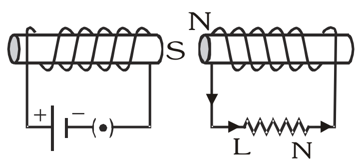
UNIT # 10

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

EXERCISE –I

1. Total change in flux = Total charge flown through the coil resistance

$= \left(\frac{1}{2} \times 4 \times 0.1\right)$ Resistance= 0.2 10 = 2 Webers

2. 

3. $e = \left|-\frac{d\phi}{dt}\right| = Na^2\frac{dB}{dt} = 5\text{volt}$

4. $e = \left|-\frac{d\phi}{dt}\right| = NA\frac{dB}{dt} = (100)(40\times10^{-4})\left(\frac{6-1}{2}\right)$

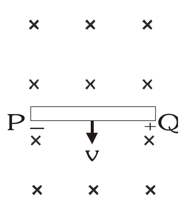
= 0.8 volt ≈ 1volt

5. $q = \frac{\Delta\phi}{R} = \frac{NBA}{R} \Rightarrow B = \frac{qR}{NA}$

6. $\phi = \pi r^2 B \Rightarrow e = \frac{d\phi}{dt} = (2\pi rB)\frac{dr}{dt}$

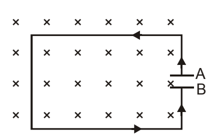
7. $W = \int Q\vec{E}.d\vec{\ell} = Q\int \vec{E}.d\vec{\ell} =$ QV

8. $\frac{d\phi}{dt} = \oint \vec{E}.d\vec{r} \Rightarrow -\alpha A = \oint \vec{E}.d\vec{r}$

9. 

e = B (2R) v = 2Brv

10. According to Lenzs law



⇒ Plate B will become positively charged.

11. $I = \frac{\frac{1}{2}B\omega\ell^2}{R}$

All spokes are in parallel

12. $W = F\ell = N(I\ell B)\ell = N\left(\frac{NB\ell v}{R}\right)\ell^2 B = \frac{N^2B^2\ell^4}{tR}$

$\because t = \frac{\ell}{v} \quad \therefore W = \frac{N^2B^2A^2}{Rt} = 0.1\text{mJ}$

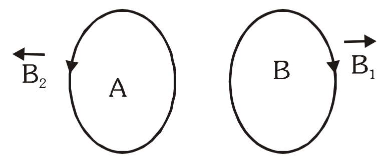
13. $I_1 = \frac{\varepsilon}{R_1} = \frac{\frac{1}{2}B\omega r^2}{R_1} = \frac{B\omega r^2}{2R_1}$

14. $I\ell B > mg \Rightarrow I\ell B < mg$

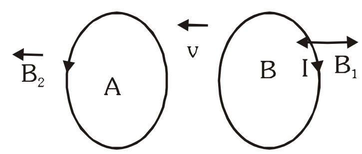
15. ϕ = NBA cosωt

$\therefore e = -\frac{d\phi}{dt} = NBA\omega \sin\omega t \Rightarrow e_{max} = NBA\omega$

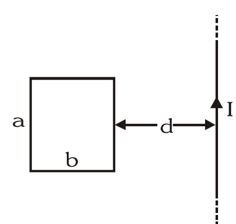
16. $e = L\frac{\Delta I}{\Delta t} \Rightarrow L = \frac{e}{\Delta I/\Delta t} = \frac{8}{(4/0.05)} = \frac{8}{80} = 0.1\text{H}$

17. 

I increases B_1 increases
So from Lenz's law
Current in A is clockwise



I same and flux linked with A increases
So from Lenz's law current in A clockwise

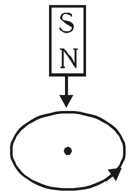
18. 

$\because \phi$ = MI but $\phi = \int_d^{d+b}\frac{\mu_0 I}{2\pi x}(adx) = \frac{\mu_0 Ia}{2\pi}\ell n\left(\frac{d+b}{d}\right)$

$\Rightarrow M = \frac{\mu_0 a}{2\pi}\ell n\left(\frac{d+b}{d}\right) \Rightarrow M \propto a$

19. Work done = LI_0^2 = (0.04) $(5)^2$ =1.0 J

20. According to Lenz law current in loop as shown in figure due to this current, a magnetic force F_m is acted on bar magnetic if ϕ is the acceleration of the bar magnet then



ma = mg – F   a = g – F/m $\therefore$ a < g

21. 100 = $V_R + V_L$ But V_R = (4000) (15 10^{-3}) =60volt
$\Rightarrow V_L$ = 40 volt

22. $\frac{L}{R_1}$ =2 10^{-3} & $\frac{L}{R_1+R_2} = 0.5\times10^{-3}$

$\Rightarrow \frac{90+R_1}{R_1}$ =4 $\Rightarrow$ R_1 = 30Ω, L = 60mH

23. Here I_0 = $\frac{E}{R}$ = constant

$\Rightarrow$ We can't change E or R.

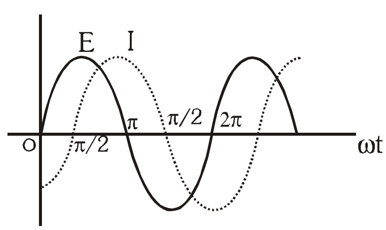
For curve 2 time constant $\frac{L}{R}$ is more

24. $I = I_0(1-e^{-t/\tau}) \Rightarrow \frac{I_0}{2} = I_0(1-e^{-t/\tau})$

$\Rightarrow t = \tau\ell n2 = \frac{L}{R}\ell n2 = \left(\frac{300\times10^{-3}}{2}\right)(0.693)$ = 0.1s

25. Average value = 0
rms value = V_0

26.



E reaches at maximum value at $\frac{\pi}{2}$ phase before I reaches at its peak value at phase π.

So E is leads I by $\frac{\pi}{2}$ So E is leads I by $\frac{\pi}{2}$

27. $I_{rms} = \sqrt{\frac{\int I^2 dt}{\int dt}} = \sqrt{\frac{\int_2^4 4t\,dt}{\int_2^4 dt}} = \sqrt{\frac{(2t^2)_2^4}{(t)_2^4}} = \sqrt{12} = 2\sqrt{3}$A

28. Ac ammeter reads rms value
dc ammeter reads average value

29. $I_{rms} = \sqrt{<(I_0 + I_1\sin\omega t)^2>} = \sqrt{<I_0^2 + I_1^2\sin^2\omega t + 2I_0I_1\sin\omega t>}$

$= \sqrt{I_0^2 + \frac{I_1^2}{2}} = \sqrt{I_0^2 + 0.5I_1^2}$

30. Impedance = $\sqrt{R^2+(\omega L)^2}$
At low frequency $\omega \to 0$
so impedance $\to$ R
At high frequency $\omega \to \infty$
so impedance $\to \omega L$

31. $R = \frac{V^2}{P} = \frac{(10)^2}{20} = 5\Omega$
In AC circuit

P = VI cos ϕ = $\frac{V^2}{z}$ cos ϕ = $\left(\frac{V^2}{z}\right)\left(\frac{R}{z}\right)$

$\Rightarrow 10 = \frac{(10)^2(5)}{(5)^2+(\omega L)^2} \Rightarrow$ ωL = 5Ω

$\Rightarrow \omega = \frac{5}{L} \Rightarrow f = \frac{\omega}{2\pi} = \frac{5}{2\pi L}$

$= \frac{5}{2\times3.14\times10\times10^{-3}}$ = 80 Hz

32. $X_c = \frac{1}{\omega c} = \frac{1}{2\pi fc}$

33. $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

34. $Z = \sqrt{R^2+X^2}$

If X is capacitive X= $\frac{1}{\omega C}$

35. $\phi = \tan^{-1}\left(\frac{x}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$= \tan^{-1}\left(\frac{200\times1}{200}\right)$ = $\tan^{-1}(1)$ = 45

36. $\because \frac{\Delta\phi}{2\pi} = \frac{\Delta T}{T} \therefore \frac{\pi/4}{2\pi} = \frac{\Delta T}{T} \Rightarrow \Delta T = \frac{T}{8}$

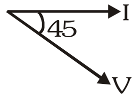
but $T = \frac{1}{50}$s $\Rightarrow \Delta T = \frac{1}{400}$s = 2.5 ms

37. $R = \frac{V}{I} = \frac{100}{25} = 4\Omega$

$\sqrt{R^2+(\omega L)^2} = \frac{V}{I} = \frac{100}{20} = 5\pi \Rightarrow$ ωL = 3Ω

38. Here $V_L = V_C$
$\Rightarrow$ Resonance condition
$\Rightarrow$ Voltage across LC combination remains same

39. tan45 = $\frac{X_C - X_L}{R}$ $\Rightarrow R = \frac{1}{2\pi fc} - 2\pi fL$



$$\Rightarrow R + 2\pi fL = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi f(2\pi fL + R)}$$

40. $V_L - V_C = 50 - 50 = 0$

41. Resonant frequency $\omega = \frac{1}{\sqrt{LC}}$

42. $V_L = IX_L = \left(\frac{V}{R}\right)(\omega L)$

$$= \left(\frac{V}{R}\right)\left(\omega \times \frac{1}{\omega^2 C}\right) = \frac{V}{\omega RC} = \frac{100}{200 \times 10^3 \times 2 \times 10^{-5}}$$

$= 250$ volt

43. $V_4 = V_L + V_C = 0$

44. $V_R = 100$ V, $I_R = \frac{V_R}{R} = \frac{100}{50} = 2A$

45. Circuit will be capacitive if $X_C > X_L$
Circuit will be inductive if $X_L > X_C$

46. Reading of voltmeter=0 as $V_L = V_C$

Reading of ammeter$= \frac{V}{R} = \frac{240}{30} = 8$ A

47. Here $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 10^{-3} \times 50 \times 10^{-6}}} = 2000 \text{rad/s}$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

so resonance condition

$$\Rightarrow I = \frac{V}{R} = \frac{20/\sqrt{2}}{6+4} = \sqrt{2} = 1.4A$$

and reading of voltmeter= (I) (4) = $4\sqrt{2}$ = 5.6V

48. $P = VI \cos\phi = \left(\frac{E_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\left(\frac{\pi}{2}\right) = 0$

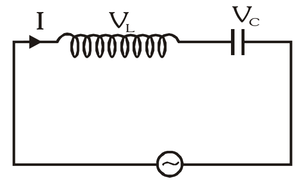
49. $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{2\pi \times 50 \times 0.7}{220}\right) = \tan^{-1}(1) = 45°$

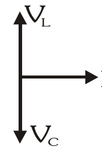
wattless current $= I_{rms} \sin\phi$

$$I = \frac{V}{Z}\frac{1}{\sqrt{2}} = \frac{220}{220\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}A = \frac{1}{2}A$$

50. $L\frac{di}{dt} = \frac{q}{c} \Rightarrow \left(\frac{di}{dt}\right)_{max} = \frac{q_0}{LC}$

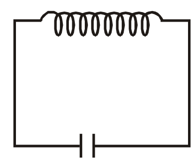
51.





52. For LC circuit $q = q_0 \cos \omega t$,

$$i = q_0\omega \cos\left(\omega t + \frac{\pi}{2}\right)$$



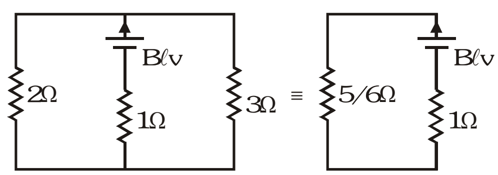
According to given condition

$$\frac{q^2}{2C} = \frac{1}{2}Li^2 \Rightarrow q = \frac{Q}{\sqrt{2}}$$

54. $\phi = (\pi R^2)B = \pi B(R_0+t)^2$

$\Rightarrow \frac{d\phi}{dt} = 2\pi B(R_0+t)$ anticlockwise

55. $I = \frac{B\ell v}{1 + 6/5} = \frac{(0.1)(0.1)(1)}{11/5} = \frac{5}{1100} = \frac{1}{220}A$



56. $e = -L\frac{di}{dt}, -\frac{di}{dt}$ is more for 1

57. Dimension based : Check yourself.

58. $I^2R = P \Rightarrow R = \frac{P}{I^2} = \frac{\frac{1}{2}LP}{\frac{1}{2}LI^2} = \frac{LP}{2U} \Rightarrow \frac{L}{R} = \frac{2U}{P}$

59. $V_R = \sqrt{10^2 - 8^2} = 6$volt

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{8}{6}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

60. $i = 2 \sin 100\pi t + 2 \sin(100\pi t) \cos 30 + 2\cos(100\pi t) \sin 30$

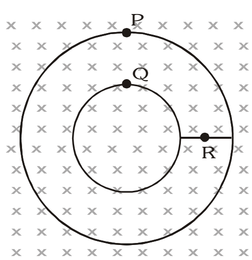
$= (2+\sqrt{3}) \sin 100\pi t + \cos(100\pi t)$

$$I_{rms} = \sqrt{<i^2>} = \sqrt{\left(2+\sqrt{3}\right)^2 \times \frac{1}{2} + 1 \times \frac{1}{2}} = \sqrt{7.5 + 4\sqrt{3}}$$

61. Old power factor $= \dfrac{R}{\sqrt{R^2+(3R)^2}} = \dfrac{1}{\sqrt{10}}$

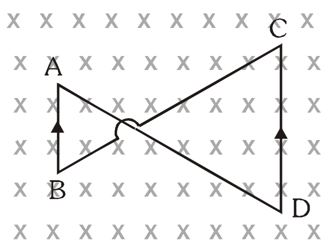
New power factor $= \dfrac{R}{\sqrt{R^2+(2R)^2}} = \dfrac{1}{\sqrt{5}}$

62.



I_P & $I_Q \rightarrow$ clockwise $I_R = 0$

63.



64. $q_A = + (B\ell v)\ C = (4)\ (1)\ (20)\ (10\ 10^{-6}) = 800\mu C$

$q_B = -q_A = -800\mu C$

65. $\dfrac{E}{R_1} = \dfrac{E}{R_2} \Rightarrow R_1 = R_2, \dfrac{L_2}{R_2} > \dfrac{L_1}{R_1} \Rightarrow L_2 > L_1$

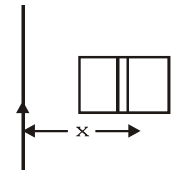
EXERCISE –II

1. $e = -\dfrac{d\phi}{dt}$ [where $\phi = \vec{B}.\vec{A}$] $\quad e = \dfrac{d}{dt}(BA\cos\theta)$

either θ,A or B should be changed for induced e.m.f.

2. Total charge transferred$= \dfrac{\Delta\phi}{R}$

It is independent of time



$\phi = \vec{B}.\vec{A} \quad \Rightarrow \int d\phi = \int_a^{2a} \dfrac{\mu_0 i}{2\pi x} a dx$

$\Delta\phi = \phi - (-\phi) = 2\phi$

$\Rightarrow \phi = \dfrac{\mu_0 i a}{2\pi} \ell n 2 \Rightarrow q = \dfrac{\mu_0 i a \ell n(2)}{\pi r}$

3. $e = Bv\ell \Rightarrow \int_0^e de = \int_a^b \dfrac{\mu_0 i}{2\pi x} v dx$

$\Rightarrow e = \dfrac{\mu_0 i v}{2\pi} \ell n(b/a) \Rightarrow i_i = \dfrac{\mu_0 i v}{2\pi R} \ell n(b/a)$

Force (बल)= $Bi\ell = b \Rightarrow \int df = \int_a^b \dfrac{\mu_0 i}{2\pi x} i_1 dx$

$f = \dfrac{\mu_0 i}{2\pi} \ell n\left(\dfrac{b}{a}\right) i_1 \quad = \left[\dfrac{\mu_0 IV}{2\pi} \ell n\left(\dfrac{b}{a}\right)\right]^2 \dfrac{V}{R}$

4. $\phi = BA = \dfrac{\mu_0 i r^2}{2(R^2+x^2)^{3/2}} \pi R^2$

$e = \dfrac{d\phi}{dt} = -\dfrac{3\mu_0 i r^2 R^2 \pi x}{(R^2+x^2)^{5/2}}$

For e to be maximum

$\dfrac{d(e)}{dx} = 0 \Rightarrow x = \dfrac{R}{2}$

5. $\phi \propto I \quad \Rightarrow LI = \phi \Rightarrow i = \dfrac{\Delta\phi}{L} \Rightarrow i = \dfrac{2B\pi R^2}{L}$

6. $e = \dfrac{B\omega L^2}{2}$

effective length of the wire frame is 2R

$\Rightarrow e = 2B\omega R^2$

7. $\omega = \dfrac{1}{2\pi}\dfrac{1}{\sqrt{LC}} \Rightarrow \omega = \dfrac{1}{6\pi\sqrt{LC}}$

8. $\tan 60 = \dfrac{X_C}{R}$ Also $\tan 60 = \dfrac{X_L}{R}$ is $X_C = X_L = \sqrt{3}\,R$

series L–C–R is in resonating condition

$I = \dfrac{V}{R} = 2A$

$P = I^2R = 4 \quad 100 = 400W$

9. $P = \dfrac{V^2}{R} \Rightarrow R = \dfrac{V^2}{P} = 100\Omega$

with AC source

$P = \left(\dfrac{V}{Z}\right)^2 R \Rightarrow Z = 200\Omega$

$Z^2 = R^2 + \omega^2 L^2 \Rightarrow L = \dfrac{\sqrt{Z^2 - R^2}}{\omega} = \dfrac{\sqrt{3}}{\pi} H$

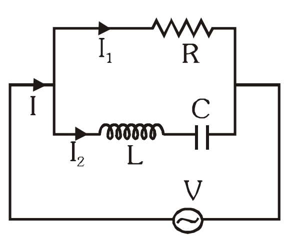
10. $\dfrac{i_1}{i_2} = \dfrac{1}{4} \Rightarrow \dfrac{W_1}{W_2} = \dfrac{1}{4}; \Rightarrow \dfrac{V_1}{V_2} = 4$

11. Minimum value of impedance is for

$X_L = X_C \Rightarrow \dfrac{1}{\omega C} = \omega L \Rightarrow L = \dfrac{1}{\omega^2 C} = 0.36$ mH

12. In DC (act as open circuit)

So $R = \dfrac{250}{1} = 250\ \Omega$



At resonance $\omega L = \dfrac{1}{\omega C}$

$\dfrac{1}{4500C} = 4500L$ ...(i)

$V = V_C - V_L = V_R$

$I_2\left(\dfrac{1}{\omega C} - \omega L\right) = V \Rightarrow I_1 R = V$

Here $I = \sqrt{I_1^2 + I_2^2} \Rightarrow I^2 = \dfrac{V^2}{R^2} + \dfrac{V^2}{\left(\dfrac{1}{\omega C} - \omega L\right)^2}$

$\Rightarrow \dfrac{(5/4)^2}{(250)^2} = \dfrac{1}{(250)^2} + \dfrac{1}{(1/\omega C - \omega L)^2}$

$\dfrac{1}{\omega C} - \omega L = \dfrac{4}{3} \times 250$ here $\omega = 2250$ rad/sec

So, $\dfrac{1}{2250C} = \dfrac{4}{3} \times 250 + 2250L$ ...(ii)

From (i) & (ii) :

$2 = \dfrac{\dfrac{4}{3} \times 250 + 2250L}{4500L}$

$9000L = \dfrac{4}{3} \times 250 + 2250L$

$L = \dfrac{4}{3} \times \dfrac{250}{6750} = 0.0494$ H

$C = \dfrac{1}{(4500)^2 \times 0.0494} = 10^{-6}$ F

13. $\phi = \left[\dfrac{\mu_0 (e/T)}{2R}\right]\pi r^2 \Rightarrow \dfrac{d\phi}{dt} = \dfrac{\mu_0 e \pi r^2}{2RT^2}$

$\Rightarrow e = \dfrac{\mu_0 e \pi r^2 \alpha}{2R(2\pi)} = \dfrac{\mu_0 e \alpha r^2}{4R}$

14. $e = \dfrac{AdB}{dt}$

$10 = (10 \times 10^{-2})^2 \dfrac{(4-0)}{\Delta t} \Rightarrow \Delta t = 20$ ms

15. $\phi = \vec{B}.\vec{A} \Rightarrow \phi = \int \dfrac{\mu_0 i_e e^{-t/\tau}}{2\pi x} b\, dx$

$\Rightarrow \phi = \dfrac{\mu_0 i_0 e^{-t/\tau}}{2\pi} b\ \ell n\left(\dfrac{a+d}{d}\right)$

$\Rightarrow e = \dfrac{d\phi}{dt} = \dfrac{\mu_0 i_0 e^{-t/\tau} b}{\tau 2\pi} \ell n\left(\dfrac{a+d}{d}\right)$

16. $\phi = at(T - t) \Rightarrow \dfrac{d\phi}{dt} = aT - 2at$

$\Rightarrow i = \dfrac{d\phi}{Rdt} = (aT - 2at)$

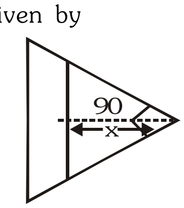
Heat $= \int_0^T (aT - 2at)^2 R dt$

$= \left[a^2T^2 t + 4a^2 \dfrac{t^3}{3} - 4a^2 T \dfrac{t^2}{2}\right]_0^T R$

$= \left[a^2T^3 + \dfrac{4a^2T^3}{3} - 2a^2T^3\right] R$ Heat $= \dfrac{a^2T^3}{3R}$

17. Area in the magnetic field is given by

$A = x^2$; $\phi = Bx^2$

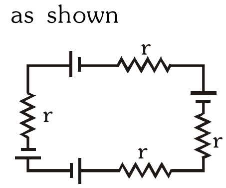


$\dfrac{d\phi}{dt} = 2Bx\left(\dfrac{dx}{dt}\right) = 2Bvx$

$i = \dfrac{1}{R}\dfrac{d\phi}{dt} = \dfrac{2Bv^2 t}{R}$, $i \propto t$

18. No induced current in the loop therefore No force acts over the loop. Work done on the loop will be zero

19. Equivalent circuit diagram is as shown



$\Rightarrow i = \dfrac{4e}{4r} = \dfrac{e}{r} = \dfrac{Bv\ell}{r\ell}$

$\Rightarrow i = \dfrac{Bv}{r}$

20. $e = B\ell v$

where all the three should be perpendicular to each other

$$e = \vec{B}(\vec{\ell} \times \vec{v}) \Rightarrow e = \left(3\hat{j} + 4\hat{k}\right)\ [(3\hat{i} + 4\hat{j}) \times 2\hat{i}]$$

$$= (3\hat{j} + 4\hat{k})(8\hat{k}) \ \Rightarrow e = 32 \text{ volt}$$

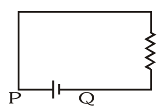
21. As capacitor blocks the current there will be no current in the circuit HKDE

22. $e = \dfrac{B\omega\ell^2}{2}$

effective length for the given diagram is

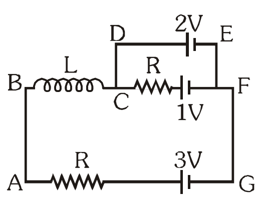
$l_{eff}^2 = \ell^2 + L^2 \Rightarrow e = \dfrac{B\omega(\ell^2 + L^2)}{2}$

23. By flemings left hand rule equivalent circuit diagram is as shown current will be P to Q In the disc

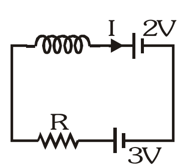


24. $e = L\dfrac{di}{dt} \Rightarrow 2 = \dfrac{4(5-0)}{\Delta t} \ \Rightarrow \Delta t = 10\text{sec}$

25. In loop CDEF current is independent of the time as current will remain $\dfrac{'V'}{R}$ all the time



$\therefore$ Circuit equivalent can be considered as current at any time $= \dfrac{V_0}{R}[1 - e^{-\frac{Rt}{L}}]$



By super imposing the two current the net current in it will be

$$i = \frac{V_0}{R} - \frac{V_0}{R}e^{-Rt/L} - \frac{V_0}{R} = -\frac{V_0}{R}e^{-Rt/L}$$

26. Decay of current in L–R circuit is given by

$$-\frac{Ldi}{dt} - iR = 0$$

$$i = -\frac{L}{R}\frac{di}{dt} \Rightarrow i = i_0(1 - e^{\frac{R_2 t}{L}})$$

Heat (ऊष्मा) $= \int L^2 R_2 dt = \dfrac{E^2}{R_1^2}R$

OR

Heat produced = energy stored in inductor

$$= \frac{1}{2}L\left(\frac{E}{R_1}\right)^2 = \frac{LE^2}{2R_1^2}$$

27. $u = \dfrac{1}{2}LI^2 \Rightarrow u = 32\text{Joule}$

L= ?, I = 4A

L = 4H, P = $i^2R \Rightarrow R = 20$ ohm

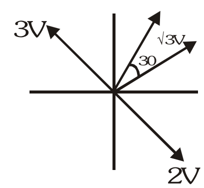
Time (समय)= $\dfrac{L}{R} = 0.2$ sec

29. As current in the circuit is given by $\dfrac{V_0}{R}$

$\Rightarrow X_L = X_C = \omega L = 100\pi \quad \dfrac{1}{\pi} = 100\Omega$

$\cos\theta = \dfrac{R}{Z} = \dfrac{1}{\sqrt{2}}$

30. $V = 2V_0\cos\left(\omega t + \dfrac{\pi}{6}\right)$ as voltage across inductor is more circuit behaves like an inductive circuit is current lags voltage by an amount 30 .



$I = I_0 \cos\left(\omega t - \dfrac{\pi}{6}\right)$

31. For (L – R) circuit

$\cos\theta = \dfrac{3}{5} \Rightarrow \theta = 53 \ \Rightarrow X_L = \dfrac{4}{3}R$

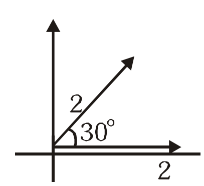
For (C – R) circuit

$\cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60 \ \Rightarrow X_C = \sqrt{3}\,R_2$

In L–C–R power factor is *i* i.e. reactance is zero

$X_L = X_C \Rightarrow \dfrac{4}{3}R_1 = \sqrt{3}R_2 \Rightarrow \dfrac{R_2}{R_1} = \dfrac{4}{3\sqrt{3}}$

32. $I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos 30}$



$$= \sqrt{4 + 4 + 8 \times \frac{\sqrt{3}}{2}}$$

$\Rightarrow I_0 = 2\sqrt{2+\sqrt{3}} \quad I_{eff} = \dfrac{I_0}{\sqrt{2}} \Rightarrow \sqrt{\dfrac{2+\sqrt{3}}{2}}$

33. $I_{rms} = \int I_0^2 \sin\omega t \ R dt \Rightarrow I_1 = I_2$

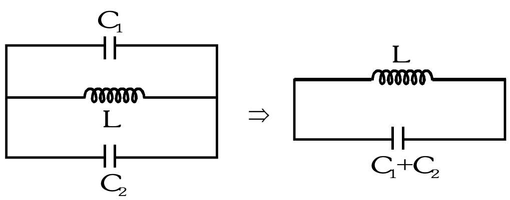
and current In the I_3 more than $I_1 = I_2$

$I_3 > I_1 = I_2 > I_4$

34. If I_2=0 and P moves towards Q, then according to Lenz law a current in opposite as I_1 is induced in Q. Same as I_1=0 and Q moves towards P when $I_1 \neq 0$, $I_2 \neq 0$ are in opposite direction then the coils repels each others.

35. No emf will be induced in any direction of its motion $\left(\vec{\ell} \| \vec{B}\right)$

36.



We know $q=q_m\cos\omega t$

$i = q_m\omega\cos\left[\omega t + \frac{\pi}{2}\right]$, $i = i_0\cos\left(\omega t + \frac{\pi}{2}\right)$

Maximum current $i_0 = q_m\omega \left[\omega = \frac{1}{\sqrt{L(C_1+C_2)}}\right]$

Maximum charge on $(C_1 + C_2)$

$$q_m = \frac{i_0}{\omega} = i_0\sqrt{L(C_1+C_2)}$$

Maximum charge on C_1

$$= \frac{C_1}{C_1+C_2} \times i_0\sqrt{L(C_1+C_2)} = i_0C_1\sqrt{\frac{L}{C_1+C_2}}$$

37. Sudden increase in the e.m.f. cause the spark in the inductor

38. In absence of L whole emf of B goes on lamp and lamp will glow with full brightness instantaneously but in presence of L some emf is induced in L. Voltage drop on P decrease and brightness $\downarrow$.

39. $\phi_1 = MI_2$, $4=2I_2 \Rightarrow I_2=2A$

40. $I = \frac{V}{Z}$

$X_L = 2\pi 50 \quad 2 = 200\pi$

$X_L = 2\pi(400) \quad 2 = 1600\pi$

Z increases 8 times current decreases by I/8

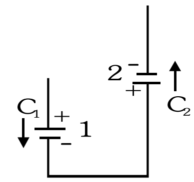
41. I_α is the induced current of i_1, so the nature of i_1 become should becomes opposite of i_α ie negative.

42. Charge on the capacitor $Q = CV = C \quad (Bv_0\ell)$

As all the quantities are constant

so $dQ = 0$ Hence $i = \frac{dQ}{dt} = 0$

43. Replace the induced emf's in the rings by cell



$e_1 = B(2r)(2V) = 4BrV$

$e_2 = B(4r)(V) = 4BrV$

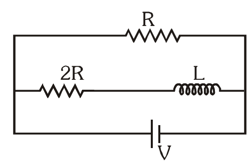
$V_2 - V_1 = e_2 + e_1 = 8BrV$

44. $e = -\frac{d\phi}{dt} = -\frac{d}{dt}[BA\cos\theta] = -\frac{d}{dt}\left(B_0\left(1+\frac{x}{a}\right)\right)d^2$

[$\because$ Area of square = d^2, $\theta = \infty$]

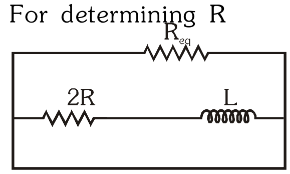
$= \frac{B_0d^2}{a}\frac{dx}{dt} = \frac{B_0d^2}{a}V_0 \quad (\vec{V} = V_0\hat{i})$

46. Charging



$\tau_1 = \frac{L}{R_{eq}}$

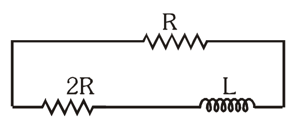
For determining R



$R_{eq} = 2R$

$\tau_1 = \frac{L}{2R}$

Discharging



$\tau_2 = \frac{1}{3R} \Rightarrow \frac{\tau_1}{\tau_2} = \frac{3}{2}$

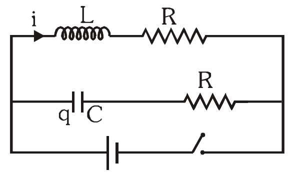
47. Energy per unit volume

$$= \frac{B^2}{2\mu_0} = \int\frac{1}{2\mu_0}\left(\frac{\mu_0 ir}{2\pi a^2}\right)^2 2\pi r dr$$

$$= \frac{1}{2\mu_0}\frac{\mu_0^2 i^2 r^2}{4\pi^2 a^4} \times 2\pi r dr = \frac{\mu_0 i^2}{4\pi a^4}\int_0^a r^3 dr$$

$$= \frac{\mu_0}{4\pi a^4} \times \frac{a^4}{4} = \frac{\mu_0 i^2}{16\pi}$$

48. Current through C and L would be equal after a time f when $\frac{1}{2}Li^2 = \frac{q^2}{2C}$



$i\sqrt{LC} = q$

$q = q_0\left(1 - e^{-t/Rc}\right)$, $i = \dfrac{q_0}{RC}e^{-t/RC}$

$\dfrac{q_0}{RC}e^{-t/RC}\sqrt{LC} = q_0\left(1 - e^{-t/RC}\right)$

$\dfrac{\sqrt{L/C}}{R}e^{-t/RC} = 1 - e^{-t/RC}$

$2e^{-t/RC} = 1 \Rightarrow e^{t/RC} = \ell n2$

$\Rightarrow \dfrac{t}{RC} = \ell n2 \Rightarrow t = RC\ell n2$

49. $I_0 = 1.57$

Instantaneous voltage across capacitor

$E = V_0\sin\omega t,\ V_0 = XI_0 = \dfrac{1}{2\pi fc}\times I_0$

$= \dfrac{1}{2\pi\times 50\times 100\times 10^{-6}}\times 1.57 \Rightarrow V_0 = 50$

$\therefore\ E = 50\sin 100\pi t$

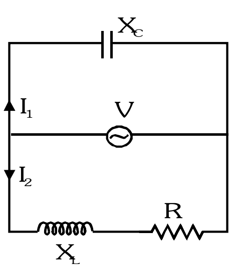
50. $I = \sqrt{I_1^2 + I_2^2}$

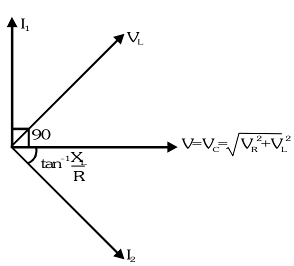
$I^2 = \dfrac{(200)^2}{(100)^2} + \dfrac{V^2}{\left(X_L - X_C\right)^2}$

$= 4 + \dfrac{(200)^2}{(100)^2} = 4 + 4 = 8$

$\Rightarrow I = 2\sqrt{2} \Rightarrow I_{rms} = \dfrac{I}{\sqrt{2}} = 2$

51.





So phase difference between I_1 and I_2

$= \dfrac{\pi}{2} + \tan^{-1}\dfrac{X_L}{R}$

52. Time constant of L–R circuit is $= \dfrac{L}{R}$

From option (A) $= 2\times\dfrac{\frac{1}{2}Li^2}{i^2R} = \dfrac{L}{R}$

53. The electric field force due to variable magnetic field$= \dfrac{1}{2}\times\vec{R}\times\dfrac{d\vec{B}}{d\ell}\times q = \dfrac{1}{2}eR\dfrac{dB}{d\ell}$

$\therefore$ Acceleration $= \dfrac{1}{2m}eR\dfrac{dB}{dt}$

54. Induced current

$I = \dfrac{e}{R} = \dfrac{1}{R}\left(-\dfrac{nd\phi}{dt}\right) = \dfrac{-n}{5R}\dfrac{(\omega_2 - \omega_1)}{t}$

56. $L = \dfrac{\mu_0 N^2 A}{L}$; N $2\pi r = L$

As wire is fixed

$\therefore\ N'\ 2\pi\ \dfrac{r}{2} - L = N\ 2\pi r \Rightarrow N' = 2N$

$\therefore\ i = \mu_0\ \dfrac{(2N)^2\times\pi\left(\frac{r}{2}\right)^2}{\frac{L}{2}} \Rightarrow L' = 2L$

58. $V_{rms} = \sqrt{\dfrac{\int_0^{T/4}V^2dt}{\int_0^{T/4}dt}} = \sqrt{\dfrac{\int_0^{T/4}\left(\frac{4V_0}{T}t\right)^2dt}{\int_0^{T/4}dt}}$

$= \sqrt{\dfrac{\frac{4\times 4V_0^2}{T^2}\times\frac{T^3}{16\times 4\times 3}}{\frac{T}{4}}} = \dfrac{V_0}{\sqrt{3}}$

59. Average value $= \dfrac{\text{Area}}{\text{time}} = 0$

(in 0–T interval)

60. $I = \dfrac{V}{\sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2}}$ & $\dfrac{I}{2} = \dfrac{V}{\sqrt{R^2 + \frac{1}{\left(\frac{\omega c}{3}\right)^2}}}$

$\therefore\ (2)^2 = \dfrac{R^2 + \frac{9}{\omega^2c^2}}{R^2 + \frac{1}{\omega^2c^2}} \Rightarrow 4R^2 + \dfrac{4}{\omega^2c^2} = R^2 + \dfrac{9}{\omega^2c^2}$

$\Rightarrow 3R^2 = \dfrac{5}{\omega^2c^2} \Rightarrow 3R^2 = 5X_c^2 \Rightarrow \sqrt{\dfrac{3}{5}} = \dfrac{X_c}{R}$

EXERCISE –III

Match the column–I

$V = 100\sin(100t) \Rightarrow i = 10\sin\left(100t - \dfrac{\pi}{4}\right)$

(A) Phase difference $\left(\dfrac{\pi}{4}\right)$

$\cos\theta = \dfrac{R}{Z} \Rightarrow Z = \sqrt{2}R$; $\tan\theta = \dfrac{X_L}{R} \Rightarrow X_L = R$

(B) $I_0 = \dfrac{V_0}{Z} \Rightarrow Z = 10\ \Omega$ $Z = \sqrt{R^2 + X_L^2}$

$\sqrt{R^2 + R^2} = 10 \Rightarrow R = 5\sqrt{2}\ \Omega = X_L$

$X_L = \omega(L) = 5\sqrt{2} \Rightarrow L = \dfrac{1}{10\sqrt{2}}$

(C) Average power = VL cos θ

$= \dfrac{100 \times 10}{2} \times \dfrac{1}{\sqrt{2}} = 250\sqrt{2}$

Match the column II

Peak value current in the circuit is given by

$$i_0 = \frac{V}{R_0}$$

⇒ If R will be less current will be maximum

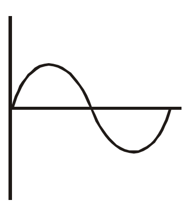
Slope of i (v/s) t graph gives time constant is $\dfrac{L}{R}$

(A) Graph III and IV denotes for 'R_0' and slope is more for III therefore R_0, L_0 represent III and (IV) represent $R_0, 2L_0$

(B) Similarly I and II denotes for '$2R_0$' and slopes is more for 'I'

Match the column III

(A) $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

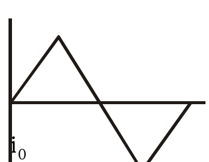


I_{mean} for full cycle = 0

I_{mean} for half cycle = 0

I_{mean} for half cycle $= \dfrac{2I_0}{\pi}$

(B) $i = \dfrac{4i_0}{T}t$

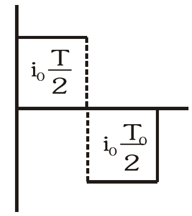


$i_r = \dfrac{16I_0^2}{T^2}\displaystyle\int_0^{T/2} t^2 R\,dt \Rightarrow i_{rms} = \dfrac{i_0}{\sqrt{3}}$

$i_{mean} = \dfrac{\text{Total Area}}{\text{Total time}} \Rightarrow i_{mean} = 0$ for full cycle

$\Rightarrow i_{mean} = \dfrac{i_0 T_0 2}{4T_0} = \dfrac{i_0}{2}$ [for half cycle]

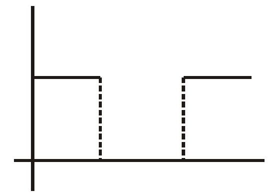
(C) $i_{mean} = \dfrac{\text{Total Area}}{\text{Total time}} = \dfrac{0}{T}$



$\Rightarrow i_{mean} = 0$

$\Rightarrow i_{mean} = \dfrac{i_0 T_0/2}{T_0/2} = i_0$

(D) $i_{mean} = \dfrac{\text{Total Area}}{\text{Total time}}$

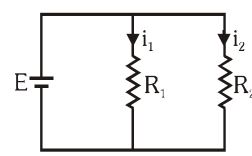


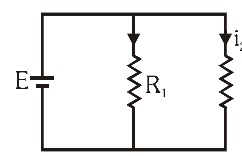
$i_{mean} = \dfrac{i_0 T_0/2}{T_0}$

$i_{mean} = \dfrac{i_0}{2} \Rightarrow i_{half\ cycle} = \dfrac{i_0 T/2}{T/2} = i_0$

Match the column (IV)

at t = 0 at t = ∞ $i_1 \dfrac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}t}\right)$





$i_1 = 0$ $I_2 = \dfrac{E}{R_2} = \dfrac{18}{3} = 6A$

$i_2 = \dfrac{E}{R_2} = \dfrac{18}{6} = 3A$ $I_1 = \dfrac{18}{6} = 3A$

Match the column V

(A) $\tan\theta = \dfrac{X_C}{R} \Rightarrow \dfrac{1}{\omega CR} = \dfrac{1}{10 \times 10^4 \times 10^{-6} \times 10}$

$\tan\theta = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

(B) $\tan\theta = \dfrac{X_L}{R}$ as $R = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

(C) $\tan\theta = \dfrac{X_C}{R}$ as $R = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

(D) $\tan\theta = \dfrac{X_C - X_L}{R}$ as R=0 $\Rightarrow \theta = \dfrac{\pi}{2}$

(E) $\tan\theta = \dfrac{X_L}{R} = \dfrac{200 \times 5}{1000} \Rightarrow \theta = \dfrac{\pi}{4}$

Match the column VI

(A) $\dfrac{d\phi}{dt} = 5 \Rightarrow i = \dfrac{5}{10} \Rightarrow \dfrac{1}{2}$ Anticlockwise

(B) $\dfrac{d\phi}{dt} = 0 \Rightarrow i = 0$ = zero

(C) $\frac{d\phi}{dt} = -5 \Rightarrow i = -\frac{5}{10} = -\frac{1}{2}$ = Clockwise

(D) $\frac{d\phi}{dt} = 5 \Rightarrow$ Anticlockwise

Compreshension–1

1. $f = Bi\ell \Rightarrow f = \frac{B^2\ell^2 v}{R}$

$\Rightarrow 3.2 \quad 10^{-5} = \frac{4\times10^{-4}\times 64\times10^{-4}\times v}{2}$

$\Rightarrow \frac{1}{40} \quad 10^3 = v \Rightarrow 25$ m/s

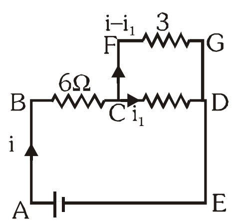
2. $e = B\ell v = 2 \quad 10^{-2} \quad 8 \quad 10^{-2} \quad 25 = 4 \quad 10^{-2}$V

3. $V = E - ir \Rightarrow V = B\ell v - \frac{B\ell v}{R}(R) \Rightarrow V = 3.6 \quad 10^{-2}$V

Comprehension–2

1. By applying K.V.L.

For ABCDE



$\Rightarrow 18 - 6i \quad \frac{di_1}{dt} = 0$ ...(i)

By applying K.V.L.

for ABCFGE

$\Rightarrow 18 - 6i - 3(i - i_1) = 0 \Rightarrow 18 - 9i + 3i_1 = 0$

$\Rightarrow i = \frac{18+3i_1}{9}$ ...(ii)

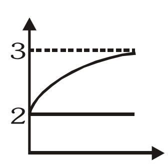
Substituting its value in equation (i)

$\Rightarrow 18 - 6\left[\frac{18+3i}{9}\right] - \frac{di_1}{dt} = 0 \Rightarrow 18 - 12 - 2i_1 = \frac{di_1}{dt}$

$\Rightarrow 2\int_0^t dt = \int_2^{i_1} \frac{di_1}{3-i_1} \Rightarrow -2t = \ell n\left(\frac{3-i_1}{3}\right) - \ell n(1)$

$\Rightarrow 3 - i_1 = e^{-2t} \Rightarrow i_1 = 3 - e^{-2t}$

2. $V_1 + V_2 = V \Rightarrow 18 - 6e^{-2t} + V_2 = 18 \Rightarrow V_2 = 6e^{-2t}$

4. 

Comprehension–3

1. M is same as that of L

$\frac{k}{m}$ is same as that of $\frac{L}{LC}$

k is same as that of $\frac{1}{C} \quad mk \rightarrow \frac{L}{C}$

Comprehension–4

1. $q = q_0 \sin(\phi)$ ...(1)

$\frac{dq}{dt} = q_0\omega \cos(\phi) \Rightarrow \sqrt{5} = q_0\ \omega \cos(\omega t + \phi)$

By dividing (1) and (2) we get

$\frac{4}{\sqrt{5}} = \frac{1}{\omega}\tan(\omega t + \phi)\ \tan(\phi) = \frac{2}{\sqrt{5}}$

from above equation $\sin(\omega t + \phi) = \frac{2}{3}$

$Q = Q_0 \sin(\omega t + \phi) \Rightarrow 4 = Q_0\left(\frac{2}{3}\right) \Rightarrow Q_0 = 6C$

2. $(\omega t + \phi) = \frac{\pi}{2} \Rightarrow \omega t = \frac{\pi}{2} - \phi \Rightarrow t = 2\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{2}{3}\right)\right]$

Comprehension–5

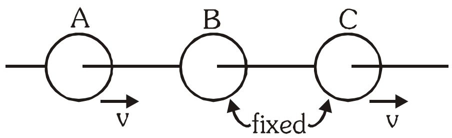
1. $\phi_1 = B(L^2 + \ell^2)$ as current in both direction are additive in nature while $\phi_2 = B(L^2 - \ell^2)$ as current in both the loops are in opposite direction.

2. By lenz law in both the loop are in clock wise direction therefore it flours from b to a and d to c in both the loop.

3. Again by lenz law current in both direction should be clockwise but it is not possible therefore it is clockwise in bigger loop and in anticlockwise in smaller loop as e.m.f. due to bigger one is greater than smaller one

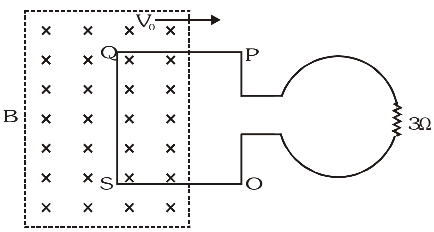
4. $I_1 > I_2$

EXERCISE –IV (A)

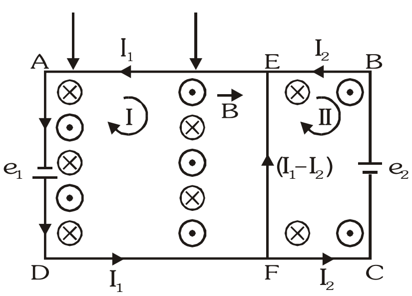
1. 

Due to current in C no change in ϕ of B. So no induced current in B. But due to I in A, ϕ is changing in B because A is moving towards B. ϕ is changing (increasing) in B.

So according to Lenz's law direction of induced current in B will be such that it will try to decrease the ϕ in B so current will be opposite in direction in B than A.

2. 

$$e = Bv\ell = IR \Rightarrow v = \frac{IR}{B\ell} = \frac{(1\times10^{-3})(4)}{(2)(10\times10^{-2})} = 0.02\ ms^{-1}$$

3. 

Refer to figure Electromotive force (emf) is induced in circuits I and II due to change of magnetic flux threading the circuits because magnetic field B is changing with time. As the areas enclosed by the circuits remain unchanged, the magnitude of the induced emf is given by

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = \frac{AdB}{dt}$$

Area enclosed by circuit I is

A_1 = AD AE = 1m 1m = 1m^2.

Therefore, the emf induced in circuit I is

e_1 = 1m^2 1 Ts^{-1} = 1 Tm^{2}s^{-1} = 1V

Area enclosed by circuit II is

A_2 = EB EF = 0.5 m 1m = 0.5 m^2

$\therefore$ Induced emf in circuit II is

e_2 = 0.5 m^2 1 Ts^{-1} = 0.5 V

Let I_1 and I_2 be the induced currents in circuits I and II respectively. From Lenz's law, the directions of these currents must be such that they oppose the increase in currents. In other words, the directions of the current in circuits I and II must be such that they produced a magnetic field which is normal to the plane of the paper but point upwards, i.e. towards the reader. This requires that current I_1 and I_2 flow in the directions shown in the figure.

Since the resistance per unit length is 1 Ωm^{-1}, the resistance of wires AD, AE, DF and EF are 1 Ω each and those of wires EB, BC and FC are 0.5 Ω each. Applying Kirchoff's loop rule to loop I (AEFDA),

$$I_1\ \ 1 + (I_1 - I_2)\times1 + I_1\times1 + I_1\times1 - e_1 = 0$$

or $4I_1 - I_2 = e_1$ = 1 volt..........(1)

Applying Kirchhoff's loop rule to loop I (BCFCB), we have

$e_2 - I_2\ \ 1 - I_2\ \ 0.5 - (I_1 - I_2)\ \ 1 - I_2\ \ 0.5 = 0$

or $3I_2 - I_1 = e_2$ – 0.5 volt ....(2)

Solving equation (1) and (2), we get

$$I_1 = \frac{7}{22} \text{ and } I_2 = \frac{3}{11}\text{A.}$$

Referring to figure, the current in segment AE = I_1 = $\frac{7}{22}$ A in the direction from E to A, the current in segment BE = I_2 = $\frac{3}{11}$ A in the direction from B to E and the current in segment EF = $I_1 - I_2$ = $\frac{7}{22}$ = $\frac{3}{11}$ = $\frac{1}{22}$ A in the direction from F to E.

4. $$\because q = \frac{\Delta\phi}{R} = \frac{NBA}{R} \quad \therefore B = \frac{qR}{NA}$$

$$= \left(\frac{\pi}{100}\right)\left(\frac{895+5}{100\times\pi(3\times10^{-2})^2}\right) = 10^2\ T$$

5. Induced emf in coil $e = M\frac{dI}{dt}$

Mutual inductance of system $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

where (जहाँ) $n = \frac{N_1}{\ell}$, $N_2 = N, A = \pi R^2$

$\Rightarrow$ M = μ_0 n N (πR^2)

Therefore $e = \mu_0 nN(\pi R^2)\frac{d}{dt}(I_0 \sin\omega t)$

$\Rightarrow \mu_0 n\ N\ (\pi R^2)\ I_0\omega\cos\omega t$

6. (i) Clockwise B$\uparrow$, $\phi\downarrow$
(ii)Anticlockwise B$\downarrow$, $\phi\downarrow$
(iii) Anticlockwise A$\downarrow$, $\phi\downarrow$
(iv) Clockwise A$\uparrow$, $\phi\uparrow$

7. Induced emf in the loop = $B_1 v\ell - B_2 v\ell = (B_1 - B_2)$

$$v\ell\left[\frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(x+a)}\right] (v)\ (a) = \frac{\mu_0 I}{2\pi}\left(\frac{a^2 v}{x(x+a)}\right)$$

$= \dfrac{2\times10^{-7}\times50\times(0.1)^2\times10}{0.2\times0.3} = \dfrac{50}{3}\,\mu V$

8. (i) Maximum Current

$$I_{max} = \frac{e_{max}}{R} = \frac{NBA\omega}{R} = \frac{50\times0.3\times2.5\times60}{500} = 4.5A$$

(ii) Flux is maximum when plane of coil is at 90 to the magnetic field.Flux is zero when plane of coil is at 0 to the magnetic field.

(iii) Yes it will work because ϕ related to coil continuous in change.

9 Induced emf in primary coil

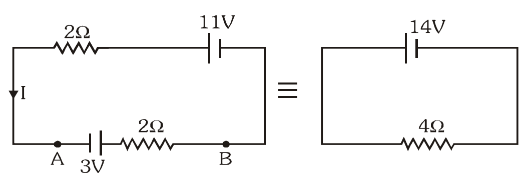
$$E_p = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_0 + 4t) = 4 \text{ volt}$$

Induced emf in secondary coil

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

$$\Rightarrow E_s = \left(\frac{N_s}{N_p}\right)E_p = \left(\frac{5000}{50}\right)(4) = 400 \text{ volt}$$

10. (i)

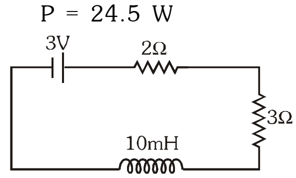


$I = \dfrac{1}{4}A$ Power in R_1

$V_A + 3 - \dfrac{7}{2}\times2 - V_B = 0$ $P_1 = I^2R = \left(\dfrac{7}{2}\right)^2\times2$

$\Rightarrow V_A - V_B = 4V$ P = 24.5 W

(ii) $i_0 = i_0\left(1-e^{-t/\tau}\right)$



$$i_0 = \frac{3}{2+3} = 0.6A$$

$$\tau = \frac{L}{R_{eq}} = \frac{10\times10^{-3}}{5} = 2\times10^{-3}\text{ sec}$$

(A) $i_0 = 0.6$ A

(B) $\dfrac{i_0}{2} = i_0\left(1-e^{-t/\tau}\right) \Rightarrow \dfrac{1}{2} = 1 - e^{-t/\tau}$

$$\Rightarrow e^{-t/\tau} = \frac{1}{2} \Rightarrow t/\tau = \ell n2$$

$t = \tau\ell n2 \Rightarrow t = 2 \quad 10^{-3} \quad 0.693$

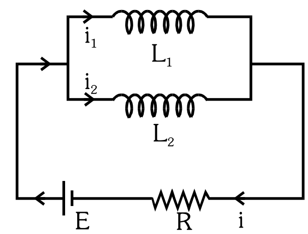
$\Rightarrow t = 1.38 \quad 10^{-3}$ sec

Energy stored in L :

$$H = \frac{1}{2}Li^2 = \frac{1}{2}\times10\times10^{-3}\left(\frac{0.6}{2}\right)^2$$

$\Rightarrow H = 4.5 \quad 10^{-4}$ J

11. Here $V_{L_1} = V_{L_2}$



$$\Rightarrow L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$$

$$\Rightarrow L_1i_1 = L_2i_2$$

But $i_1 + i_2 = \dfrac{E}{R} = i \Rightarrow i_1 = \left(\dfrac{L_2}{L_1+L_2}\right)\left(\dfrac{E}{R}\right)$

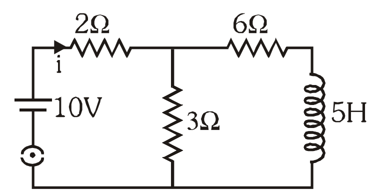
Hence

(i) Current in $L_1 = \left(\dfrac{L_2}{L_1+L_2}\right)\left(\dfrac{E}{R}\right)$

(ii) Current in $R = \dfrac{E}{R}$

12. (i) At t=0

L act as open circuit



So $i = \dfrac{10}{2+3} \Rightarrow i = 2$ A

(ii) After some time

L act as short circuit

$$i = \frac{10}{2+\frac{6\times3}{6+3}} = \frac{10}{2+2} \Rightarrow i = 2.5A$$

13. (i) After 100 ms wave is repeated so time period is T = 100 ms. $\Rightarrow f = \dfrac{1}{T} = 10$ Hz

(ii) Average value = Area/time period

$$= \frac{(1/2)\times100\times10}{(100)} = 5 \text{ volt}$$

14. $I_{rms} = \sqrt{<\left(I_1\cos\omega t + I_2\sin\omega t\right)^2>}$

$$= \sqrt{I_1^2\left\langle\cos^2\omega t\right\rangle + I_2^2\left\langle\sin^2\omega t\right\rangle + 2I_1I_2\left\langle\sin\omega t\cos\omega t\right\rangle}$$

$$= \sqrt{I_1^2\left(\frac{1}{2}\right) + I_2^2\left(\frac{1}{2}\right) + 2I_1I_2(0)} = \sqrt{\frac{I_1^2+I_2^2}{2}}$$

15. (i) $T = \dfrac{2\pi}{\omega} = \dfrac{2\times3.14}{628} = \dfrac{1}{100} = 0.01$ s

(ii) 60 (30 – C– 30)

16. (i) Impedance

$$Z = \frac{V_0}{I_0} = \frac{110}{5} = 22\Omega \quad \text{(ii)}$$

Power factor

$= \cos\phi = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ (lagging)

17. $R = 100\ \Omega$; $f = 1000$ Hz, $\phi = 45$

$\tan\phi = \frac{X_L}{R} \Rightarrow X_L = R\tan\phi = 100 \quad 1 = 100\ \Omega$

$\because X_L = 2\pi f L = 100 \quad \therefore L = \frac{100}{2\times 3.14\times 1000}$

$= 0.0159$ H $= 15.9$ mH

18. (i) X is resistor and Y is a capacitor
(ii) Since the current in the two devices is the same (0.5A at 220 volt)
When R and C are in series across the same voltage then

$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2+X_C^2}}$

$= \frac{220}{\sqrt{(440)^2+(440)^2}} = \frac{220}{440\sqrt{2}} = 0.35\text{A}$

19. (i) resistor (ii) inductor

20. (i) At resonance condition $X_L = X_C$

$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

(ii) $\because \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

$\therefore \phi = 0$

No, It is always zero.

21. (i) Impedance of the circuit

$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{(4)^2+(7-4)^2} = 5\Omega$

current flow in ckt.

$I_{rms} = \frac{V_{rms}}{Z} \quad \frac{25/\sqrt{2}}{5} = \frac{5}{\sqrt{2}}$ A

$\therefore$ Heat developed

$= I^2_{rms} Rt = \left(\frac{5}{\sqrt{2}}\right)^2 \quad 4 \quad 80 = 4000$ joule

OR

$H = Pt = (VI\cos\phi)\,t = \left(\frac{25}{\sqrt{2}}\cdot\frac{5}{\sqrt{2}}\times\frac{4}{5}\right) \quad 10 = 4000$

J $\left(\text{Use } \cos\phi = \frac{R}{Z}\right)$

(ii)Wattless current $= I_{rms}\sin\phi$

$= \frac{5}{\sqrt{2}} \quad \frac{3}{5} = 2.12$ A $\left(\because \sin\phi = \frac{X_L-X_C}{Z} = \frac{3}{5}\right)$

22. Impedance of circuit

$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{(45)^2+(4-4)^2} = 45\Omega$

Total current in circuit $I = \frac{V}{Z} = \frac{90}{45} = 2\text{A}$

(Reading of ammeter)
Voltmeter connect across L and C
so reading of voltmeter $= V_L - V_C$
Now $X_L = X_C \Rightarrow V_L = V_C$
So reading of voltmeter $= 0$

23. Power dissipation

$= VI\cos\phi = V.\frac{V}{Z}.\frac{R}{Z} = \frac{V^2R}{Z^2}$ ....(1)

$\left[\because \cos\phi = \frac{R}{Z}, I = \frac{V}{Z}\right]$

$V = 100$ V, $R = 10\ \Omega$, $Z = \sqrt{R^2+(X_L)^2} = 10\sqrt{2}\Omega$

$\left(\begin{matrix} X_L = \omega L = 2\pi\times 50\times\frac{1}{10\pi} \\ X_L = 10\Omega \end{matrix}\right)$

Put all these value in eq (1)

$P_{loss} = \frac{100\times 100\times 10}{(10\sqrt{2})^2} = 500$ watt

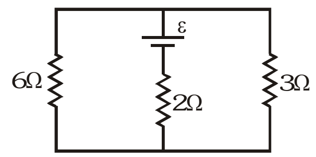
24. Mutual inductance

$M = \frac{\phi_s}{I_p} = \frac{B(\pi r^2)}{I} = \frac{\frac{\sqrt{3}\mu_0 I}{\pi a}(\pi r^2)}{I} = \frac{\sqrt{3}\mu_0 r^2}{a}$

25. $E = \vec{B}.\vec{L}\times\vec{v} = -B_0\hat{k}.\lambda\hat{i}\times(v_x\hat{i}+v_y\hat{j})$

$= -B_0\hat{k}.\lambda v_y\hat{k} = -\lambda v_y B_0$

26. $R_{eff} = \dfrac{3\times 6}{3+6} + 2 = 4\Omega$

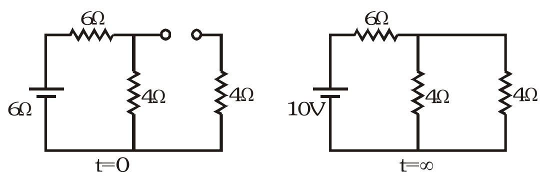


$\therefore\ I = \dfrac{\varepsilon}{R_{eff}} = \dfrac{BLv}{R_{eff}} = \dfrac{2\times 1\times 2}{4} = 1A$

$\therefore\ F = ILB = 1 \quad 1 \quad 2 = 2N$

27. $mg = ILB = \left(\dfrac{BLv_T}{R}\right)LB \Rightarrow v_T = \dfrac{mgR}{L^2B^2}$

28.



$I_1 = \dfrac{10}{6+4} = 1A$ $\qquad I_2 = \dfrac{10}{6+2} = \dfrac{5}{4}A$

$\therefore\ \dfrac{I_1}{I_2} = 0.8$

29. $I = \dfrac{E}{R}[1-e^{-t/\tau}]\left(\tau = \dfrac{L}{R}\right)$

$\therefore$ Charge passed through the battery

$$= \int Idt = \frac{E}{R}\int_0^{\tau}\left(1-e^{-t/\tau}\right)dt = \frac{E}{R}\left[t+\tau e^{-t/\tau}\right]_0^{\tau}$$

$$= \frac{E}{R}\left[(\tau+\tau/e)-(0+\tau)\right] = \frac{EL}{eR^2}$$

30. $\left(\dfrac{1}{2}Li^2\right) = \dfrac{1}{4}\left(\dfrac{1}{2}LI^2\right) \Rightarrow i = \dfrac{I}{2}$

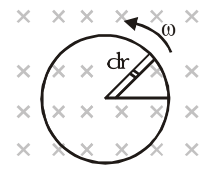
where $i = \dfrac{I}{2} = Ie^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R} \Rightarrow \dfrac{t}{\tau} = \ell n2$

$\therefore$ Charge flown $= \displaystyle\int_0^{t=\tau\ell n2} idt = \int (Ie^{-t/\tau})dt$

$$= I\tau\left[-e^{-t/\tau}\right]_0^{\tau\ell n2} = I\tau\left(\frac{1}{2}\right) = \frac{IL}{2R}$$

EXERCISE –IV (B)

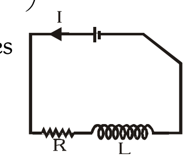
1.



(i) $e = \int de = \int_0^r B\omega r dr = \dfrac{1}{2}B\omega r^2$

(ii) $I = I_0\left(1-e^{-\frac{Rt}{L}}\right) = \dfrac{B\omega r^2}{2R}\left(1-e^{-\frac{Rt}{L}}\right)$

Now torque required for power loses

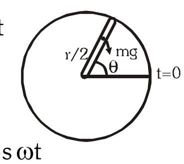


$\left(P = I^2R = \dfrac{B^2r^4\omega^2}{4R}\right)$

$\tau_1 = \dfrac{P}{\omega} = \dfrac{B^2r^4\omega^2}{4R\omega} = \dfrac{B^2r^4\omega}{4R}$

Torque required to move the rod in circular motion against gravitational field

$\tau_2\,(mg)\left(\dfrac{r}{2}\cos\theta\right) = \dfrac{1}{2}mgr\cos\omega t$

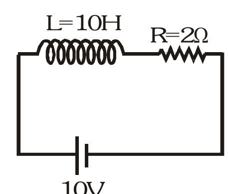


The total torque

$\tau = \tau_1 + \tau_2 = \dfrac{B^2r^4\omega}{4R} + \dfrac{1}{2}mgr\cos\omega t$

Direction of torque : clockwise

2. $I_0 = \dfrac{10}{2} = 5$



Magnetic energy $\dfrac{1}{2}LI^2$

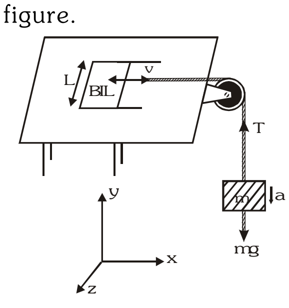
The current in the circuit for one forth of magnetic energy $I = \dfrac{I_0}{2}$

But $I = I_0\left(1-e^{-\frac{Rt}{L}}\right) \Rightarrow \dfrac{I_0}{2} = I_0\left(1-e^{-\frac{Rt}{L}}\right)$

$\Rightarrow e^{-t/5} = \dfrac{1}{2} \Rightarrow e^{t/5} = 2 \Rightarrow \dfrac{t}{5} = \ell n2$

$\Rightarrow t = 5\ell n2 = 5 \quad 0.693 = 3.47$ s

3. Refer to figure.



Let v be the velocity of the rod along the positive x–direction at an instant of time and let the magnetic field B act perpendicular to the table along the positive y–direction.

The emf induced in the rod is e = BLv.

Therefore, the induced current is

$$I = \frac{e}{R} = \frac{BLv}{R}$$

The rod of length L carrying a current I in magnetic field will experience a force F = BIL ...(2)

along the negative x–direction. Since the rod is massless, this force will also be equal to the tension T in the string acting along the positive x–direction, i.e.

$$T = F = BIL$$

Let a be the acceleration of mass m moving in the downward direction, then ma = net force acting on

$$m = mg - T = mg - F \text{ or } a = g - \frac{F}{m} \quad ...(3)$$

Using (1) , (2) and (3), we have

$$a = g - \frac{B\ell L}{m} = g - \frac{B \times BL^2 v}{mR} = g - \frac{B^2L^2v}{mR} \quad ....(4)$$

(i) The rod will acquire terminal velocity v_t when a = 0.

Putting a =0 and v = v_t in eq. (4)

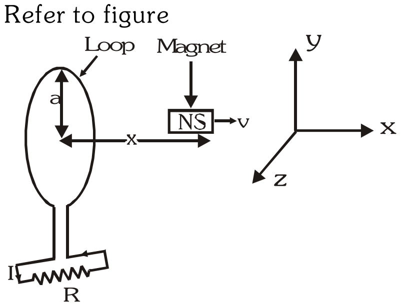
we have $0 = g - \frac{B^2L^2v_t}{mR}$ or $v_t = \frac{mgR}{B^2L^2}$

(ii) When the velocity of the rod is half the terminal velocity i.e. when $v = \frac{v_t}{2} = \frac{mgR}{2B^2L^2}$

then from equation (4), we have

$$a = g - \frac{B^2L^2v_t/2}{mR} = g - \frac{B^2L^2}{2mR}\;\frac{mgR}{B^2L^2} = g - \frac{g}{2} = \frac{g}{2}$$

4. Refer to figure



The magnetic field at distance x on the axis of a magnetic of length 2ℓ and dipole moment M is given by

$$B = \frac{\mu_0}{2\pi}\frac{Mx}{\left(x^2 - \ell^2\right)^2}$$

$\therefore$ x >> ℓ, we have $B = \frac{\mu_0 M}{2\pi x^3}$

Due to B, the flux through the loop is

$$\phi = BA = B\left(\pi a^2\right) = \frac{\mu_0 M}{2\pi x^3} \quad \pi x^2 = \frac{\mu_0 M a^2}{2x^2}$$

Induced emf in the loop is

$$e = -\frac{d\phi}{dt} = \frac{dx}{dt}\frac{d\phi}{dx} = v\frac{d\phi}{dx}$$

$$= \frac{\mu_0 M a^2 v}{2}\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{3}{2}\frac{\mu_0 M a^2 v}{x^4}$$

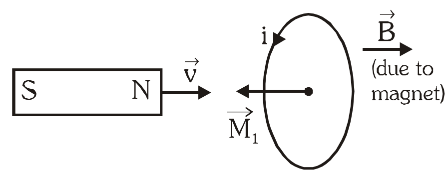
$\therefore$ Induced current int he loop is

$$I = \frac{e}{R}\frac{3}{2}\frac{\mu_0 M a^2 v}{x^4 R}$$

Magnetic moment of the loop is

$M_0 = I$ area enclosed by the loop = $I\left(\pi a^2\right)$

$$= \frac{3}{2}\frac{\mu_0 M a^2 v}{x^4 R}$$



Potential energy

$$U = -\vec{M}\cdot\vec{B} = -MB\cos 180° \Rightarrow U = M'B$$

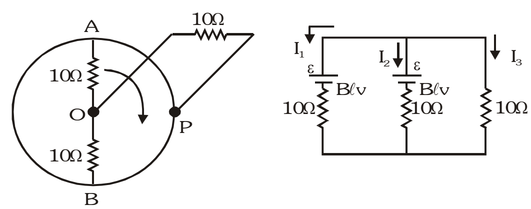
$$= \frac{3}{2}\frac{\mu_0 \pi M a^2 V}{Rx^4}\left(\frac{\mu_0}{2\pi}\cdot\frac{M}{x^3}\right)$$

$$U = \frac{3}{4}\frac{\mu_0 M^2 a^2 V}{R} \times \frac{1}{x^7}$$

$$F = -\frac{dU}{dx} \Rightarrow F = \frac{21}{4}\frac{\mu_0^2 M^2 a^4 V}{Rx^8}$$

This force is caused by the moving magnet. From Lenz's law, this force opposes the motion of the magnet.

5.



$$\varepsilon = \frac{B\omega L^2}{2} = \frac{50 \times 20 \times 0.1 \times 0.1}{2} = 5V$$

For the circuit $I_1 + I_2 + I_3 = 0$

$$\Rightarrow \frac{V-5}{10} + \frac{V-5}{10} + \frac{V}{10} = 0 \Rightarrow V = \frac{10}{3}$$

$$\therefore I_3 = \frac{V}{10} = \frac{1}{3}A$$

6. Initial current through L for switch in position = $\frac{\varepsilon}{R_1}$

$\therefore$ Energy stored in inductor = $\frac{1}{2}L\left(\frac{\varepsilon}{R_1}\right)^2 = \frac{L\varepsilon^2}{2R_1^2}$

$\therefore$ Heat developed across R_2 = $\frac{L\varepsilon^2}{2R_1^2}$

7. $\varepsilon = E2\pi r = \pi r^2 \frac{dB}{dt} \Rightarrow E = \frac{r}{2}k$

$\therefore$ For an electron $a = \frac{f}{m} = \frac{eE}{m} = \frac{erK}{2m}$

8. $\phi = MI \Rightarrow \varepsilon = \left|-\frac{d\phi}{dt}\right| = M(2kt)$

$\therefore \Delta Q = \int_0^T \frac{\varepsilon}{R}dt = \frac{kMT^2}{R}$

9. $E2\pi R = \pi R^2 \frac{dB}{dt} \Rightarrow E = \frac{R}{2}\frac{d(0.2t)}{dt} = \frac{R}{2}(0.2) = \frac{R}{10}$

$\therefore$ Torque on coil

$= qER = q\left(\frac{R}{10}\right)R = I\alpha = mR^2\alpha$

$\Rightarrow$ Angular velocity attained

$\omega = \alpha\Delta T = \left(\frac{qER}{mR^2}\right)\Delta t = 40$ rad/sec

10. For LC circuit $\frac{q}{c} + \frac{Ldi}{dt} = 0$

$\Rightarrow q = Q_0\sin(\omega t + \phi)$ $\left(\omega = \frac{1}{\sqrt{LC}}\right)$

At t=0, $q = Q_0 \Rightarrow Q_0 = Q_0 \sin\phi \Rightarrow \phi = \pi/2$

$\therefore q = Q_0 \sin\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right)$

11. $\phi = BA = (Kt - C)(\pi a^2)$

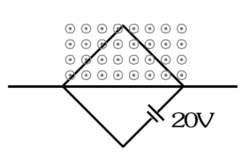
Charge $q = \frac{\Delta\phi}{R} = \frac{\phi_2 - \phi_1}{R}$

At t=0; $\phi_1 = -C(\pi a^2)$

At $t = \frac{C}{k}$ $\phi_2 = 0$

So $q = 0 - = \left(\frac{-C\pi a^2}{R}\right) \Rightarrow q = \frac{C\pi a^2}{R}$

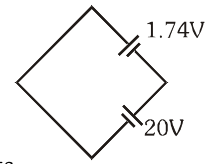
13. B= 0.042 – 0.87 t



(ii) B is decreasing with B so induced current will try to increase (B) (Lenz's law). So direction of current anticlockwise

(i) For upper half area

$e = -\frac{AdB}{dt} = -\left(\frac{2\times 2}{2}\right)(-0.87)$

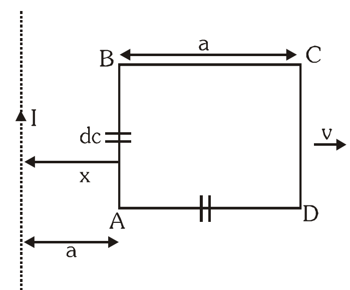


$\Rightarrow e = 1.74$ Volt

Total emf in circuit

E = 1.74 + 20 $\Rightarrow$ E = 21.74 volts

14. $e = Bv\ell$, $de = (dB)vdx$



induced current will be clockwise

in AB : $e_1 = \int_0^{2a} \frac{\mu_0 I}{2\pi x} vdx \Rightarrow e_1 = \frac{\mu_0 Iv}{2\pi}\ell n2$

in CD : $e_2 = \int_{2a}^{3a} \frac{\mu_0 I}{2\pi x} vdx \Rightarrow e_2 = \frac{\mu_0 Iv}{2\pi}\ell n\frac{3}{2}$

So net $e = e_2 - e_1$ $\frac{\mu_0 Iv}{2\pi}\ell n\frac{3}{4} \Rightarrow i = \frac{e}{R} = \frac{\mu_0 Iv}{2\pi R}\frac{\ell n3}{4}$

Force F = $i\ell B$, dF = i (dx) dB)

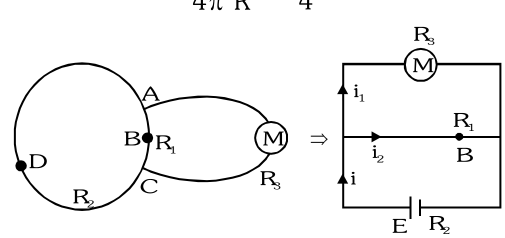
Work done dW = dF.x.

$W = \int_0^{2a} i(dx)\left(\frac{\mu_0 I}{2\pi x}\right)\cdot x$

$\Rightarrow W = \frac{i\mu_0 I}{2\pi}\int_0^{2a} dx = \frac{i\mu_0 I}{2\pi}(2a - a)$

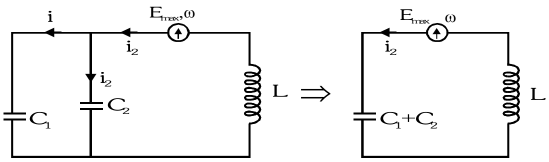
$\Rightarrow W = \frac{\mu_0 I^2 Va}{4\pi^2 R}\ell n\frac{3}{4}$

16.



$$i = \frac{E}{\frac{R_1R_3}{R_1+R_3} + R_2} = \frac{E(R_1+R_3)}{R_1R_2 + R_2R_3 + R_1R_3}$$

$$i = \frac{R_1}{R_1+R_3} = \frac{ER_1}{R_1R_2 + R_2R_3 + R_1R_3}$$

17. 

$$i_{max} = \left(\frac{E_{max}}{\omega L - \frac{1}{\omega(C_1+C_2)}}\right) = \frac{E_{max}}{X_L - X_C}$$

$$\Rightarrow (i_1)_{max} = \frac{C_1}{C_1+C_2} i_{max}$$

$$\Rightarrow (i_1)_{max} = \frac{C_1 I_{max}}{(C_1+C_2)\left(\omega L - \frac{1}{\omega(C_1+C_2)}\right)}$$

and $(i_2)_{max} = \frac{C_2}{C_1+C_2} i_{max}$

$$(i_2)_{max} = \frac{C_2 E_{max}}{(C_1+C_2)\left(\omega L - \frac{1}{\omega(C_1+C_2)}\right)}$$

18. $i = 3 + 5t$, $R = 4\Omega$, $L = 6H$

$\because E = iR + L\frac{di}{dt} = (3+5t)\ 4 + 6(5)$

$E = 42 + 20t$

19. (i) Let v be the velocity of the rod MN at an instant of time t when it is at a distance x from R. Then, the induced emf at that instant is e = Bvd. Since λ is the resistance per unit length of each wire, the total resistance in series with R at that instant is

$\lambda x + \lambda x = 2\lambda x$.

Thus the total resistance of the circuit at time t

$= R + 2\lambda x$.

Hence the current in the circuit is

$$I = \frac{\text{induced emf}}{\text{total resistance}} = \frac{Bvd}{(R+2\lambda x)} = \text{constant (given)}$$

$$\therefore v = \frac{I(R+2\lambda x)}{Bd} \quad ...(1)$$

The magnetic force acting on the rod is $F_m = BId$ directed to the left.

The net force acting on the rod is

$F_{net} = F - F_m = F - BId$ ...(2)

F_{net} is the force experienced by the rod MN at time t when its velocity is v at a distance x from R.

From Newton's law

$$F_{net} = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt}$$

Now $\frac{dx}{dt} = v$. Using (1), we have

$$F_{net} = mv\frac{d}{dt}\left\{\frac{I(R+2\lambda x)}{Bd}\right\} = \frac{2mv\lambda I}{Bd}$$

($\because$ I, R, B and d are constants)

$$= \frac{2m\lambda I^2}{B^2d^2}(R+2\lambda x) \text{ [Use equation (1)]}$$

Using this in equation (2), we get

$$F = BId + \frac{2m\lambda I^2}{B^2d^2}(R+2\lambda x) ...(3)$$

(ii) Work done in time dt = Fdx.

Therefore, work done per second i.e., power is

$$P = \frac{d}{dt}(Fdx) = \frac{Fdx}{dt} = Fv$$

Using Equation (1), we have

$$P = \frac{FI(R+2\lambda x)}{Bd}$$

Heat produced per second is

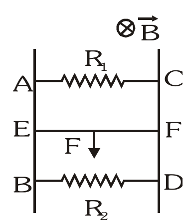
$Q = I^2(R+2\lambda x)$

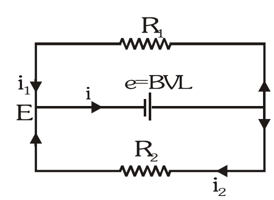
$$\therefore \text{Ratio } \frac{Q}{P} = \frac{I^2(R+2\lambda x)Bd}{FI(R+2\lambda x)} = \frac{IBd}{F} ...(4)$$

Using (3) in (4), we get

$$\frac{Q}{P} = \frac{B^3d^3}{B^3d^3 + 2m\lambda I(R+2\lambda x)}$$

20. Let the magnetic field be perpendicular to the plane of rails and inwards ⊗. if V be the terminal velocity of the rails, then potential difference across E and F would be BVL with E at lower potential and F at higher potential. The equivalent circuit is shown in figure (2). In figure (2).

(1) 

(2) 

$$i_1 = \frac{e}{R_1} ...(1)$$

$i_2 = \dfrac{e}{R_2}$ ...(2)

Power dissipated in R_1 is 0.76 watt

Therefore (इसलिए) ei_1 = 0.76 watt ...(3)

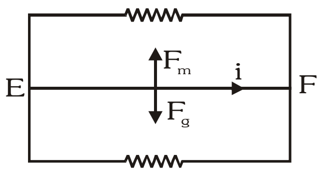
Similarly (इसी प्रकार) ei_2 = 1.2 watt ...(4)

Now the total current in bar EF is

$i = i_1 + i_2$ (From E to F) ...(5)

Under equilibrium condition, magnetic force (F_m) on bar EF = weight (F_g) of bar EF

i.e., $F_m = F_g$ or iLB = mg ...(6)



From equation (6)

$i = \dfrac{mg}{LB}$ $\dfrac{0.2(9.8)}{(1.0)(0.6)}$A or i = 3.27 A

Multiplying equation (5) by e, we get

$ei = ei_1 + ei_2$= (0.075 + 1.2) watt

(From equation 3 and 4) = 1.96 watt

$e = \dfrac{1.96}{i}$ volt = $\dfrac{1.96}{3.27}$V or e = 0.6 V

But since e = BVL

$V = \dfrac{e}{BL} = \dfrac{(0.6)}{(0.6)(1.0)}$ m/s= 1.0 m/s

Hence, terminal velocity of bar is 1.0 m/s.

Power in R_1 is 0.76 watt

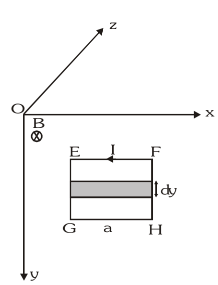
$\therefore\ 0.76 = \dfrac{e^2}{R_1}$ $\therefore\ R_1 = \dfrac{e^2}{0.76} = \dfrac{(0.6)^2}{0.76}\Omega = 0.47\Omega$

$R_1 = 0.47\ \Omega$

Similarly $R_2 = \dfrac{e^2}{1.2} = \dfrac{(0.6)^2}{1.2}\Omega = 0.3\Omega$

$R_2 = 0.3\ \Omega$

21. Refer to figure



(i) Consider a small element of the loop of width dy and side a. The area of the element dA = ady. Since the area vector and the magnetic field vector point in the same direction, i.e. the angle θ between the normal to the plane of the loop and the magnetic field is zero, the magnetic flux through the element

$d\phi = BdA\cos\theta = BdA\cos 0^0 = BdA$

The total magnetic flux linked with the loop is

$\phi = \int BdA$ $\quad\therefore B = B_0\left(\dfrac{y}{a}\right)\tilde{k}$

depends upon y, we have

$\phi = \int_y^{y+a} B_0\left(\dfrac{y}{a}\right) ady = B_0 = \int_y^{y+a} ydy = \dfrac{B_0}{2}$

$\left|y_2\right|_y^{y+a} = \dfrac{B_0}{2}$ $[(y+a)^2 - y^2]$......(1)

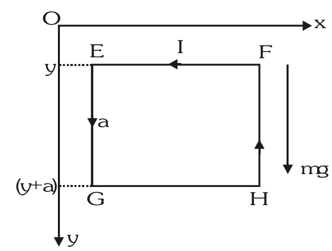
$\therefore$ Induced emf is

$e = -\dfrac{d\phi}{dt} = -\dfrac{B_0}{2}\dfrac{d}{dt}$ $[(y+a)^2 - y^2] = -\dfrac{B_0}{2}$

$\left[2(y+a)\dfrac{dy}{dt} - 2y\dfrac{dy}{dt}\right] = -B_0\ a\ \dfrac{dy}{dt}$

The induced current is $I = \dfrac{|e|}{R} = \dfrac{B_0 a}{R}\dfrac{dy}{dt}$ ......(2)

Since the magnetic field B points in the positive z–direction, it follows from Lenz's law that the direction of the induced current will be along the negative z–axis. Thus the current in the loop will flow in the counterclockwise direction as shown in figure.



(ii) The Lorentz force acting on a current element I dI of length $d\ell$ in magnetic field B is given by

$$F = I\oint_L (d\ell \times B)$$

Now, the forces acting on sides EG and FH of the loop are equal and opposite. Hence they cancel each other. The forces acting on sides EF and GH are in opposite directions but their magnitudes are different since B depends upon y. Thus, force acting on side

EF is $F_{EF} = I\left[(a\tilde{i}) \times \left(\dfrac{B_0 y}{a}\tilde{k}\right)\right] = IB_0 y\tilde{j}$

Similarly, force acting on side GH is

$F_{GH} = I\left[(a\tilde{i}) \times \left\{\dfrac{B_0(y+a)}{a}\right\}\tilde{k}\right] = -IB_0(y+a)\tilde{j}$

Therefore, the net force acting on the loop is

$F_{net} = F_{EF} + F_{GH} = IB_0 y\tilde{j} - IB_0(y+a)\tilde{j} = -IB_0 a\tilde{j}$

Substituting the expression for I from equation (2), we get

$$F_{net} = -\left(\frac{B_0^2a^2}{R}\right)\frac{dy}{dt}\tilde{j}$$

The negative sign $\left(-\tilde{j}\right)$ indicates that the force acts along the negative y–direction.

(iii) As force F_{net} is directed along the negative y–axis and the gravitational force mg is along the positive y–direction, the total force on the loop in the downward (positive y) direction is

$$F = mg - \left(\frac{B_0^2a^2}{R}\right)\frac{dy}{dt}$$

where $\frac{dy}{dt}$ = v, the speed with which the loop is falling downwards. From newton's second law, the equation of motion of the loop is

F = mass acceleration or $F = m\frac{dv}{dt}$

$$\therefore\ m\frac{dv}{dt} = mg - \left(\frac{B_0^2a^2}{R}\right)v$$

$$\text{or } \frac{dv}{dt} = g - \left(\frac{B_0^2a^2}{mR}\right)v \text{ or } \frac{dv}{dt} = g - kv \quad ....(3)$$

where $k = \frac{B_0^2a^2}{mR}$

Rearranging expression (3), we have $\frac{dv}{(g-kv)} = dt$

Intergrating, we have

$$\int_0^v \frac{dv}{(g-kv)} = \int_0^t dt \text{ or } -\frac{1}{k}\log_e\left[\frac{(g-kv)}{g}\right] = t$$

which gives

$$v = \frac{g}{k}\left(1-e^{-kt}\right) = \frac{gmR}{B_0^2a^2}\left[1-\exp\left(-\frac{B_0^2a^2t}{mR}\right)\right]$$

This is the required expression for the speed v of the loop as a function of time t. When the loop acquires terminal velocity v_0 no force acts on it. Hence its acceleration dv/dt is zero.
Using this in equation (3), we have

$$0 = g - kv_t \text{ or } v_t = \frac{g}{k} = \frac{mgR}{B_0^2a^2}$$

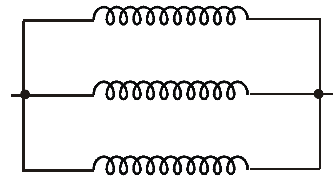
22. flux $\phi = BA$; $d\phi = (dB)(dA) \Rightarrow d\phi = \left(\frac{\mu_0 I}{2\pi x}\right)(\ell dx)$

$$\phi = \int_a^{a+b} d\phi = \frac{\mu_0 I\ell}{2\pi}\int_a^{a+b}\frac{1}{x}dx$$

$$\frac{\mu_0 I\ell}{2\pi}(\ell n x)_a^{a+b} \Rightarrow \phi = \frac{\mu_0 I\ell}{2\pi}\ell n\left(\frac{a+b}{a}\right)$$

EXERCISE –V(A)

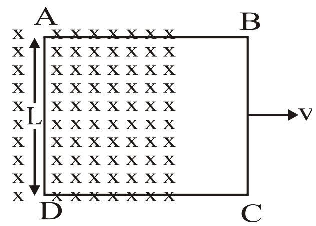
1.



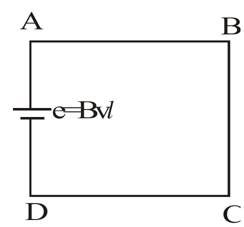
In parallel $\frac{1}{L} = \frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}$

$\frac{1}{L} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3}{3} = 1$ as L=1H

2.



The equivalent circuit of the conductor will be



4. The core of transformer is laminated so as to reduce the energy loss due to eddy currents.

6. I_1=2A
I_2=–2A
t=0.05, e= 8V

$$e = \frac{-Ldi}{dt} = -L\left[\frac{-2A-2A}{0.05}\right]$$

$$8 = L\left[\frac{4}{0.05}\right]$$

L = 2 0.05 = 0.1 H

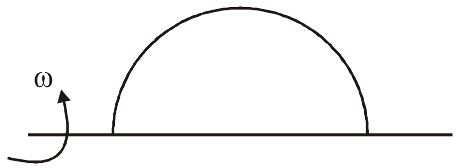
7. Number of turns = n
Resistance of coil = RΩ
Resistance of galvanometer= 4RΩ

Induced current = $\frac{e}{R}$

$$= -\left[\frac{\phi_2-\phi_1}{t}\right]\frac{1}{\text{Resitance of circuit}}$$

$$= -\left[\frac{nW_2-nW_1}{t}\right]\frac{1}{5R}$$

$$= -n\left[\frac{W_2-W_1}{5Rt}\right]$$

8. 

The flux associated with coil = NBAcosωt

$\phi = B\dfrac{\pi r^2}{2}\cos\omega t$

$e = \dfrac{-d\phi}{dt} + \dfrac{B\pi r^2\omega}{2}\sin\omega t$

$P_{inst.} = \dfrac{e^2}{R} = \left(\dfrac{B\pi r^2\omega}{2}\right)\dfrac{\sin^2\omega t}{R}$

$P_{av} = \dfrac{\int P dt}{\int dt} = \dfrac{B^2\pi^2 r^4\omega^2}{4R}\dfrac{\int \sin^2\omega t dt}{\int dt}$

$\int_0^T \sin^2\omega t dt = \dfrac{1}{2}T$

Hence $P_{av} = \dfrac{B^2\pi^2 r^4\omega^2}{4R}\dfrac{1}{2}$

$P_{av} = \dfrac{B^2\pi^2 r^4\omega^2}{8R}$

$P_{av} = \dfrac{\left(B\pi r^2\omega\right)^2}{8R}$

9. The emf developed across the ends of the pivoted rod is $e = \dfrac{B\ell^2\omega}{2}$

$\omega = 5\dfrac{rad}{sec}$, B = 0.2 10^{-4}T, L=1m

$e = \dfrac{0.2\times10^{-4}\times1^2\times5}{2}$ =5 10^{-5} = 50μV

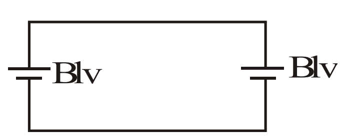
10. $I_{steady} = \dfrac{V}{R} = \dfrac{2}{2} = 1A$; $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\dfrac{R}{L} = \dfrac{2}{300\times10^{-3}} = \dfrac{2000}{300}$

$\dfrac{I_0}{2} = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\dfrac{1}{2} = 1 - e^{-\frac{R}{L}t} \Rightarrow t = \dfrac{\ell n2}{L/R} \approx 0.1s$

11. The vertical arm of the both tubes will becomes a battery of emf Blv.



The emf induced in the circuit is 2 Blv.

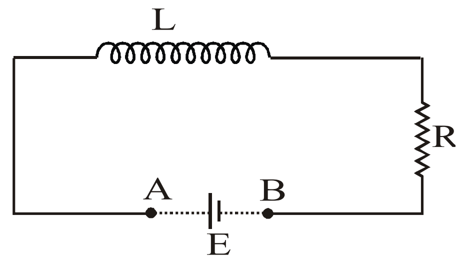
12. $\phi = 10t^2 - 50t + 250$

$\dfrac{d\phi}{dt} = 20t - 50$

$e = \dfrac{-d\phi}{dt} = 50 - 20t$

$e_{(t=3)}$ = 50 – 20 3 = – 10V

13. L = 100 mH; R=100Ω; E=100V



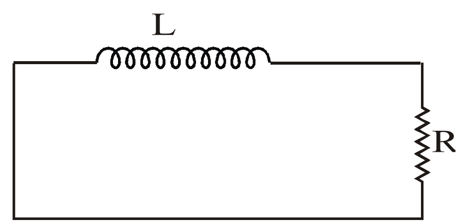
Long time after, current in the circuit is

$I_0 = \dfrac{E}{R} = 1A$

On short circuiting $I = I_o e^{-\frac{R}{L}t}$

$I = 1e^{-\frac{100}{100\times10^{-3}}\times10^{-3}}$

$1 = \left(\dfrac{1}{e}\right)A$



14. RMS value of electric field = 720 N/C

Peak value of electric field = $\sqrt{2}\times720$ N/C

the average total energy density of electromagnetic wave = $\dfrac{1}{2}\varepsilon_0E_0^2$; $u_{av} = \dfrac{1}{2}\varepsilon_0E_0^2$

On solving we get

u_{av} = 4.58 10^{-6} J/m^3

15. Inductance of the coil L = 10 H

Resistance of the coil R=5Ω

As R-L is connected across battery, hence nature of current that will flow through the circuit will be transient current, i.e., $I = I_0(1 - e^{-t/\tau})$

where $\tau = \dfrac{L}{R} = \dfrac{10}{5}$ =2s and $I_0 = \dfrac{V}{R} = \dfrac{5}{5}$ =1A

hence $I = 1(1 - e^{-2/2})$; $I = (1 - e^{-1})A$

16. $M = \dfrac{\mu_0 N_1 N_2 A}{\ell} = 2.4\pi \times 10^{-4} H$

17. Equivalent resistance across L(by short circuiting battery) = R_{eq} = 2Ω

then current through L $\quad i = i_0(1 - e^{-Rt/L})$

where R = 2Ω, L = $\dfrac{4}{10}$ H, $i_0 = \dfrac{E}{R} = \dfrac{12}{2} = 6A$

$i = 6(1 - e^{-5t})$ .... (i)

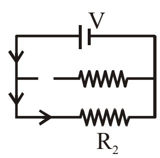
∴ p.d. across L is

$V_L = L\dfrac{di}{dt} = \dfrac{4}{10}\dfrac{d}{dt}[6(1 - e^{-5t})] = 12\ e^{-5t}$

This question contains Statement–1 and Statement–2. Of the four choices given after the statements, choose the one that best describes the two statements.

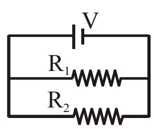
18. At t = 0 inductor behaves as broken wire then

$i = \dfrac{V}{R_2}$

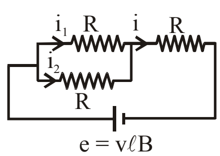


at t = ∞ Inductor behaves as conducting wire

$i = \dfrac{V}{R_1R_2/(R_1+R_2)}$



19. Circuit can be reduced as

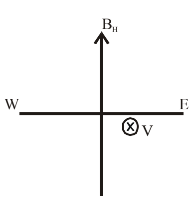


$i = \dfrac{e}{3R/2} = \dfrac{2v\ell B}{3R}$

$i_1 = i_2 = \dfrac{i}{2} = \dfrac{v\ell B}{3R}$

20. e = Bℓv = (5 $\quad 10^{-5}$) (2) (1.50) = 0.15 mV

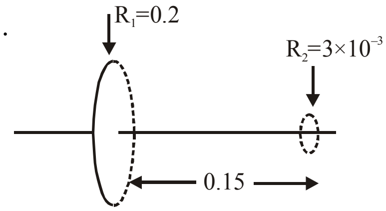
21. B = 0.3 $\quad 10^{-4}$ wb/m²



e = vℓB= 5 $\quad$ 20 $\quad$ 0.3 $\quad 10^{-4}$ = 3mV

22. Due to conducting nature of Al eddy currents are produced

23.



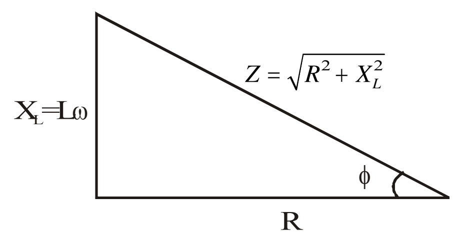
Mutual inductance = $\dfrac{\mu_0 R_1^2 \pi R_2^2}{2(R_1^2 + X^2)^{3/2}}$

so flux through trigger coil $= \left(\dfrac{\mu_0}{4\pi}\right) \cdot \dfrac{2\pi^2 R_1^2 R_2^2}{(R_1^2 + X^2)^{3/2}} i$

$= \dfrac{10^{-7} \times (3.14)^2 (2 \times 10^{-1})^2 (3 \times 10^{-3})^2 \times 2 \times 2}{((0.2)^2 + (0.15)^2)^{3/2}}$

ALTERNATING CURRENT

24. On drawing the impedance triangle; we get



The power factor $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + \omega^2 L^2}}$

26. Let $Q_{max} = Q_{max}$

$\dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}LI^2$ ...(i)

[Given that energy stored in capacitor= Energy stored in conductor].

According to the conservation of energy we know that $\dfrac{1}{2}\dfrac{Q_{max}^2}{C} = \dfrac{1}{2}\dfrac{Q^2}{C} + \dfrac{1}{2}LI^2$

$\dfrac{1}{2}\dfrac{Q_{max}^2}{C} = \dfrac{1}{2}\dfrac{Q^2}{C} + \dfrac{1}{2}\dfrac{Q^2}{C}$ [from eq. (i)]

$\dfrac{Q_{max}^2}{C} = \dfrac{2Q^2}{C}$

$Q^2 = \dfrac{Q_{max}^2}{C}$

$Q = \dfrac{Q_{max}}{\sqrt{2}}$

27. A DC meter measure the average value and the average value of AC over one full cycle is zero. Hence, DC meters can't measure AC.

28. Voltage across LC combination = $|V_L - V_C|$
Voltage across LC combination = $|50-50|=0V$

29. The resonant frequency $\omega_r = \dfrac{1}{\sqrt{LC}}$

For $\omega_1=\omega_2$ $\dfrac{1}{\sqrt{L_1 C}} = \dfrac{1}{\sqrt{L_2 2C}}$

On squaring both sides, we get

$$\frac{1}{L_1 C} = \frac{1}{L_2(2C)};\ \frac{L_2}{L_1} = \frac{1}{2};\ L_2 = \frac{L_1}{2}$$

30. In order to transfer maximum power the generator should work at resonant frequency, i.e., C should be such so that

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f^2 = \frac{1}{4\pi^2 LC}$$

$$\Rightarrow C = \frac{1}{4\pi^2 L f^2} = \frac{1}{4\times 10\times 10\times 2500}$$

$$\Rightarrow C = 10^{-6}F = 1\mu F$$

31. Power factor= $\cos\phi = \dfrac{R}{Z}$

$$\cos\phi = \frac{12}{15} = \frac{4}{5} = 0.8$$

32. If resistance will be the part of circuit, phase difference between voltage and current can not be $\dfrac{\pi}{2}$.

33. The voltage across L $=V_L = IX_L$

At resonance, current = $\dfrac{\text{Voltage}}{\text{Resistance}}$;

$$I = \frac{100V}{10^3\Omega} = 0.1$$

Also at resonance $X_L = X_C$

$$X_C = \frac{1}{\omega_C} = \frac{1}{200\times 2\times 10^{-6}} = \frac{10^4}{4}\Omega$$

$$V_L = IX_L = 0.1\times\frac{10^4}{4} = 250V$$

34. Magnetic flux associated with rotating coil
= $NBA\cos\omega t$
$\phi = NBA\cos\omega t$

$$\frac{d\phi}{dt} = -(NBA\omega)\sin(\omega t)$$

$e = e_0\sin\omega t$
Maximum value of emf generated in coil
= $NBA\omega$

35. Given that
$E=E_0\sin(\omega t)$ and $I\backslash I_0\sin(\omega t-\pi/2)$

The phase difference between E and I is $\dfrac{\pi}{2}$.

Power dissipated in an AC circuit
$P = E_{rms} I_{rms} \cos\phi = 0$
So, power dissipated for this situation where phase difference between voltage and current is $\dfrac{\pi}{2}$ will be zero.

36. $\tan\phi = \dfrac{X_L - X_C}{R}$

$\tan 30 = \dfrac{X_C}{R} \Rightarrow X_C = \dfrac{R}{\sqrt{3}}$

$\tan 30 = \dfrac{X_L}{R} \Rightarrow X_L = \dfrac{R}{\sqrt{3}}$

$X_L = X_C \Rightarrow$ Condition for resonance
So $\phi = 0$
$P = VI\cos 0$

$$P = \frac{V^2}{R} = \frac{(220)^2}{200} = 242W$$

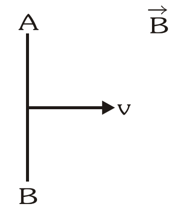
37. Energy is shared equally between L and C at

$t = \dfrac{T}{8}, \dfrac{3T}{8}$...... where $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC}$

so $t = \dfrac{T}{8} = \dfrac{2\pi\sqrt{LC}}{8} = \dfrac{\pi}{4}\sqrt{LC}$

EXERCISE –V-B

1. A motional emf, $e = B\ell v$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



2. Magnetic field produced by a current i in a large square loop at its centre.

$$B \propto \frac{i}{L} \text{ say } B = K\frac{i}{L}$$

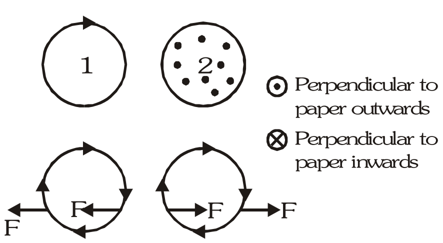
$\therefore$ Magnetic flux linked with smaller loop,

$$\phi = B.S \Rightarrow \phi = \left(K\frac{i}{L}\right)(\ell^2)$$

Therefore, the mutual inductance

$$M = \frac{\phi}{i} = K\frac{\ell^2}{L} \Rightarrow M \propto \frac{\ell^2}{L}$$

3. For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce * magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produce ⊗ magnetic field passing through it i.e., induced current in loop 2 will also be clokwise as shown in the figure.



The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in the opposite directions.

4. The current–time (i–t) equation in L–R circuit is given by [Growth of current in L–R circuit]

$$i = i_0(1 - e^{-t/\tau_L}) \quad \text{...(i)}$$

where $i_0 = \frac{V}{R} = \frac{12}{6} = 2A$

and $\tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3}$ s

and i = 1A (given)

Substituting these values in equation (i), we get

t = 0.97 10^{-3}s

$\Rightarrow$ t = 0.97 ms $\Rightarrow$ t $\approx$ 1ms

5. $\oint \vec{E}.d\vec{\ell} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$

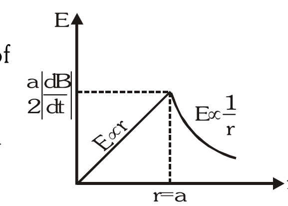
$\Rightarrow E(2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$ for $r \geq a$ $\therefore$ $E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$

$\therefore$ Induced electric field $\propto \frac{1}{r}$

For $r \leq a$; $E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right| \Rightarrow E = \frac{r}{2}\left|\frac{dB}{dt}\right| \Rightarrow E \propto r$

At $r = a$, $E = \frac{a}{2}\left|\frac{dB}{dt}\right|$

Therefore, variation of E with r(distance from centre)will be as follows :



6. The equations of $I_1(t)$, $I_2(t)$ and B(t) will take the following forms :

$I_1(t) = K_1(1-e^{-k_2t}) \rightarrow$ current growth in L–R circuit

$B(t) = K_3(1-e^{-k_2t}) \rightarrow B(t) \rightarrow I_1(t)$; $B = \mu_0 Ni$ in case of solenoid coil and $\frac{\mu_0 Ni}{2R}$ in case of circular coil i.e.,

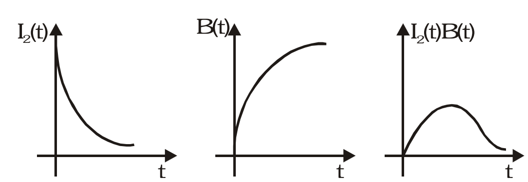
$B \propto i \Rightarrow I_2(t) = K_4e^{-k_2t}$

$$\left[I_2(t) = \frac{e_2}{R} \text{ and } e_2 \propto \frac{dI_1}{dt} : e_2 = -M\frac{dI_1}{dt}\right]$$

Therefore the product $I_2(t)\,B(t) = K_5e^{-k_2t}(1-e^{-k_2t})$.

The value of this product is zero at t=0 and t=∞. Therefore, the product will pass through a maximum value ($K_1 : K_2 : K_3 : K_4$ and K_5 are positive constants and M is the mutual inductance between the coil and the ring).

The corresponding graph will be as follows :



7. Electric field will be induced in both AD and BC.

8. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

9. When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law induced current in Q i.e., I_{Q_1} will flow in such a direction, so that the magnetic field lines due to I_{Q_2} passes from left to right through Q. This is possible when I_{Q_1} flows in anticlockwise direction as seen by E. Opposite is the case when switch S is opened i.e., I_{Q_2} will be clockwise as seen by E.

10. Power $P = \dfrac{e^2}{R}$

Here, e = induced emf = $\left(\dfrac{d\phi}{dt}\right)$

where $\phi = NBA$ $\quad e = -NA\left(\dfrac{dB}{dt}\right)$

Also, $\quad R \propto \dfrac{1}{r^2}$

where R = resistance, r = radius, ℓ = length

$\therefore\ P \propto \dfrac{N^2r^2}{\ell}\ \therefore\ \dfrac{P_1}{P_2} = 1$

11. As the current i leads the emf e by

$\dfrac{\pi}{4}$, it is an R–C circuit. $\tan\phi = \dfrac{X_C}{R}$

or $\tan\dfrac{\pi}{4} = \dfrac{\frac{1}{\omega C}}{R}\ \therefore\ \omega CR = 1$

As ω = 100 rad/s

The product of C–R should be $\dfrac{1}{100}\ s^{-1}$.

12. Polarity of emf will be opposite in the two case while entering and while leaving the coil. Only in option (b) polarity is changing.

13. In uniform magnetic field, change in magnetic flux is zero.

Therefore, induced current will be zero.

14. As area of outer loop is bigger therefore emf induced in outer loop is dominant and therefore according to lenz law current in outer loop is Anticlockwise and inner loop is clockwise

MCQ

1. Electrostatic and gravitational field do not make closed loops.

3. When resistivity is low current induced will be more; therefore impulsive force on the ring will also be more and it jumps to higher levels. [But for this mass should be either less or equal to the other]

Comprehension#1

1. Charge on capacitor at time t is :

$q = q_0(1 - e^{-t/\tau})$

Here, $\quad q_0 = CV$ and $t = 2\tau$

$\therefore \quad q = CV\,(1 - e^{-2\tau/\tau})$

$= CV\,(1 - e^{-2})$

2. From conservation of energy,

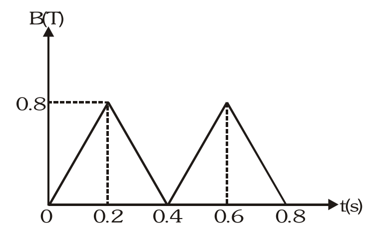
$\dfrac{1}{2}LI^2_{max} = \dfrac{1}{2}CV^2\ \therefore \qquad I_{max} = V\sqrt{\dfrac{C}{L}}e$

3. Comparing the LC oscillations with normal SHM we get,

$\dfrac{d^2Q}{dt^2} = -\omega^2 Q$ Here, $\omega^2 = \dfrac{1}{LC}\ \therefore\ Q = -LC\dfrac{d^2Q}{dt^2}$

Subjective

1. Magnetic field (B) varies with time (t) as shown in figure.



$\left|\dfrac{dB}{dt}\right| = \dfrac{0.8}{0.2} = 4\,T/s$

Induced emf in the coil due to change in magnetic flux passing through it,

$e = \left|\dfrac{d\phi}{dt}\right| = NA\left|\dfrac{dB}{dt}\right|$

Here, $\quad$ A = Area of coil = 5 $\quad 10^{-3}\ m^2$

N = Number of turns = 100

Substituting the values, we get

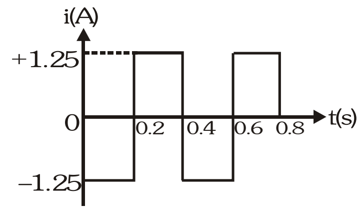
$e = (100)\,(5 \quad 10^{-3})\,(4)\ V = 2V$

Therefore, current passing through the coil

$i = \dfrac{e}{R}$ (R = Resistance of coil = 1.6 Ω)

$\Rightarrow i = \dfrac{2}{1.6} = 1.25A$

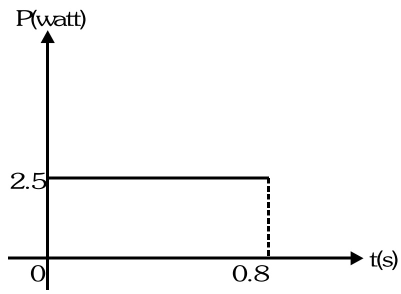
Note that from 0 to 0.2 s and from 0.4s to 0.6s, magnetic field passing through the coil increases, while during the time 0.2s to 0.4s and from 0.6s to 0.8s magnetic field passing through the coil decreases. Therefore, direction of current through the coil in these two time intervals will be opposite to each other. the variation of current (i) with time (t) will be as follows :



Power dissipated in the coil is

$P = i^2 R = (1.25)^2 (1.6)W = 2.5W$

Power is independent of the direction of current through the coil. Therefore, power (P) versus time (t) graph for first two cycles will be as follows :



Total heat obtained in 12,000 cycles will be

$$H = P.t = (2.5)\,(12000)\,(0.4) = 12000\ J$$

This heat is used in raising the temperature of the coil and the water. Let θ be the final temperature.

Then $H = m_w S_w(\theta - 30) + m_c S_c(\theta - 30)$

Here m_w = mass of water = 0.5 kg

S_w = specific heat of water = 4200 J/kg–K

m_c = mass of coil = 0.06 kg

and S_c = specific heat of coil = 500J/kg–K

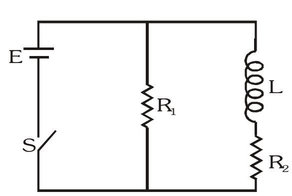
Substituting the values, we get

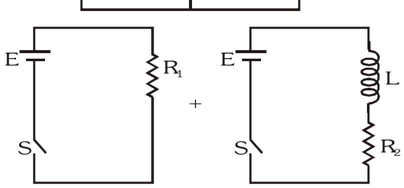
$1200 = (0.5)\,(4200)\,(\theta-30) + (0.06)\,(500)\,(\theta-30)$

$\Rightarrow \qquad \theta = 35.6\ C$

2. (a) Given $R_1 = R_2 = 2\Omega$, E = 12V

and L=400 mH = 0.4 H. Two parts of the circuit are in parallel with the applied battery. So, the upper circuit can be broken as :





(a) (b)

Now refer figure (b) :

This is a simple L–R circuit, whose time constant

$$\tau_L = L/R_2 = \frac{0.4}{2} = 0.2s$$

and steady state current $i_0 = E/R_2 = 12/2 = 6A$

Therefore, if switch S is closed at time t=0, then current in the cicuit at any time t will be given by

$i(t) = i_0\,(1-e^{-t/\tau_L})$

$i(t) = 6\,(1-e^{-t/0.2}) = 6(1-e^{-5t}) = i \qquad$ (say)

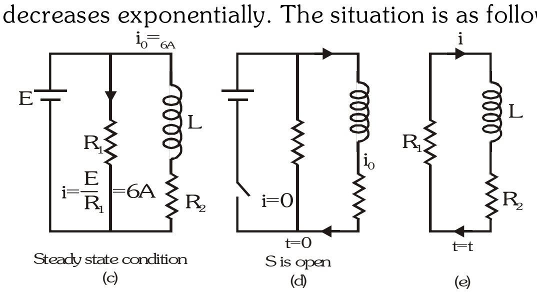
Therefore, potential drop across L at any time t is:

$$V = \left|L\frac{di}{dt}\right| = L(30e^{-5t}) = (0.4)(30)e^{-5t}$$

$\Rightarrow \qquad V = 12e^{-5t}$ volt

(b) The steady state current in L or R_2 is i = 6A

Now, as soon as the switch is opened, current in R_1 is reduced to zero immediately. But in L and R_2 it decreases exponentially. The situation is as follows:



Refer figure (e) :

Time constant of this circuit would be

$$\tau_L' = \frac{L}{R_1 + R_2} = \frac{0.4}{(2+2)} = 0.1s$$

$\therefore$ Current through R_1 at any time t is

$i = i_0 e^{-t/\tau_L'} = 6e^{-t/0.1} \Rightarrow\ i = 6e^{-10t}$ A

Direction of current in R_1 is as shown in figure or clockwise.

3. (i) Applying Kirchhoff' second law :

$$\frac{d\phi}{dt} - iR - L\frac{di}{dt} = 0 \Rightarrow \frac{d\phi}{dt} = iR + L\frac{di}{dt} \quad ....(i)$$

This is the desired relation between i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$.

(ii) equation (1) can be written as

$$d\phi = iRdt + Ldi$$

Integrating we get, $\Delta\phi = R.\,\Delta q + Li$

$$\Delta q = \frac{\Delta\phi}{R} - \frac{Li_1}{R} \quad ...(ii)$$

Here, $\Delta\phi = \phi_f - \phi_i = \int_{x=2x_0}^{x=x_0} \frac{\mu_0}{2\pi}\frac{I_0}{x}\ell dx = \frac{\mu_0 I_0 \ell}{2\pi}\ell n(2)$

So, from Equation (ii) charge flown through the resistance upto time t=T, when current is i_1, is–

$$\Delta q = \frac{1}{R}\left[\frac{\mu_0 I_0 \ell}{2\pi}\ell n(2) - Li_1\right]$$

(iii) This is the case of current decay in an L–R circuit.

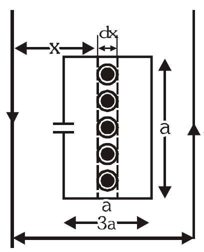
Thus, $i = i_0 e^{-t/\tau_L}$ ....(iii)

Here, $i = \frac{i_1}{4}$, $i_0 = i_1$, $t = (2T - T) = T$ and $\tau_L = \frac{L}{R}$

Substituting these values in equation (3), we get :

$$\tau_L = \frac{L}{R} = \frac{T}{\ell n 4}$$

4. (i) For a element strip of thickness dx at a distance x from left wire, net magnetic field (due to both wires)



$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{3a - x}$$

(outwards)

$$= \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a - x}\right)$$

Magnetic flux in this strip,

$$d\phi = BdS = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a - x}\right) a\,dx$$

$$\therefore \text{ total flux } \phi = \int_a^{2a} d\phi = \frac{\mu_0 Ia}{2\pi}\int_a^{2a}\left(\frac{1}{x} + \frac{1}{3a - x}\right)dx$$

$$\phi = \frac{\mu_0 Ia}{\pi}\ell n(2) \qquad \phi = \frac{\mu_0 a \ell n(2)}{\pi}(I_0 \sin\omega t) \quad ...(i)$$

Magnitude of induced emf,

$$e = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0 \omega \ell n(2)}{\pi}\cos\omega t = e_0 \cos\omega t$$

where $e_0 = \frac{\mu_0 a I_0 \omega \ell n(2)}{\pi}$

Charge stored in the capacitor,

$q = Ce = Ce_0 \cos\omega t$ ...(ii)

and current in the loop $i = \frac{dq}{dt} = C\omega e_0 \sin\omega t$...(iii)

$$i_{max} = C\omega e_0 = \frac{\mu_0 a I_0 \omega^2 C \ell n(2)}{\pi}$$

(ii) Magnetic flux passing through the square loop

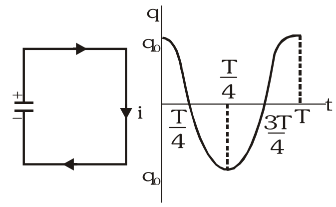
$\phi \propto \sin\omega t$ [From equation (i)]

i.e., * magnetic field passing through the loop is increasing at t=0. Hence, the induced current will produce ⊗ magnetic field (from Lenz's law). Or the current in the circuit at t=0 will be clockwise (or negative as per the given convention). Therefore, charge on upper plate could be written as,

$q = +q_0 \cos\omega t$ [From equation (ii)]

Here, $q_0 = Ce_0 = \frac{\mu_0 a C I_0 \omega \ell n(2)}{\pi}$

The corresponding q–t graph is shown in figures.



5. After a long time, resistance across an inductor becomes zero while resistance across capacitor becomes infinite. Hence, net external resistance,

$$R_{net} = \frac{\frac{R}{2} + R}{2} = \frac{3R}{4}$$ Current through the batteries,

$$i = \frac{2E}{\frac{3R}{4} + r_1 + r_2}$$ Given that potential across the terminals of cell A is zero.

$$\therefore E - ir_1 = 0 \Rightarrow E - \left(\frac{2E}{3R/4 + r_1 + r_2}\right)r_1 = 0$$

Solving this equation, we get $R = \frac{4}{3}(r_1 - r_2)$

6. Inductive reactance

$X_L = \omega L = (50)(2\pi)(35\ \ 10^{-3}) \approx 11\Omega$

Impedance

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\Omega$$

Given $v_{rms} = 220V$ Hence, amplitude of voltage

$$v_0 = \sqrt{2}\,v_{rms} = 220\sqrt{2}V$$

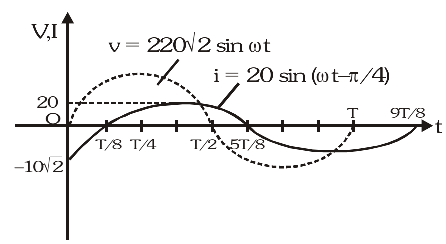
$$\therefore \text{ Amplitude of current } i_0 = \frac{v_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20A$$

$$\text{Phase difference } \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11}{11}\right) = \frac{\pi}{4}$$

In L–R circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$i = (20A)\sin(\omega t - \pi/4)$

Corresponding i–t graph is shown in figure.



7. Out side the solenoid net magnetic field zero. It can be assumed only inside the solenoid and equal to $\mu_0 nI$.

induced $e = -\frac{d}{dt} = -\frac{d}{dt}(BA) = -\frac{d\phi}{dt}(\mu_0 n / \pi a^2)$

or $|e| = (\mu_0 n\pi a^2)(I_0\omega\cos\omega t)$

Resistance of the cylindrical vessel $R = \frac{\rho\ell}{s} = \frac{\rho(2\pi R)}{Ld}$

$\therefore$ Induced current $i = \frac{|e|}{R} = \frac{\mu_0 Ldna^2 I_0\omega\cos\omega t}{2\rho R}$

8. This is a problem of L–C oscillations.

Charge stored in the capacitor oscillates simple harmonically as $Q = Q_0\sin(\omega t + \phi)$

Here, Q_0 = max. value of Q = 200 μC= 2 10^{-4} C

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3}H)(5.0\times10^{-6}F)}} = 10^4 s^{-1}$

Let at t=0, $Q=Q_0$ then

$Q(t) = Q_0\cos\omega t$...(i) $I(t) = \frac{dQ}{dt} = -Q_0\omega\sin\omega t$ ...(ii)

and $\frac{dI(t)}{dt} = -Q_0\omega^2\cos(\omega t)$...(iii)

(i) Q = 100 μC $\Rightarrow \frac{Q_0}{2}$ At $\cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$

At $\cos(\omega t) = \frac{1}{2}$ 1, from equation (iii) :

$\left|\frac{dI}{dt}\right| = (2.0\times10^{-4}C)(10^4 s^{-1})^2\left(\frac{1}{2}\right) = 10^4 A/s$

(ii) Q=200μC or Q_0 when cos(ωt) =1 i.e. ωt=0, 2π...

At this time $I(t) = -Q_0\omega\sin\omega t \Rightarrow I(t) = 0$

[sin 0 =sin 2π=0]

(iii) $I(t) = -Q_0\omega\sin\omega t$

$\therefore$ Maximum value of I is $Q_0\ \omega$

$I_{max} = Q_0\omega$ = (2.0 10^{-4}C) ($10^4 s^{-1}$) $\Rightarrow I_{max}$ = 2.0A

(iv) From energy conservation

$\frac{1}{2}LI_{max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$

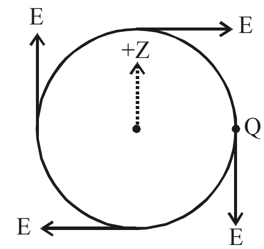
$\Rightarrow Q = \sqrt{LC(I_{max}^2 - I^2)}$ $\left[I = \frac{I_{max}}{2} = 1.0A\right]$

$\therefore Q = \sqrt{(2.0\times10^{-3})(5.0\times10^{-6})(2^2 - 1^2)}$

$Q = \sqrt{3}\times10^{-4}C$ =1.732 10^{-4}C

11. induce electric field $= \frac{R}{2}\frac{dB}{dt} = \frac{BR}{2}$

torque on charge $= \frac{QBR^2}{2}(-\tilde{k})$



by $\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow \int d\vec{L} = \int_0^1 \vec{\tau}dt$

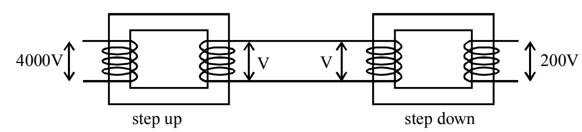
$\Delta\vec{L} = \frac{QBR^2}{2}(-\tilde{k})$

Change magnetic dipole moment $= \gamma\Delta\vec{L}$

$\frac{\gamma QBR^2}{2}(-\tilde{k})$

12. Magnitude of induced electric field =

$\frac{R}{2}\frac{dB}{dt} = \frac{BR}{2}$

13. 

for step up transformer

$\frac{V}{4000} = \frac{10}{1} \Rightarrow$ V = 40,000 Volt

for step down transformere

$\frac{N_1}{N_2} = \frac{V}{200} = \frac{4000}{200} = 200$

14. Current in transmission line

$= \frac{\text{Power}}{\text{Voltage}} = \frac{600\times10^3}{40,000} = 150A$

Resistance of line = 0.4 20 = 8Ω

Power loss in line = i^2R = $(150)^2 8$= 180 KW

percentage of power dissipation in during transmission $= \frac{180\times10^3}{600\times10^3}\times100 = 30\%$