Poor magnetisation in same direction. Here $B_m > B_0$ 	Strong magnetisation in same direction. Here $B_m \gg B_0$ 
$I - H$ curve	$I \rightarrow$ Small, negative, varies linearly with field 	$I \rightarrow$ Small, positive, varies linearly with field 	$I \rightarrow$ very large, positive & varies non-linearly with field 
$\chi_m - T$ curve	$\chi_m \rightarrow$ small, negative & temperature independent $\chi_m \propto T$ 	$\chi_m \rightarrow$ small, positive & varies inversely with temp. $\chi_m \propto \frac{1}{T}$ (Curie law) 	$\chi_m \rightarrow$ very large, positive & temp. dependent $\chi_m \propto \frac{1}{T - T_c}$ (Curie Weiss law) (for $T > T_c$ ) ( $T_c$ = Curie temperature)  $T_{C(\text{Iron})} = 770 \text{ C or } 1043 \text{ K}$
$\mu_r$	$(\mu < \mu_0) \quad 1 > \mu_r > 0$	$2 > \mu_r > 1 \quad (\mu > \mu_0)$	$\mu_r \gg 1 \quad (\mu \gg \mu_0)$
Magnetic moment of single atom	Atoms do not have any permanent magnetic moment	Atoms have permanent magnetic moment which are randomly oriented. (i.e. in absence of external magnetic field the magnetic moment of whole material is zero)	Atoms have permanent magnetic moment which are organised in domains.

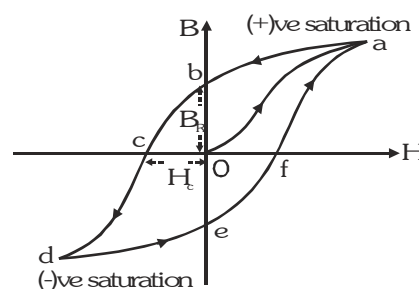
PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Behaviour of substance in Nonuniform magnetic field	<p>It moves from stronger to weaker magnetic field.</p>  <p>Weak Field</p>  <p>Strong field</p>  <p>Level depressed in that limb</p>	<p>It moves with weak force from weaker magnetic field to stronger magnetic field.</p>  <p>Weak Field</p>  <p>Strong field</p>  <p>Level slightly rises</p>	<p>Strongly attract from weaker magnetic field to stronger magnetic field.</p>  <p>Weak Field</p>  <p>Strong field</p>
When rod of material is suspended between poles of magnet.	<p>It becomes perpendicular to the direction of external magnetic field.</p> 	<p>If there is strong magnetic field in between the poles then rod becomes parallel to the magnetic field.</p> 	<p>Weak magnetic field between magnetic poles can make rod parallel to field direction.</p> 
Magnetic moment of substance in presence of external magnetic field	Value $\vec{M}$ is very less and opposite to $\vec{H}$ .	Value $\vec{M}$ is low but in direction of $\vec{H}$ .	$\vec{M}$ is very high and in direction of $\vec{H}$ .
Examples	Bi, Cu, Ag, Pb, H <sub>2</sub> O, Hg, H <sub>2</sub> , He, Ne, Au, Zn, Sb, NaCl, Diamond. (May be found in solid, liquid or gas).	Na, K, Mg, Mn, Sn, Pt, Al, O <sub>2</sub> (May be found in solid, liquid or gas.)	Fe, Co, Ni all their alloys, Fe <sub>3</sub> O <sub>4</sub> Gd, Alnico, etc. (Normally found only in solids) (crystalline solids)

### Magnetic susceptibility of some Elements at 300 K

Diamagnetic Substance			$\chi$	Paramagnetic Substance			$\chi$
Bismuth	–	1.66	$10^{-5}$	Aluminium	2.3	$10^{-5}$	
Copper	–	9.8	$10^{-6}$	Calcium	1.9	$10^{-5}$	
Diamond	–	2.2	$10^{-5}$	Chromium	2.7	$10^{-4}$	
Gold	–	3.6	$10^{-5}$	Lithium	2.1	$10^{-5}$	
Lead	–	1.7	$10^{-5}$	Magnesium	1.2	$10^{-5}$	
Mercury	–	2.9	$10^{-5}$	Niobium	2.6	$10^{-5}$	
Nitrogen (STP)	–	5.0	$10^{-9}$	Oxygen (STP)	2.1	$10^{-6}$	
Silver	–	2.6	$10^{-5}$	Platinum	2.9	$10^{-4}$	
Silicon	–	4.2	$10^{-6}$	Tungsten	6.8	$10^{-5}$	

## MAGNETIC HYSTERESIS

Only Ferromagnetic materials show magnetic hysteresis, when Ferromagnetic material is placed in external magnetic field for magnetisation then  $B$  increases with  $H$  non-linearly along  $Oa$ . If  $H$  is again bring to zero then it decreases along path  $ab$ . Due to lagging behind of  $B$  with  $H$  this curve is known as hysteresis curve.[Lagging of  $B$  behind  $H$  is called hysteresis]



**Cause of hysteresis :** By removing external magnetising field ( $H = 0$ ), the magnetic moment of some domains remains aligned in the applied direction of previous magnetising field which results into a residual magnetism.

- ⊙ **Residual magnetism (ob) =  $B_r$   $\equiv$  retentivity  $\equiv$  remanence**

Retentivity of a specimen is a measure of the magnetic field remaining in the ferromagnetic specimen when the magnetising field is removed.

- © **Coercivity (oc)** : Coercivity is an measure of magnetising field required to destroy the residual magnetism of the ferromagnetic specimen.

## Ferromagnetic materials

## Soft magnetic materials

Low retentivity, low coercivity and small hysteresis loss.

suitable for making electromagnets, cores of transformers etc. Ex. Soft iron, (used in magnetic shielding)

## Hard magnetic materials

High retentivity, high coercivity  
and large hysteresis loss  
suitable for permanent magnet  
Ex. Steel, Alnico





**Solution**

The volume of the cubic domain  $V = (10^{-6})^3 = 10^{-18} \text{ m}^3 = 10^{-12} \text{ cm}^3$

mass = volume density =  $7.9 \times 10^{-12} \text{ g}$

An Avogadro number ( $6.023 \times 10^{23}$ ) of iron atoms has a mass of 55g.

The number of atoms in the domain  $N = \frac{7.9 \times 10^{-12} \times 6.023 \times 10^{23}}{55} = 8.65 \times 10^{10} \text{ atoms}$

The maximum possible dipole moment  $M_{\text{max}}$  is achieved for the (unrealistic) case when all the atomic moments are perfectly aligned.

$$M_{\text{max}} = (8.65 \times 10^{10}) (9.27 \times 10^{-24}) = 8.0 \times 10^{-13} \text{ Am}^2$$

The consequent magnetisation is  $I_{\text{max}} = \frac{M_{\text{max}}}{\text{domain volume}} = \frac{8.0 \times 10^{-13}}{10^{-18}} = 8.0 \times 10^5 \text{ Am}^{-1}$

**Example**

Relation between permeability  $\mu$  and magnetising field  $H$  for a sample of iron is  $\mu = \left(\frac{0.4}{H} + 12 \times 10^{-4}\right)$  henry/meter. where unit of  $H$  is A/m. Find value of  $H$  for which magnetic induction of  $1.0 \text{ Wb/m}^2$  can be produce.

**Solution**

Magnetic induction for medium  $B = \mu H \Rightarrow 1 = \left(\frac{0.4}{H} + 12 \times 10^{-4}\right) H$

$$\Rightarrow 1 = 0.4 + 12 \times 10^{-4} H \Rightarrow H = \frac{1 - 0.4}{12 \times 10^{-4}} = 500 \text{ A/m}$$

**Example**

When a rod of magnetic material of size  $10 \text{ cm} \times 0.5 \text{ cm} \times 0.2 \text{ cm}$  is located in magnetising field of  $0.5 \times 10^4 \text{ A/m}$  then a magnetic moment of  $5 \text{ A-m}^2$  is induced in it. Find out magnetic induction in rod.

**Solution**

$$\begin{aligned} \text{Total magnetic induction } B &= \mu_0 (I + H) = \mu_0 \left( \frac{M}{V} + H \right) \quad \left( \because I = \frac{M}{V} \right) \\ &= 4\pi \times 10^{-7} \left( \frac{5}{10^{-6}} + 0.5 \times 10^4 \right) = 6.28 \text{ Wb/m}^2 \end{aligned}$$

**Example**

A rod of magnetic material of cross section  $0.25 \text{ cm}^2$  is located in  $4000 \text{ A/m}$  magnetising field. Magnetic flux passes through the rod is  $25 \times 10^{-6} \text{ Wb}$ . Find out for rod

(i) permeability (ii) magnetic susceptibility (iii) magnetisation

**Solution**

$$(i) \quad \text{Magnetic flux } \phi = BA \Rightarrow B = \phi/A = \frac{25 \times 10^{-6}}{0.25 \times 10^{-4}} = 1 \text{ wb/m}^2$$

$$B = \mu H \Rightarrow \mu = \frac{B}{H} = \frac{1}{4000} = 2.5 \times 10^{-4} \text{ wb/A-m}$$

$$(ii) \quad \chi_m = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \frac{2.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 199 - 1 = 198 \quad (\because \mu_r = \frac{\mu}{\mu_0})$$

$$(iii) \quad \text{Magnetization } I = \chi_m H = 198 \times 4000 = 7.92 \times 10^5 \text{ A/m}$$