UNIT # 01 (PART – II)

KINEMATICS

EXERCISE –I

1. $\tilde{v} = \dfrac{(4-1)\tilde{i}+(2+2)\tilde{j}+(3-3)\tilde{k}}{\sqrt{3^2+4^2+0^2}} = \dfrac{3\tilde{i}+4\tilde{j}}{5}$

$\vec{v} = |\vec{v}|\ \tilde{v} = 10\left(\dfrac{3\tilde{i}+4\tilde{j}}{5}\right) = 6\tilde{i}+8\tilde{j}$

2. Avg. velocity $= \dfrac{20\times3+4\times20+5\times20}{20+20+20} = 4$ m/s

3. $v_i = 2\tilde{i}$

$v_f = 4\cos60°\tilde{i} + 4\sin60°\tilde{j}$

$= \dfrac{4}{2}\tilde{i} + \dfrac{4\sqrt{3}}{2}\tilde{j}$

$= 2\tilde{i} + 2\sqrt{3}\tilde{j}$

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 2\tilde{i}+2\sqrt{3}\tilde{j}-2\tilde{i} = 2\sqrt{3}\tilde{j}$

$\langle\vec{a}\rangle = \dfrac{2\sqrt{3}\tilde{j}}{2} = \sqrt{3}\tilde{j}$ m/s²

4. For v = 0, x = 1,4 and $a = v\dfrac{dv}{dx}$

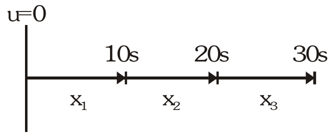
so $a|_{x=1} = 0 \quad \dfrac{dv}{dx} = 0$; $a|_{x=4} = 0 \quad \dfrac{dv}{dx} = 0$

5. $|\vec{v}| = \sqrt{v_x^2+v_y^2}$ here $v_x = \dfrac{dx}{dt} = 2ct$; $v_y = \dfrac{dy}{dt} = 2bt$

Therefore $|\vec{v}| = \sqrt{4t^2(c^2+b^2)} = 2t\sqrt{(c^2+b^2)}$

6. $\vec{v}_{(1)} = (3 + 4 \quad 1)\,\tilde{i} + (4 + (-3) \quad 1)\,\tilde{j} = 7\,\tilde{i} + \tilde{j}$

$|\vec{v}_{(1)}| = \sqrt{49+1} = 5\sqrt{2}$ m/s

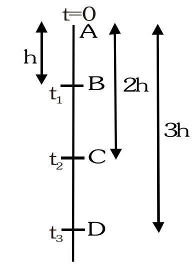
7. 

$x_1 = \dfrac{1}{2}a(10)^2$

$x_1 + x_2 = \dfrac{1}{2}a(20)^2$

$x_1 + x_2 + x_3 = \dfrac{1}{2}a(30)^2 \Rightarrow x_1 : x_2 : x_3 = 1 : 3 : 5$

8. $t_1 = \sqrt{\dfrac{2h}{g}}$

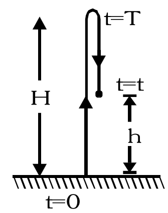


$t_2 = \sqrt{\dfrac{2\times2h}{g}}$

$t_3 = \sqrt{\dfrac{2\times3h}{g}}$

Required ratio $t_1 : (t_2 - t_1) : (t_3 - t_2)$

$= 1 : \left(\sqrt{2}-1\right) : \left(\sqrt{3}-\sqrt{2}\right)$

9. 

$h = H - \dfrac{1}{2}g(t-T)^2$

10. Velocity after 10 sec is equal to

0 + (10) (10) = 100 m/s

Distance covered in 10 sec is equal to

$\dfrac{1}{2}(10)(10)^2 = 50$m

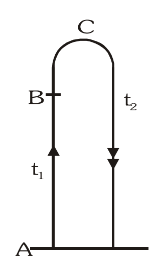
Now from $v^2 = u^2 + 2as$.

$\Rightarrow v^2 = (100)^2 - 2(2.5)(2495-400) = 25 \Rightarrow v = 5\text{ms}^{-1}$

11. It happens when in this time interval velocity becomes zero in vertical motion

$\Rightarrow \dfrac{u}{g} = 5 \Rightarrow u = 5 \quad 9.8 = 49$ m/s

12. $t_{AC} = \dfrac{t_1+t_2}{2}$; $t_{BC} = \dfrac{t_2-t_1}{2}$



$\therefore$ AB = AC – BC

$= \dfrac{1}{2}g\left(\dfrac{t_1+t_2}{2}\right)^2 - \dfrac{1}{2}g\left(\dfrac{t_2-t_1}{2}\right)^2$

$= \dfrac{1}{2}gt_1t_2$

13. Displacement = $\frac{1}{2}[4+2]\quad 4 - \frac{1}{2}[4+3]\quad 2$

$= 12 - 7 = 5$ m

Distance = 12 + 7 = 19 m

14. $S_B = S_A + 10.5$

$\frac{t^2}{2} = 10t + 10.5$

$t^2 = 20t + 21$

$t^2 - 20t - 21 = 0$

t = 21 sec

15. When the secant from P to that point becomes the tangent at that point

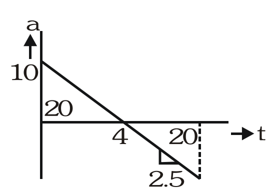
16. Two values of velocity (at the same instant) is not possible.

17. $a = \frac{d^2x}{dt^2}$ = change in velocity w.r.t. the time

For OA $\rightarrow$ velocity decreases so a is negative
For AB $\rightarrow$ velocity constant so a is zero.
For BC $\rightarrow$ velocity constant so a is zero.
For CD $\rightarrow$ velocity increases so a is positive.

18. Initially velocity increases downwards (negative) and after rebound it becomes positive and then speed is decreasing due to acceleration of gravity ($\downarrow$)

20. Upward area of a-tgraph gives the change in velocity = 20 m/s for acquiring initial velocity, it again changes by same amount in negative direction. Slope of curve = – 10/4 = – 2.5



$\therefore$ time $= \sqrt{\frac{2 \times 20}{2.5}}$ = 4sec

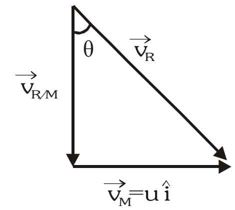
Total time = 4 + 4 =8 sec

21. Initially the speed decreases and then increases.

22. $\Rightarrow \vec{v}_{R/M} = \frac{u}{\tan\theta} - \hat{j}$

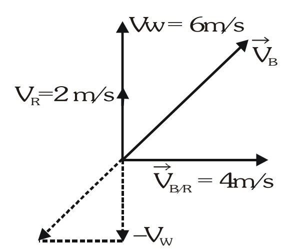
$\therefore \vec{v}_R = \vec{v}_{R/M} + \vec{v}_M$

$\Rightarrow \vec{v}_R = u\hat{i} - \frac{u}{\tan\theta}\hat{j}$



23. For shortest time to cross, velocity should be maximum towards north as river velocity does not take any part to cross.

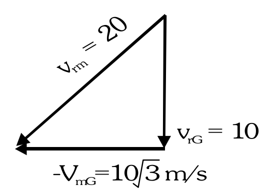
24. Flag blows in the direction of resultant of $\vec{V}_W$ & $-\vec{V}_B$



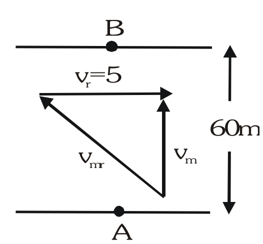
$\vec{V}_W - \vec{V}_B = 6\hat{j} - (4\hat{i} + 2\hat{j}) = 4(-\hat{i} + \hat{j})NW$

$\Rightarrow$ N-W direction.

25. $v_{mG} = \sqrt{(v_{rm})^2 - (v_{rG})^2} = \sqrt{(20)^2 + (10)^2} = 10\sqrt{3}$ m/s

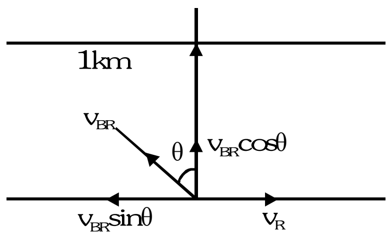


26. The resultant velocity should be in the direction of resultant displacement



So time $= \frac{60}{\sqrt{v_m^2 - 5^2}} = 5$ $\quad \because v_{rm}$ = 13 m/s

27.



s = ut

$1 = v_{BR}\cos\theta\ \ t$

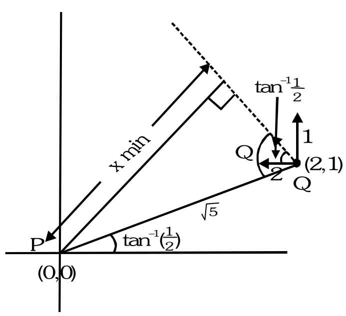
$1 = 5\ \cos\theta\ \ \frac{1}{4}$

$\cos\theta = \frac{4}{5} \Rightarrow \theta = 37$

$v_R = v_{BR}\sin 37 = 5\ \ \frac{3}{5} = 3$ km/hr

28. For shortest time then maximum velocity is in the direction of displacement.

29. $\vec{v}_{QP} = -\tilde{i} + 2\tilde{j} - \tilde{i} - \tilde{j} = -2\tilde{i} + \tilde{j}$



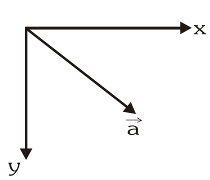
So from sine rule $\frac{\sqrt{5}}{\sin 90^\circ} = \frac{x_{min}}{\sin\theta} \Rightarrow x_m$

$= \sqrt{5} \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sqrt{5} \times 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} == \frac{4}{\sqrt{5}}$

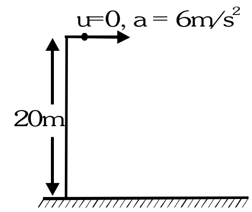
30. Time of collision of two boat = 20/2 = 10 sec. As given in question i.e. the time of flight of stone is also equal to 10 sec. so vertical component of stone initially is 50 m/s and the horizontal component w.r.t. motorboat equals to 2 m/s.

Hence $\vec{v}_{BG} = 3\tilde{i} + 50\tilde{j}$

31. $\vec{a}_x = a_1\hat{i}\ ;\ \vec{a}_y = -a_2\hat{j}$

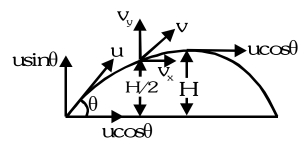


32. Time to reach the ground $= \sqrt{\frac{2 \times 20}{10}} = 2$ sec



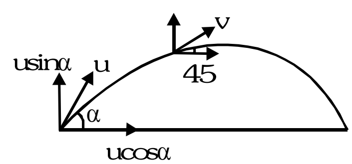
So horizontal displacement = $0 + \frac{1}{2}\ \ 6\ \ \ 4 = 12$m

33. $v_y^2 = u^2\sin^2\theta - 2g\ \ \frac{H}{2}$; $v_x^2 = u^2\cos^2\theta$



$\therefore\ u\cos\theta = \sqrt{\frac{6}{7}}\left[\sqrt{v_x^2 + v_y^2}\right] \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = 30^\circ$

34. $\vec{v} = u\cos\alpha\,\tilde{i} + (u\sin\alpha - gt)\,\tilde{j}\ \ \because\ \vec{v} = \vec{v}_x = \vec{v}_y$



ucosα = usinα - gt $\Rightarrow t = \frac{u}{g}$ (sinα – cos α)

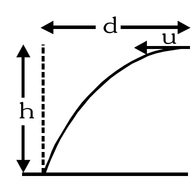
35. $\vec{v} = a\tilde{i} + (b - ct)\tilde{j}$

Time to reach maximum height (when $\tilde{j}$ comp. of velocity becomes zero)

$\therefore\ b - ct = 0 \Rightarrow t = \frac{b}{c}$ $\therefore$ Time of flight $= \frac{2b}{c}$

range = horizontal velocity Time of flight = a $\frac{2b}{c}$

36. Time to reach at ground $= \sqrt{\frac{2h}{g}}$



In this time horizontal displacement

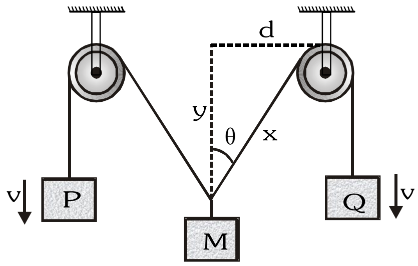
$d = u\ \ \sqrt{\frac{2h}{g}} \Rightarrow d^2 = \frac{u^2 \times 2h}{g}$

37. $-1500 = \frac{-500}{3}\sin 37 \quad t - \frac{1}{2} \quad 10 \quad t^2$; t = ?

Distance $= \frac{500}{3}\cos 37 \quad t$ (Horizontal)

$\Rightarrow x = \frac{4000}{3}$ m

38.



Here $x^2 = y^2 + d^2$.

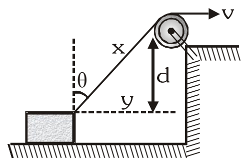
So $2x\frac{dx}{dt} = 2y\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \left(\frac{x}{y}\right)\left(\frac{dx}{dt}\right) = \left(\frac{x}{y}\right)(v) = \frac{v}{\cos\theta}$

OR

Component of velocity along string must be same

so $v_M \cos\theta = v \Rightarrow v_M = \frac{v}{\cos\theta}$

39. $x^2 = y^2 + d^2$



$\Rightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

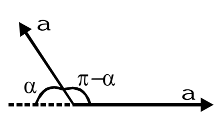
$\Rightarrow \frac{dy}{dt} = \left(\frac{x}{y}\right)\left(\frac{dx}{dt}\right) = \frac{v}{\sin\theta}$

OR

Component of velocity along string must same so

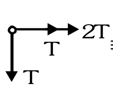
$v_M \cos\theta(90-\theta) = v \Rightarrow v_M = \frac{v}{\sin\theta}$

40. Net acceleration of load

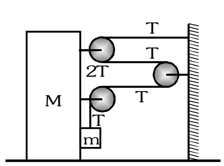


$= 2a\cos\left(\frac{\pi-\alpha}{2}\right) = 2a\sin\left(\frac{\alpha}{2}\right)$

41. Net tension on M



$\equiv \sqrt{(3T)^2 + T^2} = \sqrt{10}T$

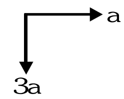


Now from acceleration Tension = constant

$\Rightarrow a_M(\sqrt{10}T) = a_m(T) \Rightarrow a_m = (\sqrt{10})a_M = \sqrt{10}a$

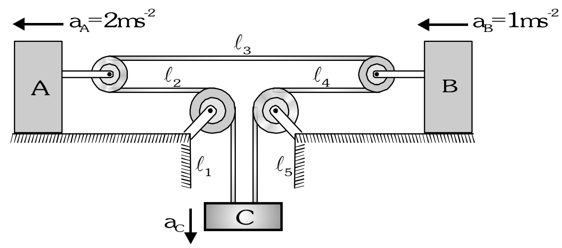
OR

Net acceleration of m



$\equiv \sqrt{a^2 + (3a)^2} = \sqrt{10}\,a$

42.



$\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5$ = constant

$\Rightarrow \quad \ddot{\ell}_1 + \ddot{\ell}_2 + \ddot{\ell}_3 + \ddot{\ell}_4 + \ddot{\ell}_5 = 0$

$\Rightarrow \quad a_C + a_A + (a_A - a_B) + (-a_B) + a_C = 0$

$\Rightarrow \quad 2a_C + 2a_A - 2a_B = 0$

$\Rightarrow \quad a_C = a_B - a_A = 1 - 2 = -1\ ms^{-2}$

$\Rightarrow$ Acceleration of C is 1 ms^{-2} upwards

43. Given $\omega = \theta^2 + 2\theta$

$\frac{d\omega}{d\theta} = 2\theta + 2$

$\alpha = \omega\frac{d\omega}{d\theta} = (\theta^2 + 2\theta)(2\theta + 2)$

at $\theta = 1$

α = 12 rad/sec^2

44. Centripetal acceleration $= \frac{v^2}{R}$

$\frac{v_1^2}{R_1} = \frac{v_2^2}{R_2} = \frac{v_1}{v_2} = \sqrt{\frac{R_1}{R_2}} = \sqrt{\frac{1}{2}}$

45. $\omega = \frac{14 \times 2\pi}{25}$

$\therefore$ magnitude of acceleration

$= \omega^2 r = \left(\frac{14 \times 2\pi}{25}\right)^2 \quad \frac{80}{100} \simeq 9.9\ m/s^2$

46. Given $r = \frac{20}{\pi}$ m

Angular velocity after second revolution

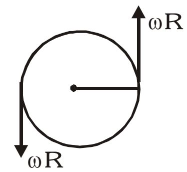
$\omega = \frac{v}{r} = \frac{50\pi}{20} = \frac{5\pi}{2}$

$\omega_{final}^2 = \omega_{initial}^2 + 2\alpha\theta$

$$\frac{25}{4}\pi^2 = 2\alpha(4\pi) \Rightarrow \alpha = \frac{25\pi}{32}$$

$$a_t = \alpha r = \frac{25\pi}{32} \times \frac{20}{\pi} = 15.6$$

47. ω = constant , $a_T = 0$



$$\left[\frac{2\omega^2 rx}{\pi}, \omega = \frac{2\pi}{T}, \frac{T}{2} = \frac{\pi}{\omega}\right]$$

$$a_{av} = \frac{2\omega R}{\pi/\omega} = \frac{2\omega^2 R}{\pi};\ a_{inst} = \omega^2 R$$

So ratio $= \frac{a_{av}}{a_{inst}} = \frac{2}{\pi}$

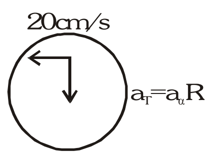
48. ℓ = 6cm, v = ?, $\omega = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s.

So $v = \omega\ell = \frac{\pi}{30} \times 6 = \frac{\pi}{5}$ cm/s = 2π mm/s

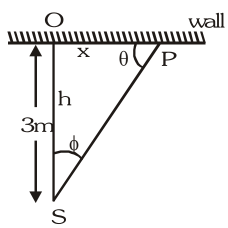
Difference = $\sqrt{2}\frac{\pi}{5}$ cm/s = $2\sqrt{2}\ \pi$ mm/s

49. ω and α remain same but v and a_T is proportional to r thus at half the radius,

$v' = \frac{v}{2}$ & $a_T' = \frac{a_T}{2}$



50. Let x is the distance of point P from O, the, from figure



$\tan\phi = \frac{x}{h}$ or $x = h\tan\phi$

$\Rightarrow \frac{dx}{dt} = h\sec^2\phi\ \frac{d\theta}{dt}$

$\left[\frac{d\phi}{dt} = \omega\right] \Rightarrow v = h\sec^2\phi\ \omega$

So putting values

h=3, ϕ = 180 – (90 + 45) = 45

we get $v = (3\sqrt{2})^2 \times 0.1 = 0.6$ m/s

51. Angular velocity ω about centre = 2ω

= 2 0.40 = 0.30 rad/sec

v = ωR

= 0.80 $\frac{1}{2}$ = 0.40 m/s

$$a = \frac{v^2}{R} = \frac{0.40 \times 0.40 \times 100}{50} = 0.32\ \text{cm/s}^2$$

EXERCISE –II

1. $u_x = u_0$; $u_y = a\omega\cos\omega t$

$x = u_0 t$; $\int_0^y dy = a\omega \int_{t=0}^{t} \cos\omega t\, dt$

$y = a\frac{w}{w}\ \sin\omega t = a\sin\left(\frac{\omega x}{u_0}\right)$

2. $\alpha = -av^2 = \int_u^v \frac{dv}{v^2} = -a\int_{t=0}^{t} dt$

$\Rightarrow -\left[\frac{1}{v}\right]_u^v = -at \Rightarrow \frac{1}{u} - \frac{1}{v} = -\ at$

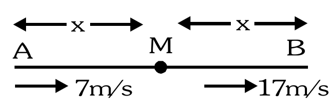
$\Rightarrow v = \frac{u}{1+aut}\quad \int_0^x dx = \int_{t=0}^{t} \frac{udt}{1+aut}$

$\Rightarrow x = \frac{u}{au}\left[\ell n(1+aut)\right]_0^t = \frac{1}{a}\ \ell n\ (1 + aut)$

3. $t = \alpha x^2 + \beta x \Rightarrow 1 = (2\alpha\ x + \beta)v \Rightarrow v = \frac{1}{\beta + 2\alpha x}$

$\therefore$ Acceleration $= \frac{2\alpha}{(\beta+2\alpha x)^2} v = 2\alpha v^3$

4.

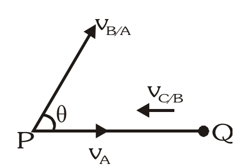


$v_m^{\ 2} = (7)^2 + 2\quad a\quad x$; v_m = 13 m/s

$(17)^2 = (7)^2 + 2\quad a\quad 2x$

$\frac{13-7}{a} = t_1$; $t_2 = \frac{17-13}{a}$; $\frac{t_1}{t_2} = \frac{6}{4} = \frac{3}{2}$

5. $|\vec{v}_A|$ = 10 m/s



$\vec{v}_C = \vec{v}_{C/B} + \vec{v}_{B/A} + \vec{v}_A$

$$= 12(-\hat{i}) + 6 \times \frac{15}{24}(\hat{i}) + \left(6 \times \frac{\sqrt{351}}{24}\hat{j}\right) + 10\hat{i}$$

$$= \left(\frac{15}{4} - 2\right)\hat{i} + \frac{\sqrt{351}}{4}\hat{j}$$

$$|\vec{v}_C| = \frac{\sqrt{7^2 + \left(\sqrt{351}\right)^2}}{4} = 3\,m/s$$

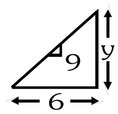
6. Time of fall of stone = $\sqrt{\frac{2\times 20}{10}}$ = 2 sec

Horizontal displacement of truck in 2 sec

$\Rightarrow S = 2 \quad 2 + \frac{1}{2} \quad 1 \quad 4$.

Length of truck = 6m

7. As given 9 =y/6 ⇒ $\boxed{y = 54m}$



Average velocity of particle

$B = \frac{\text{Displacement}}{\text{time}} = \frac{54}{6} = 9\text{m/s}$

8. Distance covered by :

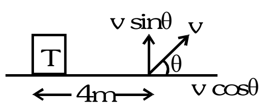
train I = (Area of $\Delta)_{\text{train I}}$ = 200 m

train II = (Area of $\Delta)_{\text{train II}}$ = 80 m

So the seperation = 300 – (200 + 80) = 20 m.

9. $\vec{r} = (t^2 - 4t + 6)\tilde{i} + t^2\tilde{j}$; $\vec{v} = (2t - 4)\tilde{i} + 2t\tilde{j}$

$\vec{a} = 2(\tilde{i} + \tilde{j})$; when $\vec{a} \perp \vec{v}$ then $\vec{a} \quad \vec{v} = 0$; t = 1s



10. Time to cross 2m is $\left(\frac{2}{v\sin\theta}\right)$ .....

To avoid an accident

Displacement = 4 + v cos θ $\quad \frac{2}{v\sin\theta}$

$8 \quad \frac{2}{v\sin\theta} = 4 + 2\cot\theta$

$v\sin\theta = \frac{16\sin\theta}{4\sin\theta + 2\cos\theta}$

$v_{min} = \frac{16}{\sqrt{4^2+2^2}} = 1.6\sqrt{5}$ m/s

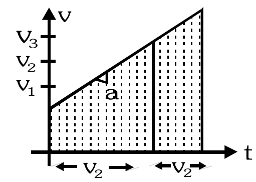
[∵ (a cos θ + b sin θ) has max. value = $\sqrt{a^2+b^2}$]

11. $\vec{v} = 4t\tilde{i} + 3t\tilde{j}$ (∵ x = at² & y = 3/2t²)

$v(1) = 4\tilde{i} + 3\tilde{j}$; $v(2) = 8\tilde{i} + 6\tilde{j}$

$\therefore \langle v \rangle = \frac{12\tilde{i} + 9\tilde{j}}{2} = (6\tilde{i} + 4.5\tilde{j})$ m/s

12. When acceleration is constant the instantaneous velocity is equal to the average velocity in mid of the time interval.



$a = \frac{v_2 - v_1}{\frac{t_1}{2} + \frac{t_2}{2}} = \frac{v_3 - v_2}{\frac{t_2}{2} + \frac{t_3}{2}}$.

13. $\langle v_{space} \rangle = \frac{\int v\,ds}{\int ds} = \frac{\int \sqrt{2as}\,ds}{\int ds} = \frac{2}{3}v$

$\langle v_{time} \rangle = \frac{\int v\,dt}{\int dt} = \frac{\int at\,dt}{\int dt} = \frac{v}{2} \quad \therefore \frac{\langle v_s \rangle}{\langle v_t \rangle} = 4 : 3$

14. $x = 40 + 12t - t^3$.

Speed $\frac{dx}{dt} = 0 + 12 - 3t^2 \Rightarrow t = \pm 2\text{sec}$

$\therefore x(2) = 40 + 12 \quad 2 - 2^3$

$= 64 - 8 = 56$ m.

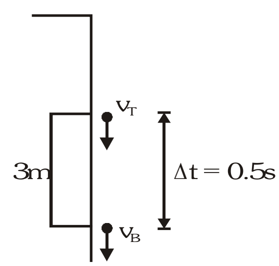
at t = 0, x(0) = 40

Δx = x(2) – x(0) = 16

15. $v_B = v_T + 9.8 \quad 0.5 = v_T + 4.9$

$v_B - v_T = 4.9$ m/s and

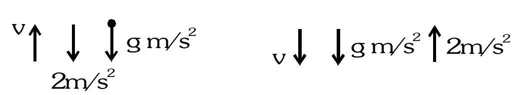
$v_B^2 - v_T^2 = 2gs = 2 \quad 9.8 \quad 3 = 58.8$



$\Rightarrow (v_B + v_T) \quad (v_B - v_T) = 2 \quad 9.8 \quad 3$

$\Rightarrow v_B + v_T = 12$ m/s

16.



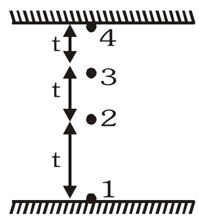
$\therefore$ time of ascent = $\sqrt{\frac{2h}{g+2}}$

time of descent = $\sqrt{\frac{2h}{g-2}}$

$\therefore \frac{t_a}{t_d} = \sqrt{\frac{8}{12}} = \sqrt{\frac{2}{3}}$

17. Time taken to reach the drop to ground

$9 = 0 + \frac{1}{2} \quad 10 \quad (3t)^2$



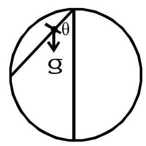
$\sqrt{\frac{9}{5}} = 3t$

$\boxed{\frac{\sqrt{1.8}}{3} = t}$

$x_2 = \frac{1}{2} \quad 10 \quad (2t)^2 = 20t^2 = 20 \times \frac{1.8}{9} = 4m$

$x_3 = \frac{1}{2} \quad 10 \quad (t)^2 = 5t^2 = 5 \times \frac{1.8}{9} = 1m$

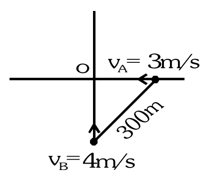
18. Time to fall = $\sqrt{\frac{2 \times 2R\cos\theta}{g\cos\theta}}$



so it does not depend on θ i.e. the chord position.

19. $300^2 = (3t)^2 + (4t)^2$
$300 \quad 300 = 25t^2$
$t = 60$

Ratio = $\frac{3 \times 2\sqrt{3}}{4 \times 2\sqrt{3}} = 3 : 4$



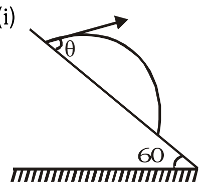
20. For man on trolley $\frac{3}{2}vt = L \Rightarrow t = \frac{2L}{3v}$

with respect to ground : $vt + \frac{3}{2}vt = L + \frac{2L}{3} = \frac{5L}{3}$

$\therefore \frac{3}{2}vt - vt = L - \frac{2L}{3} = \frac{L}{3} \therefore \Delta S = \frac{5L}{3} - \frac{L}{3} = \frac{4L}{3}$

21. Time of flight $\boxed{4 = \frac{2u\sin\theta}{g\cos 60^\circ}}$ ...(i)

(angle of projection = θ)



Distance travelled by Q on incline in 4 secs is

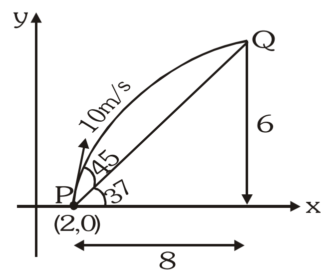
$= 0 + \frac{1}{2} \quad \frac{\sqrt{3}g}{2} \quad 4^2 = 40\sqrt{3}$

& the range of particle 'P' is $40\sqrt{3}$

$= u\cos\theta \quad 4 + \frac{1}{2} \frac{\sqrt{3}g}{2} \quad 4^2 = 40\sqrt{3}$

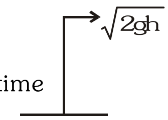
$= u\cos\theta = 0$; so $\theta = 90$
from equation (i) $u = 10$ m/s

22. $PQ = R = \frac{u^2\sin 90^\circ}{g} = \frac{100 \times 1}{10} = 10 \Rightarrow PQ = 10$



(x,y) = (10, 6, 0)

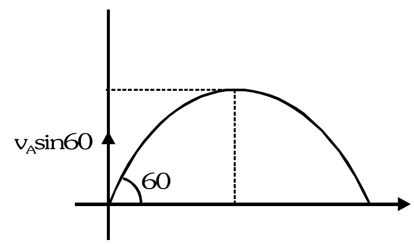
23. Time to fall = $\sqrt{\frac{2 \times h}{g}}$



Range = Horizontal velocity time

$x = \sqrt{2gh} \quad \sqrt{\frac{2h}{g}} = 2h$

24. At maximum height vertical component of velocity becomes zero.
$v^2 = u^2 + 2as$



For A : $0 = v_A^2\sin^2 60 - 2gh$
$2gh = v_A^2\sin^2 60 = v_A^2\ (3/4)$

$v_A = \sqrt{\frac{8gh}{3}}$

For B : $0 = v_B^2 - 2gh$

$v_B = \sqrt{2gh}$; $\frac{v_A}{v_B} = \frac{2}{\sqrt{3}}$

25. $x = 10\sqrt{3}t$; $y = 10t - t^2$; $\frac{dx}{dt} = 10\sqrt{3}$

$v_y = \frac{dy}{dt} = 10 - 2t \Rightarrow$ at t = 5 sec.
v_y becomes zero at maximum height
$\Rightarrow y = 10 \quad 5 - 5^2 = 25m.$

26. $\vec{r} = t^2\tilde{i} + (t^3 - 2t)\tilde{j}$;

$\vec{v} = \frac{d\vec{r}}{dt} = 2t\tilde{i} + (3t^2 - 2)\tilde{j}$

$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 2\tilde{i} + 6t\tilde{j}$

$\vec{a}.\vec{v} = 4t + 18t^3 - 12t = 0$ (For $\perp$)

$\therefore\ t = \pm 2/3,\ 0.$

For parallel to x-axis $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{3t^2 - 2}{2}$

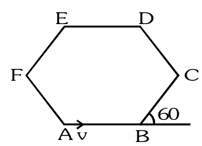
$\therefore$ at $t = \sqrt{\frac{2}{3}}$ sec it becomes zero so (c)

$\vec{a}_{(4,4)} = 2\tilde{i} + 6 \times 2\tilde{j} = 2\tilde{i} + 12\tilde{j}$

27. Area of the curve gives distance.

28. Acceleration = Rate of change of velocity i.e. velocity can be changed by changing its direction, speed or both.

29. Av. velocity $= \frac{\text{Displacment}}{\text{time}}$



30. $x = t^3 - 3t^2 - 9t + 5$. $x(5) > 0$ and $x(3) > 0$

so [A] $v = dx/dt = 3t^2 - 6t - 9$

$\Rightarrow t = -1, 3$ so $t = 3$

Hence particle reverses its direction only once average acc. = change in velocity /time.

In interval (t = 3 to t = 6), particle does not reverse its velocity and also moves in a straight line so distance = displacement.

31. Motion A to C $\Rightarrow 17^2 = 7^2 + 2as$

Motion A to B $\Rightarrow v_B{}^2 = 7^2 + 2a\left(\frac{s}{2}\right) = \frac{17^2 + 7^2}{2}$

(A) $v_B = \sqrt{\frac{289 + 49}{2}} = 13$ m/s

(B) $\langle v_{AB} \rangle = \frac{7 + 13}{2} = 10$ m/s

(C) $t_1 = \frac{13 - 7}{a}$, $t_2 = \frac{17 - 13}{a}$, $\frac{t_1}{t_2} = \frac{6}{4} = \frac{3}{2}$

(D) $\langle v_{BC} \rangle = \frac{13 + 17}{2} = 15$ m/s

32. $x = u(t - 2) + a(t - 2)^2$ ...(i)

$v = \frac{dx}{dt} = u + 2a(t - 2)$

Therefore $v(0) = u - 4a$

$a = \frac{d^2x}{dt^2} = 2a.$ Hence [C]

$x(2) = 0$ [From (i)]. Hence [D]

33. $x = 2 + 2t + 4t^2$, $y = 4t + 8t^2$

$v_x = \frac{dx}{dt} = 2 + 8t$, $v_y = \frac{dy}{dt} = 4 + 16t$

$a_x = 8$; $a_y = 16$; $\vec{a} = 8\tilde{i} + 16\tilde{j}$ = constant

$y = 2(2t + 4t^2)$; $y = 2(x - 2)$ ($\because x = 2 + 2t + 4t^2$) which is the equation of straight line.

34. $\vec{v}(t) = (3 - 1 \times t)\tilde{i} + (0 - 0.5t)\tilde{j}$ ...(i)

For maximum positive x coordinate when v_x becomes zero

$\therefore\ 3 - t = 0 \Rightarrow t = 3$ sec

then $\vec{r}(3) = 4.5\tilde{i} - 2.25\tilde{j}$.

35. [A] $\because$ Distance $\geq$ Displacement

$\therefore$ Average speed $\geq$ Average velocity

[B] $|\vec{a}| \pm 0 \Rightarrow \Delta\ \vec{v} \pm 0$

velocity can change by changing its direction

[C] Average velocity depends on displacement in time interval e.g. circular motion $\rightarrow$ after one revolution displacement become zero hence average velocity but instantaneous velocity never becomes zero during motion.

[D] In a straight line motion ; there must be reversal of the direction of velocity to reach the destination point for making displacement zero and hence instantaneous velocity has to be zero at least once in a time interval.

36. $\vec{v} = |\vec{v}|\tilde{v}$; $[|\vec{v}| \rightarrow$ speed]

Velocity may change by changing either speed or direction and by both.

37. $v = \sqrt{x}$; $\int_4^x \frac{dx}{\sqrt{x}} = \int_{t=0}^{t} dt \Rightarrow [2\sqrt{x}]_4^x = t$

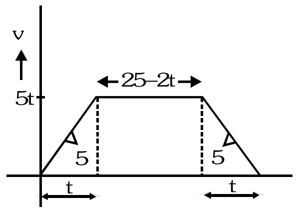
$\Rightarrow x = \left(\frac{t + 4}{2}\right)^2$ at $t = 2 \Rightarrow x = 9$m

$a = v\frac{dv}{dx} = \sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2}$ m/s^2

at $x = 4 \Rightarrow v = 2$m/s & it increases as x increases so it never becomes negative.

38. Average velocity

$= \frac{\text{Displacement}}{\text{time interval}} = \frac{\text{Area under v} - \text{t curve}}{\text{time}}$



$$20=\frac{\frac{1}{2}[25+25-2t]\times 5t}{25} \Rightarrow t = 5,\ 20$$

39. For returning, the starting point
Area of (ΔOAB) = Area of (ΔBCD)

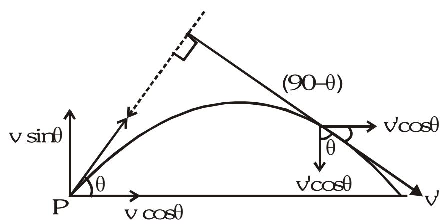
$\frac{1}{2}$ 20 25 $=\frac{1}{2}$ t 4t $\Rightarrow$ t = $5\sqrt{5} \simeq 11.2$

$\therefore$ Required time = 25 + 11.2 = 36.2

40. As air drag reduces the vertical component of velocity so time to reach maximum height will decrease and it will decrease the downward vertical velocity hence time to fall on earth increases.

41. $\because$ Horizontal component of velocity remains constant

$\therefore$ v'sin θ = v cos θ (from figure) $\therefore$ $\boxed{v' = v\cot\theta}$



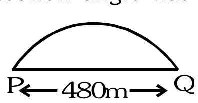
So from $v_y = u_y + a_y t \rightarrow -$ v' cosθ

$= \sin\theta - gt - v\frac{\cos^2\theta}{\sin\theta}$ = v sinθ – gt $\therefore$ $\boxed{t = \frac{v}{g}\operatorname{cosec}\theta}$

43. As given horizontal velocity = 40m/s
u cos θ t = 40; t = 1 sec
At t = 1, height = 50 m
$\therefore$ 50 = u sin θ 1 – 1/2 g 1 $\Rightarrow$ u sin θ = 55
$\therefore$ Initial vertical component = u sinθ = 55 m/s
As hoop is on same height of the trajectory.
So by symmetry *x* will be 40 m.

44. Range = $\frac{u^2 \sin 2\theta}{g} \Rightarrow 480 = \frac{4900}{980}\times \sin 2\theta$

(90 – θ) projection angle has same range.



Time of flight :

$T_1=\frac{2u\sin\theta}{g}$; $T_2=\frac{2u\sin(90-\theta)}{g}$

45. Range = $\frac{u^2\sin 2\theta}{g}$

For θ & (90 – θ) angles, range will be same so for 30 & (90 – 30)$\equiv$60 , projections both strike at the same point. For time of flight, vertical components are responsible

$$\frac{h_1}{h_2} = \frac{u^2\sin^2\theta_1}{u^2\sin^2\theta_2} = \frac{\sin^2 30}{\sin^2 60} = \frac{1}{3}$$

46. $y = x^2$, $y_{x=\frac{1}{2}} = \frac{1}{4}$; $\frac{dy}{dt} = 2x\ \frac{dx}{dt} = 2x\ v_x$

$v_y = 2\ \frac{1}{2}\ 4$ (at $x = \frac{1}{2}$, $v_x = 4$)

v_y = 4m/s ; $\vec{v}_{x=\frac{1}{2}} = 4\tilde{i}+4\tilde{j}$; $|\vec{v}| = 4\sqrt{2}$

Slope of line 4x – 4y – 1 = 0 is tan 45 = 1 and also the slope of velocity is 1.

47. After t = 1 sec, the speed increases with
a = g sin 37 = 6 m/s^2
$\therefore$ v_Y = g sin37 1 = 6 m/s
$\therefore$ speed = $\sqrt{8^2+6^2}$ = 10m/s

48. New horizontal range

$= R + \frac{1}{2}\ \frac{g}{2}\ T^2 = R + \frac{g}{4}\ \frac{4u^2\sin^2\theta}{g^2}$

= R + 2H ($\because$ $H=\frac{u^2\sin^2\theta}{2g}$)

49. $h_{max} = \frac{u^2}{2g} \Rightarrow$ u = 12 10 5 = 10 m/s

$t_H = \sqrt{\frac{2\times 5}{10}}$ = 1s so no. of balls in one min.

= 1 60 = 60

50. a = – kv + c [k > 0, c > 0]

$\int\frac{dv}{-kv+c} = \int dt \Rightarrow -\frac{1}{k}\ell n$ (– kv + c) = t

$\Rightarrow$ kv = c – e^{-kt}

51. Let acceleration of B $\vec{a}_B = a_B\ \tilde{i}$
Then acceleration of A w.r.t.

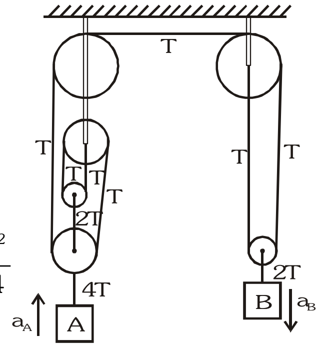
B = $\vec{a}_A - \vec{a}_B = (15-a_B)\tilde{i}+15\tilde{j}$

This acceleration must be along the inclined plane

so tan 37 = $\frac{15}{15-a_B} \Rightarrow \frac{3}{4} = \frac{15}{15-a_B}$ a_B = –5

$\Rightarrow \vec{a}_B = -5\tilde{i}$

52. (4T) a_A = (2T) (a_B)



$\Rightarrow a_A = \frac{a_B}{2}$

but $a_B = \frac{dv_B}{dt}$

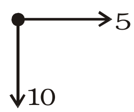
$= t + \frac{t^2}{2} \Rightarrow a_A = \frac{t}{2} + \frac{t^2}{4}$

At t = 2s,

$a_A = \frac{2}{2} + \frac{(2)^2}{4}$ 1+1=2 ms^{-2}

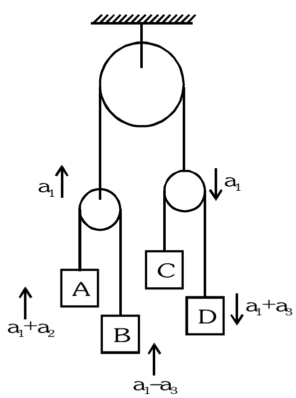
53. For B :

Net acceleration



$= \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}$ ms^{-2}

54. $a_1 + a_2 = 1$



$a_1 - a_2 = 7$

$a_3 - a_1 = 2$

$\Rightarrow a_1 = 4,$

$a_2 = -3,$

$a_3 = 6$

Acceleration of D = $a_1 + a_3$

= 4 + 6 = 10 ms^{-2} downwords

55. Block B will again comes to rest if

$v_A = v_c$ i.e. 3t = (12t)t $\Rightarrow$ t = ½ s

56. Given $\frac{dv}{dt} = \frac{v^2}{r} \Rightarrow \frac{dv}{ds} = \frac{v^2}{r}$; $-\int_{v_0}^{v} \frac{1}{v} dv = \int_{o}^{s} \frac{ds}{r}$

$\Rightarrow \ell n\left[\frac{v_0}{v}\right] = \frac{S}{r} \Rightarrow \frac{v_0}{v} = e^{S/r}$

$\Rightarrow v_0 = ve^{S/r} \Rightarrow v = v_0 e^{-S/r}$

57. $\tan\alpha = \frac{v^2}{R} \times \frac{1}{dv/dt} = \frac{a^2 s}{Rav/2\sqrt{s}} = \frac{2s}{R}$

EXERCISE –III

TRUE/FALSE

1. ∵ Acceleration depends on change in velocity not on the velocity.

2. Velocity and displacement are in same direction.

3. $S_{3^{rd}} = S_3 - S_2 = \frac{1}{2}$ g $(3)^2 - \frac{1}{2}$ g$(2)^2$=25m

4. Initially packet acquires balloon velocity which is in upwards direction so it moves upwards for some time & then in downward.

5. Because all bodies having same acceleration g in dow2 nwards direction.

6. At highest point, vertical velocity becomes zero and total velocity due to horizontal component of velocity & acceleration due to gravity which acts always vertically downwards.

7. Greatest height

$H = \frac{u^2}{2g}$ and.....horizontal displacement = $\frac{u^2}{g}$

$\therefore$ $\boxed{R = 2H}$

8. Instantaneous velocity is tangential to the trajectory.

9. Trajectory of particle depends on the instantaneous velocity not on acceleration.

10. $a_t = \frac{dv}{dt}$ v = speed of particle $a_N = \frac{v^2}{R}$ where always acts towards the centre or ⊥ to the instantaneous velocity.

11. No, because all masses having same acceleration g is in downward direction.

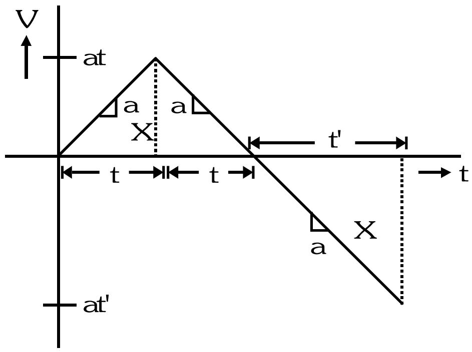
12. Firstly gravity decreases the speed when particle moves upwards and then again increases by same amount in downward direction.

13. When the vertical velocity component becomes zero, then the particle is at the top i.e. it has only horizontal component at that time which never changes so it is min. at the top.

FILL IN THE BLANKS

1. X= $\frac{1}{2}$ 2t at = $\frac{1}{2}$ t' at' ⇒ t' = $\sqrt{2}$ t

∴ Total time = 2t+ $\sqrt{2}$ t = (2 + $\sqrt{2}$)t



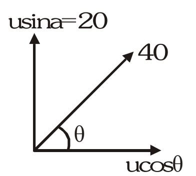
2. y = $\sqrt{3}$ x – $\frac{gx^2}{2}$

Trajectory equation is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$u^2\cos^2\theta = 1 \Rightarrow u^2\cos^2 60 = 1 \Rightarrow u = 2$ m/s

3. usinθ 1 – $\frac{1}{2}$ g 1^2 = usinθ 3 – $\frac{1}{2}$ g 3^2

2u sinθ = 40 ⇒ usinθ = 20



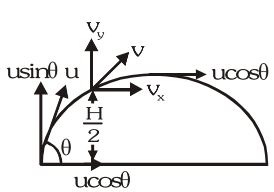
Time of flight = $\frac{2 \times u\sin\theta}{g}$ = 4 sec

40 sinθ = 20 ⇒ sinθ = $\frac{1}{2}$ ⇒ θ = 30

∴ h = 20 1 – $\frac{1}{2}$ g 1^2 = 15m

4. Due to gravity, it acquires vertical velocity and due to horizontal force it acquires horizontal component of force and when a velocity having both components then the path of the particle becomes parabolic.

5.



$v_y^2 = (u\sin\theta)^2 - 2g\frac{H}{2}$ and $v_x^2 = u^2\cos^2\theta$

$$\Rightarrow u^2\cos^2\theta = \frac{2}{5}\left(u^2 - gH\right) \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

As given ucosθ = $\sqrt{\frac{2}{5}} \times \left[\sqrt{u^2\sin^2\theta - gH + u^2\cos^2\theta}\right]$

$u^2\cos^2\theta = \frac{2}{5}(u^2 - gH)$

MATCH THE COLUMN

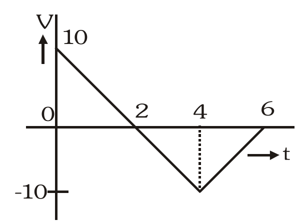
1. [A] $X = 3t^2 + 2 \Rightarrow V = \frac{dx}{dt} = 6t \Rightarrow a = \frac{d^2x}{dt^2}$

[B] $V = 8t \Rightarrow a = \frac{dv}{dt} = 8$

[D] For changing the direction $6t - 3t^2 = 0$
⇒ t = 0, 2 sec

2. Slope of v.t. curve gives acceleration (instantaneous) at that point $\vec{a} = \frac{d\vec{v}}{dt}$

3. At t = 0, v(0) = 10 m/s; t = 0 ; v(6) = 0
Change v(6) – v(0); Δv =0 – 10 = –10 m/s



Average acceleration

$= \frac{\text{charge in velocity}}{\text{time}} = \frac{-10}{6} = \frac{-5}{3}$ m/s^2

Average velocity = $\frac{\text{Displacement}}{\text{time interval}}$

Total displacement = Area of Δ's (with +ve or –ve)

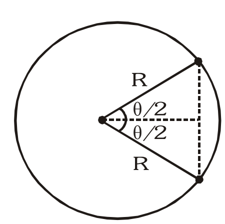
= $\frac{1}{2}$ 2 10 – $\frac{1}{2}$ 4 10 = –10 m (units)

∴ Average velocity = $\frac{-10}{6} = -\frac{-5}{3}$ m/s

a(3) = slope of line which exist at t = 0 I_0t = 4

a = tanθ = $\frac{-10}{2}$ = –5

4. Rθ = vt ; θ = $\frac{4 \times 1}{1}$ = 4 radian



$\therefore$ Displacement = 2R sin θ/2 = 2 sin 2
Distance = vt = 4m

Average velocity = $\frac{\text{Displacement}}{\text{time}}$ = 2 sin 2

Average acceleration =

$\frac{\text{Change in velocity}}{\text{time}} = \frac{2\times 4\sin 2}{1}$ = 8 sin 2

5. Velocity & height of the balloon after 2 sec:
v = 0 + 10 2 = 20 m/s ↑
h = 1/2 10 4 = 20 m
Initial velocity of drop particle is equals to the velocity of balloon = 20 m

$\because$ u_s = 20 m/s $\boxed{a_s = g\downarrow}$

After further 2s $\boxed{v_s = 0}$

$\therefore$ height = $\frac{u_s + v_s}{2}$ 2 = 20m from initial position of balloon

$\therefore$ Height from ground = 20 + 2v = 40m

ASSERTION & REASON

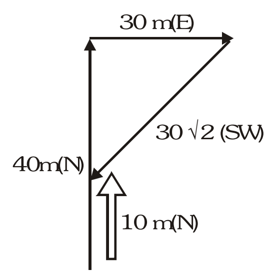
1. For max. range $\left(\frac{u^2\sin 2\theta}{g}\right)$, the projection angle(θ) should be 45 .

So initial velocity ai + bj ⇒tan 45 = $\frac{b}{a}$ ⇒ a = b

2. Whenever a particle having two ⊥ components of velocity then the path of projectile will be parabolic, if particle is projects vertically upwards then the path of projectile will be straight.
3. Acceleration depends on change in velocity not on velocity.
4. If displacement is zero in given time interval then its average velocity also will be zero. e.g. particle projects vertically upwards.
5. To meet, co-ordinates must be same. So in frame of one particle, second particle should approach it.
6. In air, the relative acceleration is zero. The relative velocity becomes constant which increases distance linearly which time.
7. Yes, river velocity does not any help to cross the river in minimum time.
8. Because initial vertical velocity component is zero in both cases.
9. Inclined plane, in downwards journey. The component of gravity is along inclined supports in displacement but not in the other case.
10. Maximum height depends on the vertical component of velocity which is equal for both.
11. Speed is the magnitude of velocity which can't be negative.
12. If the acceleration acts opposite to the velocity then the particle is slowing down.
13. Free fall implies that the particle moves only in presence of gravity.

Comprehension#1

1. $\frac{\text{Distance}}{\text{Displacement}} = \frac{\pi d/2}{d} = \frac{\pi}{2}$

2. 

3. x_1 = 1, y_1 = 4; x_2 = 2, y_2 = 16

$\therefore$ Displacement = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

= $\sqrt{1^2+12^2}$ = $\sqrt{145}$ ≅ 12m

Comprehension #2

1. Positive slopes have positive acceleration, negative slopes have negative accleration.
2. Accelerated motion having positive area on v-t graph has concave shape.
3. Maximum displacement = total area of graph
 = 20 + 40 + 60 + 80 – 40 = 160 m
4. Average speed

$= \frac{\text{Distance}}{\text{time}} = \frac{20+40+60+80+40}{70} = \frac{24}{7}\text{m/s}$

5. Time interval of retardation = 30 to 70.

Comprehension # 3

1. $y = \sqrt{3}x - 2x^2$

Trajectory equation is $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$\tan\theta = \sqrt{3} \Rightarrow \boxed{\theta = 60^\circ}$ & $\dfrac{g}{2u^2\cos^2\theta} = 2$

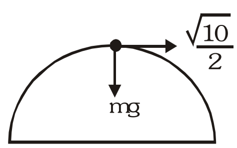
$\Rightarrow u = \dfrac{5}{2 \times \frac{1}{4}} = \sqrt{10}$

2. Max. height $H = \dfrac{u^2\sin^2\theta}{2g}$, $\dfrac{10 \times \left(\frac{\sqrt{3}}{2}\right)^2}{2 \times 10} = \dfrac{3}{8}$ m

3. Range of A = $\dfrac{u^2\sin 2\theta}{g} = \dfrac{10 \times \sin 120^\circ}{10} = \dfrac{\sqrt{3}}{2}$

4. Time of flight $= \dfrac{2u\sin\theta}{g} = \dfrac{2 \times \sqrt{10} \times \frac{\sqrt{3}}{2}}{10} = \dfrac{\sqrt{3}}{10}$

5. At the top most point $v = u\cos\theta = \sqrt{10}\cos 60 = \dfrac{\sqrt{10}}{2}$



$\therefore\ mg = \dfrac{mv^2}{R}$; $R = \dfrac{\left(\frac{\sqrt{10}}{2}\right)^2}{10} = \dfrac{10}{40}$ $\boxed{R = \frac{1}{4}\text{m}}$

Comprehension #4

1. $R = Cv_0^n$

Putting data from table: $8 = C\ \ 10^n$

$\Rightarrow 31.8 = C\ \ 20^n \Rightarrow \dfrac{31.8}{8} = 3.9 \cong 4 = 2^n \Rightarrow n=2$

2. C depends on the angle of projection.

3. $R = C\ \ v_0^n \Rightarrow 8 = C\ \ 10^n$ and

$R = C\ \ 5^n \Rightarrow R = \dfrac{8}{2^2} = 2$m

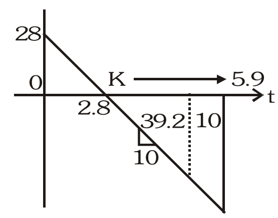
Comprehension # 5

1. If the projection angle is increased, maximum height will increase.

2. Projection angle is 45 & V_y = 21 m/s, projection speed is V_0 sin 45 =21 $\Rightarrow V_0 = 21\sqrt{2}$ =30m/s

3. By the v_y – t graph the acceleration is

$\dfrac{-21}{2.1} = -10 = -g$

4. 

5. Initial kinetic energy = 1/2 mV_0^2

If mass doubles, then we can sec from (v_y – t) curve then velocity becomes half of previous.

$\therefore \dfrac{1}{2}\ \ 2m\ \left(\dfrac{v_0}{2}\right)^2 = \dfrac{1/2mv_0^2}{2}$ Hence [B]

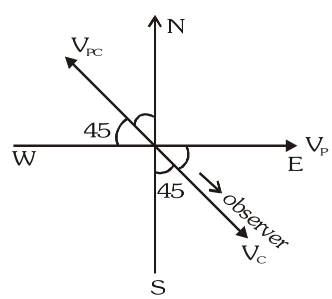
6. Position of the cable at the max. height point.

$H = \dfrac{(V_0\sin 45)^2}{2g} = \dfrac{V_0^2}{4g}$

Comprehenison # 6

1. In ground frame [A] it is simply a projectile motion. But in [B] frame horizontal component of the displacement is zero i.e. in this frame only vertical comp. appear which is responsible for the maximum height.

2. As observer observes that particle moves north-wards.



3. Frame [D], which is attached with particles itself so the minimum distance is equal to zero.

4. a_b = 20 m/s² ; a_D = 10 m/s²

a_{bD} = 30 m/s² ↑

$\therefore$ Force acting on a body = 10 20 = 200N

Comprehension#7

1. In vertical direction $h = (u\sin\theta)t - \frac{1}{2}gt^2$

$$\Rightarrow t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

$$\Rightarrow t_1 + t_2 = \frac{2u\sin\theta}{g} \quad .....(i)$$

In horizontal direction $x = (u\cos\theta)t - \frac{1}{2}at^2$

$$\Rightarrow t^2 - \left(\frac{2u\cos\theta}{a}\right)t + \frac{2x}{a} = 0$$

$$\Rightarrow t_3 + t_4 = \frac{2u\cos\theta}{a} \quad ....(ii)$$

From (i) and (ii) $\theta = \tan^{-1}\left[\frac{g(t_1+t_2)}{a(t_3+t_4)}\right]$

2. At maximum height $v_y = 0$

$$\Rightarrow H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{g}{8}(t_1+t_2)^2$$

3. At maximum range vertical displacement = 0

$\Rightarrow t = \frac{2u\sin\theta}{g}$. So range R

$$= (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right) - \frac{1}{2}a\left(\frac{2u\sin\theta}{g}\right)^2$$

$$= \frac{2u^2\sin\theta\cos\theta}{g}\left(\frac{g}{a} - \tan\theta\right)$$

EXERCISE –IV A

1. By observation, for equal interval of time the magnitude of slope of line in x-t curve is greatest in interval 3.

2. By observing the graph, position of A (Q) is greater than position of B (P) i.e. B lives farther than A and also the slope of x-t curve for A & B gives their velocities $v_B > v_A$.

3. $a = a_0\left(1 - \frac{t}{T}\right)$ where a_0 & T are constants

$$\int_0^v dv = a_0\int_{t=0}^{t}\left(1-\frac{t}{T}\right)dt \Rightarrow v = a_0\left[t - \frac{t^2}{2T}\right]$$

$$\Rightarrow \int dx = a_0\int_{t=0}^{t}\left[t - \frac{t^2}{2T}\right]dt$$

For $a = 0 \Rightarrow 1 - \frac{t}{T} = 0$ $\boxed{t = T}$ $= a_0\left[\frac{t^2}{2} - \frac{t^3}{6T}\right]$

$$\therefore \langle v \rangle = \frac{\int_0^T v\,dt}{\int_0^T dt} = \frac{a_0\left[\frac{T^2}{2} - \frac{T^3}{6T}\right]}{T} = \frac{a_0T}{3}$$

4. $S_n = u + \frac{a}{2}(2n-1)$ by putting the value of n=7 and 9, find the value of u & a, u=7 m/s & a =2 m/s^2.

5. After 3 sec distance covered =1/2 2 9 = 9m velocity of lift = 2 3 = 6 m/s↓ $\therefore u_p$ = 6m/s↓, a = g ↓ height = (100–9) = 91 m

$\therefore$ Time to reach the ground

$= 91 = 6t + \frac{1}{2}$ g t^2 t = 3.7 sec

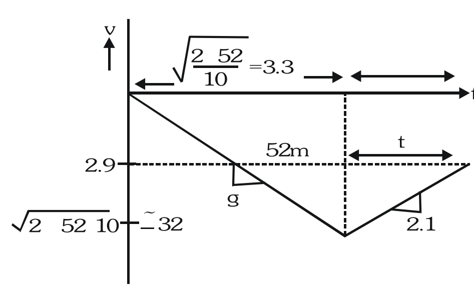
Total time taken by object to reach the ground = 3 + 3.7 = 6.7 sec.

Time to reach on the ground by lift

$= \frac{1}{2}\times 2\times t^2 = 100 \Rightarrow t = 10$ sec.

So interval = 10 – 6.7 = 3.3 sec

6. (i)



$2.1 = \frac{32-2.9}{t}$; $t = \frac{29}{2.1} \simeq 14$ $\therefore$ $14 + 3.3 \simeq 17$

(ii) Height= $52 + \frac{1}{2}$ [32 + 2.9] 14 = 293.8

7. Deaceleration of train ,

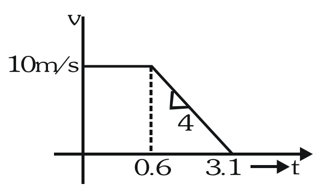
$a = \left|\frac{v^2 - u^2}{2s}\right| = \frac{20 \times 20}{2 \times 2}$ = 100 km/hr^2

Time to reach platform = $\frac{20}{100} = \frac{1}{5}$hr

$\therefore$ Total distance travelled by the bird

= vt = 60 $\frac{1}{5}$ =12km

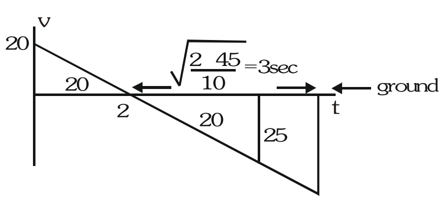
8. $\Delta t = t - 0.6 = \frac{0-10}{-4} = 2.5$



Stopping distance = 0.6 10 + $\frac{1}{2}$ 2.5 10

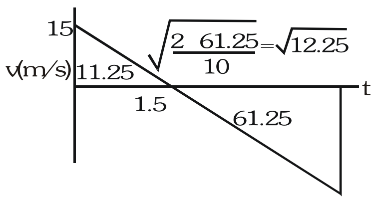
6 + 12.5 m= 18.5 m

9. (i) Height = upward area under v–t curve = 20m



(ii) Total time of flight = 2 + 3 = 5sec

10. Total time = 1.5 + 3.5 = 5s



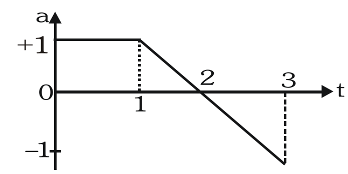
11. From given situation :

(i) $a_{avg} = \frac{60-20}{1.00-0.75} = \frac{4000}{25}$ = 160 km/hr^2

(ii) Area = $\frac{1}{2}$ [20 + 60] 0.25

= 40 $\frac{25}{100}$ = 10km

12.



u = 0

(i) Area under a – t curve the change in velocity

Δu = 1 1 + $\frac{1}{2}$ 1 1; $u_2 - u_0$ = 1.5 m/s

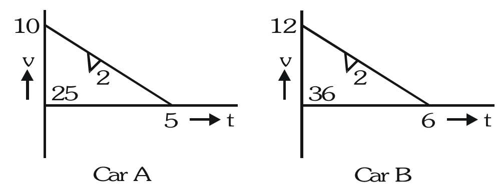
$\boxed{u_2 = 1.5\,m/s}$ ($\because u_0$ = 0)

upto 3 sec : Δu = 1.5 – $\frac{1}{2}$ 1 1 =1m/s

$u_3 - u_0$ = 1m/s $\Rightarrow$ u_3 = 1m/s ($\because u_0$ = 0)

13.

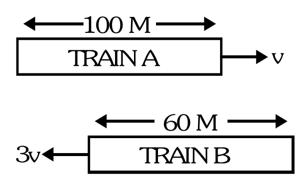
Distance travelled to stop



Total distance = 25 + 36 = 61 m covers by both car

$\therefore$ Remaining distance = 150 – 61 = 89 m

14. Let v_{AB} = v – (–3v) = 4v



time $= \frac{160}{4v}$ = 4 sec $\boxed{v = 10\,m/s}$

velocity of train v_A = 10 m/s

v_B = 3 v = 30 m/s

15. Direction of flag = Resultant direction of the wind velocity and the opposite of boat velocity

$\Rightarrow \vec{v}_W - \vec{v}_B = \frac{72}{\sqrt{2}}(\hat{i}+\hat{j}) - 51\hat{j}$

$= 36\sqrt{2}\hat{i} + (36\sqrt{2} - 51)\hat{j} == 36\sqrt{2}\hat{i}$ (EAST)

16. **For A :** $30t_1$ = S/2 = 60 (2 – t_1) $\Rightarrow$ t_1 = 4/3 hr (Here S is the total distance and t_1 is time up to which A's speed is 30 km/hr)

For B : $\frac{1}{2}$ a $2^2 = \left(30 \times \frac{4}{3}\right)$ 2 = S

$\Rightarrow a = 40$ km/hr^2

(i) (a) $v_B = 40t = 30 \Rightarrow t = 0.75$ hr

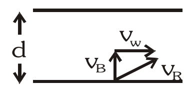
(b) $v_B = 40t = 60 \Rightarrow t = 1.5$ hr

(ii) There is no overtaking.

17. Relative velocity of A w.r to B,

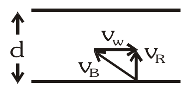
V_{AB} time $= \frac{a}{v - v\cos\theta} = \frac{a}{v(1-\cos\theta)}$ $\boxed{\theta = \frac{2\pi}{n}}$

18. $t = \frac{d}{v_B} = 600s$, drift$= v_w$ $\frac{d}{v_B}$



$120 = v_w$ 600s; $v_w = \frac{1}{5}\ \frac{m}{sec}$

$t = \frac{d}{\sqrt{v_B^2 - v_w^2}} = 750$

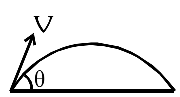


$\sqrt{1 - \left(\frac{v_w}{v_B}\right)^2} = \frac{4}{5} \Rightarrow \left(\frac{v_w}{v_B}\right)^2 = \frac{9}{25}$

$\boxed{\frac{v_w}{v_B} = \frac{3}{5}} \Rightarrow \boxed{v_B = \frac{1/5}{3/5} = \frac{1}{3} m/sec}$

$\frac{d}{V_B} = 600 = \boxed{d = 600 \times \frac{1}{3} = 200m}$

19. $\vec{v}(0) = v\cos\theta\tilde{i} + v\sin\theta\tilde{j}$



$\vec{v}(t) = v\cos\theta\tilde{i} + (v\sin\theta - gt)\tilde{j}$

$|\vec{v}(t)| = \sqrt{v^2\cos^2\theta + (v\sin\theta - gt)^2}$

$< \vec{v}(t) > = \frac{\vec{v}(t) + \vec{v}(0)}{2} = v \cos\theta\ \tilde{i} + \frac{(2v\sin\theta - gt)}{2}\tilde{j}$

According to question $\sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$

$= \sqrt{(v\cos\theta)^2 + \left(\frac{2v\sin\theta - gt}{2}\right)^2}$

$v^2\cos^2\theta + (v\sin\theta - gt)^2 = v^2\cos^2\theta + \left(\frac{2v\sin\theta - gt}{2}\right)^2$

$v\sin\theta - gt = -v\sin\theta + \frac{gt}{2} \Rightarrow \frac{3gt}{2} = 2v\sin\theta$

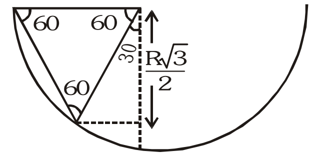
$\boxed{t = \frac{4}{3}\left(\frac{v\sin\theta}{g}\right)}$

20. u sinθ 1 − $\frac{1}{2}$g$(1)^2$ = u sinθ 3 − $\frac{1}{2}$ g $(3)^2$

2u sinθ = 40 $\Rightarrow$ usin θ = 20m/s

Max. height$= \frac{u^2\sin^2\theta}{2g} = \frac{20 \times 20}{20} = 20m$

21. $\therefore$ Vertical displacement of particle $= \frac{R\sqrt{3}}{2}$



Time for this $= \sqrt{\frac{2 \times R\frac{\sqrt{3}}{2}}{g}} = \sqrt{\frac{\sqrt{3}R}{g}}$

$\vec{v}(t) = u\tilde{i} + gt\tilde{j} = u\tilde{i} + g \times \sqrt{\frac{\sqrt{3}R}{g}}\tilde{j} = u\tilde{i} + \sqrt{\sqrt{3}Rg}\,\tilde{j}$

22. 780 = u sin θ 6 + $\frac{1}{2}$ g 36

780 − 180 = u sin θ 6

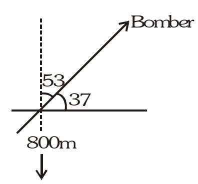
usin θ $= \frac{600}{6}$ =100 m/sec

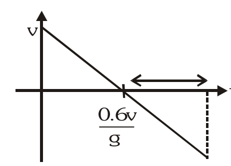
i.e. food package dropped before 10 secs

1000 = u 10 $\Rightarrow$ u = 100 m/s

$\therefore$ h $= \frac{g \times (16)^2}{2}$ = 1280 m.

23.





$20 = \frac{0.6v}{g} + \sqrt{\frac{2}{g} \times \left[\frac{(0.6v)^2}{2g} + 800\right]}$ ....(i)

(i) By solving equation (i), we get v = 100 m/s.

(ii) Maximum height :

$= 800 + \frac{(0.6v)^2}{2g} = 800 + \frac{(0.6 \times 100)^2}{20} = 980m$

(iii) horizontal distance

= Horizontal velocity time of flight

= 100 cos 37 20 = 1600m

(iv) horizontal component

$v_H = u_H$ = 100 cos 37 = 80 m/s

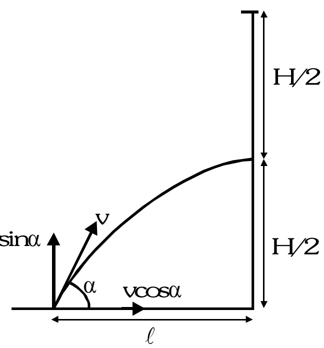
$v_v = u_v$ − 10 20 = 100 sec 37 − 200

= 140 m/s

$\therefore v_{strike} = 80\tilde{i} - 140\tilde{j}$, $|\vec{v}| = \sqrt{80^2 + 140^2}$

24. $\frac{H}{2}$ distance covered by free falling body

$$\frac{H}{2}=\frac{1}{2}gt^2 \quad ; \quad t=\sqrt{\frac{H}{g}}$$



In same time, projectile also travel vertical distance

$\frac{H}{2}$, then $\frac{H}{2}=v\sin\alpha\sqrt{\frac{H}{2}}-\frac{1}{2}g\frac{H}{g}$

$v\sin\alpha=\sqrt{gH}$ ...(i)

also $\ell=v\cos\alpha\sqrt{\frac{H}{g}}$; $v\cos\alpha=\ell\sqrt{\frac{g}{H}}$ ...(ii)

From equation (i) and (ii)

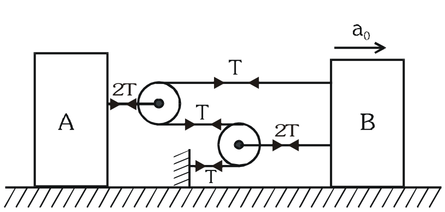
$$\tan\alpha=\frac{H}{\ell}v^2\sin^2\alpha+v^2\cos^2\alpha=gH+\ell^2\frac{g}{H}$$

$$v=\sqrt{gH\left(1+\frac{\ell^2}{H^2}\right)}t$$

25. $\frac{d}{10\sqrt{2}\cos 45°+10}=\frac{10}{10\sqrt{2}\sin 45°}$

d = 20 1 = 20 m.

26. Here a_B (3T) = (a_A) (2T) $a_A=\frac{3}{2}a_B$



$$a_{AB}=a_A-a_B=\tfrac{3}{2}a_0-a_0=\frac{a_0}{2}$$

27. $\vec{a}_t=6\tilde{i}=\vec{\alpha}\times\vec{R}=\vec{\alpha}\times 2\tilde{j} \Rightarrow \vec{\alpha}=-3\tilde{k}$ rad/s²

$\vec{a}_r=-8\tilde{j}=\vec{\omega}\times\vec{v}=\vec{\omega}\times(\omega R)\tilde{i}\Rightarrow\vec{\omega}=-2\tilde{k}$ rad/s

28. $v=2t^2$; $a_T=\frac{dv}{dt}=4t \Rightarrow a_T(1)=4$

$$a_N=\frac{v^2}{R}=\frac{(2\times1^2)^2}{1}=4$$

$$a=\sqrt{a_T^2+a_N^2}=\sqrt{(4)^2+4^2}=\sqrt{32}$$

$$\boxed{a=4\sqrt{2}}$$

29. $(v_A+v_B)\,t=2\pi R$, $(0.7+1.5)\,t=2\ \frac{22}{7}\ 5$

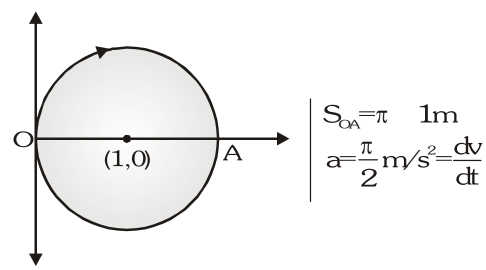
$$t=\frac{2\times22\times5}{7\times2.2}\times10=\frac{100}{7}\text{sec}=14.3\text{ sec}$$

Acceleration of B = $\frac{v_B^2}{R}=\frac{1.5^2}{5}=0.45$ m/s²

30. $a_t=ar$; $\alpha r=\omega^2 r$; $\alpha=\alpha^2t^2\Rightarrow\alpha=\frac{1}{t^2}$

31. (a) $\pi=0+\frac{1}{2}\times\frac{\pi}{2}t^2 \Rightarrow t=2\text{sec}$

(b) $v=0+\frac{\pi}{2}\times2=\pi$ m/s



32. $r=2.5$ m, $a_{net}=25$ m/s²

(a) Radial acceleration = 25 cosθ = 25 $\frac{\sqrt{3}}{2}$ m/s²

(b) $25\frac{\sqrt{3}}{2}=\frac{v^2}{25}\Rightarrow v=\left(125\frac{\sqrt{3}}{4}\right)^{1/2}$ m/s

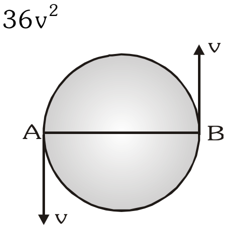
(c) Tangential acceleration = 25 sinθ = 25 $\frac{1}{2}$ m/s²

33. According to

$$\theta=\frac{1}{2}\times\frac{72v^2}{25\pi R}\times t^2=\pi R\Rightarrow t=\frac{5\pi R}{6v}$$

Using $R\theta=vt+\left(\frac{1}{2}\right)\frac{72v^2}{25\pi R}\times\frac{25\pi^2R^2}{36v^2}$

$$a_T=\frac{72v^2}{25\pi R}$$



$$R\theta=\frac{v5\pi R}{6v}+\pi R=\frac{11}{6}\pi$$

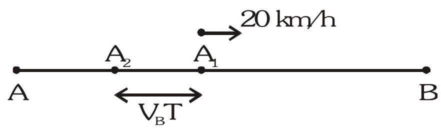
Angular velocity : $\omega=\omega_v+\alpha t$

$$=\frac{v}{R}+\frac{72v^2}{25\pi R^2}\times\frac{5\pi R}{6v}=\frac{v}{R}+\frac{12v}{5R}=\frac{17v}{5R}$$

Angular acceleration $\alpha=\omega^2R=\frac{289v^2}{25R}$

EXERCISE –IV B

1. $\frac{V_B T}{V_B - 20} = 18,\ \frac{V_B T}{V_B + 20} = 6;\ \frac{V_B + 20}{V_B - 20} = 3$

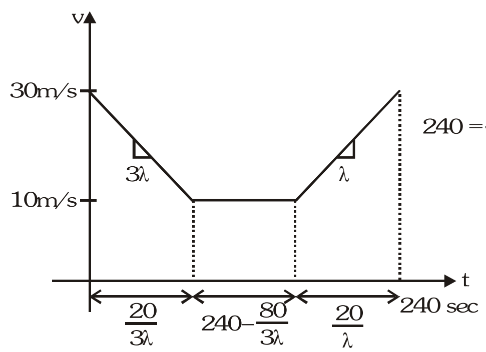


$\Rightarrow V_B + 20 = 3V_B - 60$ $\boxed{v_B = 40 km/h}$

$\therefore\ T = \frac{6(V_B + 20)}{V_B} = \frac{6 \times 60}{40} = 9$ min

2. (i) Area = $\frac{1}{2}$ [10 + 30] $\frac{20}{3\lambda}$ + 10 $\left(240 - \frac{80}{3\lambda}\right)$

$+ \frac{1}{2}$ [10 + 30] $\frac{20}{\lambda}$ = 4000



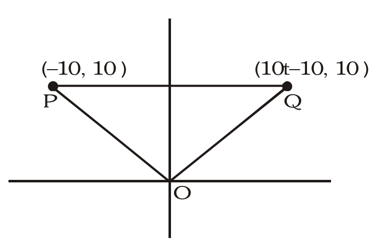
$\frac{400}{3\lambda} + 2400 - \frac{800}{3\lambda} + \frac{400}{\lambda} = 4000$

$\frac{400 - 800 + 1200}{3\lambda} = 1600$

$3\lambda = \frac{800}{1600} = \frac{1}{2};\ \lambda = \frac{1}{6}$

(ii) Dist. travelled = $10\left(240 - \frac{80}{3 \times 1/6}\right) = 800$ m

3. It's velocity is $10\hat{i}$



$\therefore$ displacement after time 't' = $10\hat{i}$ t

Velocity of second ship = $u \times \frac{(\hat{i} + 2\hat{j})}{\sqrt{5}}$

$\tan\theta = \frac{2u/5}{\left(10 - \frac{u}{\sqrt{5}}\right)} = \frac{2 \times 10\sqrt{5}}{10\sqrt{5} - 10\sqrt{5}}$

(i) $t = \frac{10}{20} = \frac{1}{2}$ sec , minimum distance = 10 km

4. –25 m/s After 5 sec
height of balloon = 25 5 = 125 m
(i) Minimum speed

$125 = \frac{(u - 25)^2}{2g} \Rightarrow (u - 25)^2 = 2500;$

$u - 25 = 50;\ u = 75$ m/s

(ii) u = 2 75 = 150 m/s
$125 = (150 - 25)\,t - 5t^2$
$125 = 125t - 5t^2 \Rightarrow t^2 - 25t + 25 = 0$

5. $v_{12} = v_1 - v_2 = v_1 - (-v_2) = v_1 + v_2$

$\boxed{\ell_{max} = \frac{(v_1 + v_2)^2}{2(a_1 + a_2)}}$

$a_{12} = -a_1 - a_2 = -(a_1 + a_2)$

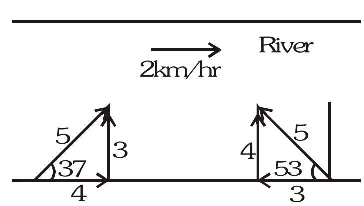
6. Let t = time of accelerated motion of the helipcopter.
Distance travelled by helicopter
= Distance travelled by sound

$\Rightarrow \frac{1}{2}$ 3 $t^2 = 320\,(30 - t) \Rightarrow t = \frac{80}{3}$ sec

Final velocity of helicopter

v = u + at = 0 + 3 $\frac{80}{3}$ = 80 m/s

7. $V_A = (4 + 2)\ \hat{i} + 3\hat{j},\ V_B = (-3 + 2)\,\hat{i} + 4\hat{j}$



Time to cross the river $t_A = \frac{100}{3};\ t_B = \frac{100}{4}$

Drift = $\frac{100}{3}$ 6 = 200 m ; Drift = –1 $\frac{100}{4}$

Remaining distance = 300 – 200 ; 25 m

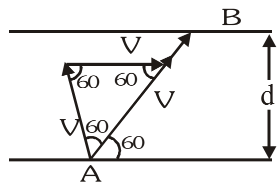
$(t_{total})_A = \frac{100}{3}+\frac{100}{8}$; $t_B = \frac{100}{4}+\frac{100}{6}$

$t_A = \frac{800+300}{24}=\frac{1100}{24}$; $t_B = \frac{600+400}{24}=\frac{1000}{24}$

$\boxed{t_A = 165\text{ sec}}$ $\boxed{t_B = 150\text{ sec}}$

8. From figure (a) 120

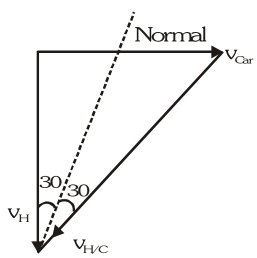
time to cross = $\frac{2d}{\sqrt{3}V}$



Minimum time t = $\frac{d}{v}$

∴ Ratio = $2\sqrt{3}$

9. tan 60 = $\frac{v_{Car}}{v_H} \Rightarrow \sqrt{3} = \frac{v_C}{10} \Rightarrow v_C = 10\sqrt{3}$ m/s



10. Range (OA) = $\frac{u^2 \sin 2\theta}{g} = \frac{1600\times\sqrt{3}}{10\times 2} = 80\sqrt{3}$

$h = 80\sqrt{3}\times\tan 60° = \frac{10\times 80\times 80\times 3}{2\times v^2\cos 60°}$

Time to strike ⇒ vcos 60 t = $80\sqrt{3}$

$\Rightarrow t = \frac{80\sqrt{3}\times 2}{v\times 1} = \frac{10\sqrt{3}}{v}$

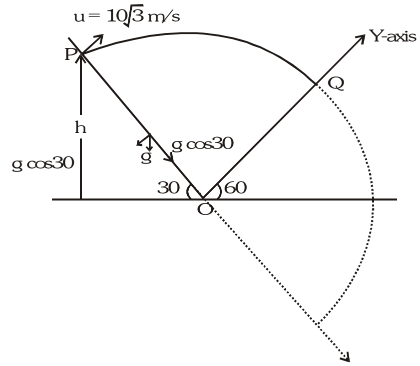
$h = 9\sqrt{3}\times\frac{160\sqrt{3}}{v}\frac{480\times 9}{v} = \frac{240^2 - 38400}{v^2}$

v^2 – 1600 – 18v = 0

$v = \frac{18\pm\sqrt{324+6400}}{2}$

$\Rightarrow \boxed{v = 50 m/s}$

11.



(i) $v(t) = (u - g\cos 30\ t)\tilde{i} - g\sin\theta\ t\tilde{j}$

From given situation

u – g cos 30 t = 0

t = 2 sec

(ii) Velocity u_x = 0, a_x = g cos30 = $\frac{g}{2}$

∴ $v_x = 0 + \frac{g}{2}$ 2 = 10 m/s

(iii) Distance PO =

$10\sqrt{3}\cos 90°\times t + \frac{1}{2}\times g\sin 30°\times(2)^2$

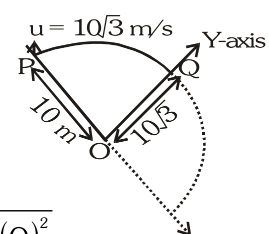
PO = 10 m ∴ h = 10 sin 30 = 5 m

(iv) Maximum height = h + $\frac{u(\sin 60°)^2}{2g}$

$= 5 + \frac{\left(10\sqrt{3}\times\frac{\sqrt{3}}{2}\right)^2}{20} = 16.25$ m

(v) Distance PQ

$OQ = \frac{(10\sqrt{3})^2}{2g\cos 30°}$



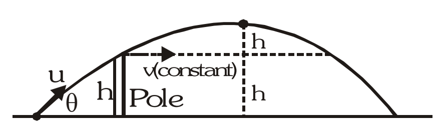
OQ = $10\sqrt{3}$

∴ PQ = $\sqrt{(PO)^2 + (O)^2}$

$= \sqrt{10^2 + (10\sqrt{3})^2}$ = 20 m

12. For stone : 2h = $\frac{(u\sin\theta)^2}{2g}$ & h = (u sin θ)t – $\frac{1}{2}gt^2$

$\Rightarrow t = \frac{\sqrt{40h}\pm\sqrt{20h}}{10} \Rightarrow \Delta t = \sqrt{0.8h} = \frac{2}{10}\sqrt{20h}$



Horizontal displacement : vt_2 = u cos θ Δt

$$\Rightarrow \frac{v(\sqrt{2}+1)\sqrt{20h}}{10} = u\cos\theta \times \frac{2\sqrt{20h}}{10}$$

$$\Rightarrow \frac{v}{u\cos\theta} = \frac{2}{\sqrt{2}+1}$$

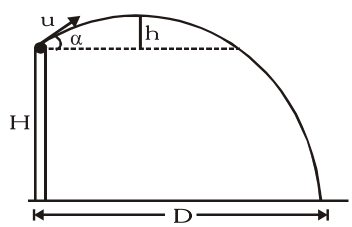
13. u cos αt = D ....(i)

$u \sin \alpha\, t - \frac{1}{2} gt^2 = -H$ ....(ii)

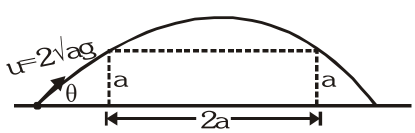
$$\Rightarrow t = \frac{2u\sin\alpha \pm \sqrt{u^2\sin^2\alpha + 2gH}}{g} = \frac{D}{u\cos\alpha}$$

$$h = \frac{(u\sin\alpha)^2}{2g} = \frac{D^2\tan^2\alpha}{4(H + D\tan\alpha)}$$

$$\therefore H_{max} = h + H = H + \frac{D^2\tan^2\alpha}{4(H+D\tan\alpha)}$$



14. $s = ut + \frac{1}{2}at^2 \Rightarrow a = (u\sin\theta)t - \frac{1}{2}gt^2$



$$\Rightarrow t = \frac{u\sin\theta \pm \sqrt{u^2\sin^2\theta - 2ag}}{g}$$

$$\Delta t = \frac{2\sqrt{u^2\sin^2\theta - 2ag}}{g}$$

For horizontal motion : $2a = u\cos\theta \quad \Delta t$

$$\Rightarrow 2a = \frac{u\cos\theta \times 2\sqrt{u^2\sin^2\theta - 2ag}}{g} \Rightarrow \theta = 60$$

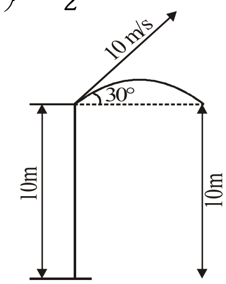
$$\therefore \Delta t = \frac{2a}{u\cos\theta} = \frac{2a}{2\sqrt{ag}\times\frac{1}{2}} = 2\sqrt{\frac{a}{g}}$$

EXERCISE –V-A

1. Kinetic energy of a projectile at the highest point $= E\cos^2(\theta)$ where E is the kinetic energy of projection, θ is the angle of projection.

$$E_{\text{highest point}} = E(\cos 45)^2 = E\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{E}{2}$$

2. $R = \frac{u^2\sin 2\theta}{g}$



$$R = \frac{(10)^2\sin 60°}{10}$$

$$R = 10\times\frac{\sqrt{3}}{2} = 5\sqrt{3} = 8.66 \text{ m}$$

3. Both horizontal direction speed is same

$v_0\cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60$

4. When a body is projected at an angles θ and 90–θ; the ranges for both angles are equal and the corresponding time of flights for the two ranges are t_1 and t_2.

$$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{1}{2}g\left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\sin(90°-\theta)}{g}\right)$$

$$= \frac{1}{2}gt_1t_2 \Rightarrow R \propto t_1t_2$$

5. $K_{\text{highest point}} = [K_{\text{Point of projection}}]\cos^2\theta$

$$K_H = K(\cos 60°) \Rightarrow K_H = \frac{K}{4}$$

6. $\vec{v} = K(y\tilde{i} + x\tilde{j})$; $v_x = Ky$; $\frac{dx}{dt} = Ky$

similarly $\frac{dy}{dt} = Kx$

Hence $\frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx$,

by integrating $y^2 = x^2 + c$.

7. $R_{max} = \frac{u^2}{g}$; Area $= \pi r^2 = \frac{\pi u^2 R^2_{max}}{g^2}$

8. $H_{max} = \frac{u^2}{2g} = 10$ m and $R_{max} = \frac{u^2}{g} = 20$ m

9. $u = \sqrt{5}$ and $\tan\theta = 2$

so by $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$

$$\Rightarrow y = 2x - \frac{10x^2}{2\times 5}(1+4) \Rightarrow y = 2x - 5x^2$$

EXERCISE –V-B

Single Choice

1. $v_{av} = \frac{\text{total displacement}}{\text{total time}} = \frac{2}{1} = 2\text{m/s}$

2. $v^2 = 2gh$ [it is parabola]
and direction of speed (velocity) changes.

3. $a = -\frac{10}{11}t + 10$ at maximum speed a =0

$\frac{10}{11}t + 10 \Rightarrow t = 11$ sec

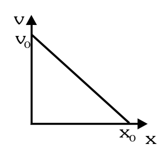
Area under the curve = $\frac{1}{2}$ 11 10 = 55

4. $S_n = u + \frac{a}{2}(2n-1) = \frac{a}{2}(2n-1)$

$S_{(n+1)} = x + \frac{a}{2}(2n+1) = \frac{a}{2}(2n+1)$

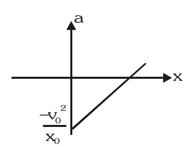
$\Rightarrow \frac{S_n}{S_{n+1}} = \frac{(2n-1)}{(2n+1)}$

5. $v = -\left(\frac{v_0}{x_0}\right)x + v_0$



$a = \left[-\frac{v_0}{x_0}n + v_0\right]\left[-\frac{v_0}{x_0}\right]$

$a = \left(-\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$



MCQ's

1. x = a cospt ; y = bsinpt; $\vec{r} = a\cos(pt)\tilde{i} + b\sin(pt)\tilde{j}$

$\because \sin^2 pt + \cos^2 pt = 1$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)

$\vec{v} = -ap\sin(pt)\tilde{i} + bp\cos(pt)\tilde{j}$; $v_t = \frac{\pi}{2p} = -ap\tilde{i}$

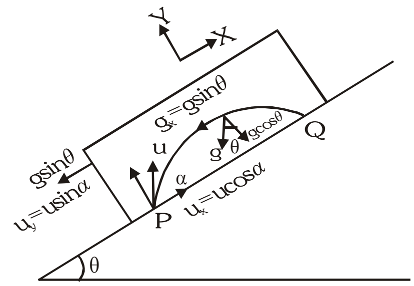
$\vec{a} = -ap^2(pt)\tilde{i} - bp^2\sin(pt)\tilde{j}$; $a_t = \frac{\pi}{2p} = -bp^2\tilde{j}$

$\vec{a}\cdot\vec{v} = 0$

$\vec{a} = -p^2\left[a\cos pt\tilde{i} + b\sin pt\tilde{j}\right] = -p^2\vec{r}$

Subjective

1.(i) u is the relative velocity of the particle with respect to the box.



u_x is the relative velocity of particle with respect to the box in x-direction. u_y is the relative veloicty with respect to the box in y-direction. Since there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground also.

Y-direction motion

(Taking relative terms w.r.t. box)

$u_y = +u\sin\alpha$; $a_y = -g\cos\theta$

$s = ut + \frac{1}{2}at^2 \Rightarrow 0 = (u\sin\alpha)t - \frac{1}{2}g\cos\theta \quad t^2$

$\Rightarrow t = 0$ or $t = \frac{2u\sin\alpha}{g\cos\theta}$

X-direction motion

(taking relative terms w.r.t box)

$u_x = +u\cos\alpha$ & $s = ut + \frac{1}{2}at^2$

$a_x = 0 \Rightarrow s_x = u\cos\alpha \quad \frac{2u\sin\alpha}{g\cos\theta} = \frac{u^2\sin 2\alpha}{g\cos\theta}$

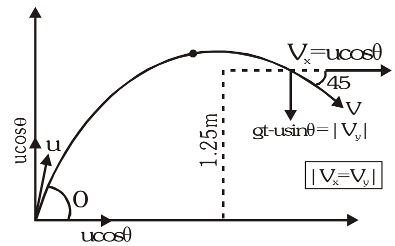
(ii) For the observer (on ground) to see the horizontal displacement to be zero, the distance travelled by the box in time $\left(\frac{2u\sin\alpha}{g\cos\theta}\right)$ should be equal to the range of the particle. Let the speed of the box at the time of projection of particle be u. Then for the motion of box with respect to ground.

$u_x = -v$, $s = vt + \frac{1}{2}at^2$, $a_x = -g\sin\theta$

$s_x = \frac{-u^2\sin 2\alpha}{g\cos\theta} = -v\left(\frac{2u\sin\alpha}{g\cos\theta}\right) - \frac{1}{2}g\sin\theta\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$

On solving we get $v = \frac{u\cos(\alpha+\theta)}{\cos\theta}$

2. Let 't' be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal. According to question, we have following three conditions.



(i) Vertical displacement of stone is 1.25 m.

$\because 1.25 = (u \sin\theta)\, t - \frac{1}{2} g t^2$ where g=10 m/s²

$\Rightarrow (u \sin\theta)\, t = 1.25 + 5t^2$ ...(i)

(ii) Horizontal displacement of stone

= 3 + displacement of object A.

Therefore $(u \cos\theta)\, t = 3 + \frac{1}{2} a t^2$

where a= 1.5 m/s² $\Rightarrow (u\cos\theta)\, t = 3 + 0.75t^2$...(ii)

(iii) Horizontal component of velocity (of stone)

= vertical component (because velocity vector is inclined) at 45 with horizontal).

Therefore $(u\cos\theta) = gt - (u\sin\theta)$ ...(iii)

The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion.

Therefore, $gt > u \sin\theta$.

In upward motion $u \sin\theta > gt$.

Multiplying equation (iii) with t we can write,

$(u \cos\theta)\, t + (u\sin\theta)\, t = 10t^2$ ...(iv)

Now (iv)-(ii)-(i) gives $4.25\, t^2 - 4.25 = 0$ or t = 1 s

Substituting t = 1s in (i) and (ii) we get

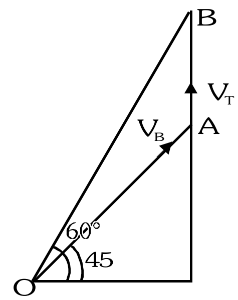
$u \sin\theta = 6.25$ m/s

$\Rightarrow u_y = 6.25$ m/s and $u \cos\theta = 3.75$ m/s

$\Rightarrow u_x = 3.75$ m/s therefore $\vec{u} = u_x \hat{i} + u_y \hat{j}$

$\Rightarrow \vec{u} = \left(3.75\hat{i} + 6.25\hat{j}\right)$ m / s

3. (a) From the diagram $\vec{V}_{BT}$ makes an angle of 45 with the x-axis.



(b) Using sine rule

$$\frac{V_B}{\sin 135°} = \frac{V_T}{\sin 15°}$$

$\Rightarrow V_B = 2$ m/s

Integer Type questions

1. With respect to train :

Time of flight : $T = \frac{2v_y}{g} = \frac{2 \times 5\sqrt{3}}{10} = \sqrt{3}$

By using $s = ut + \frac{1}{2} a t^2$

we have $1.15 = 5T - \frac{1}{2} a T^2 \Rightarrow a = 5$ m/s²