UNIT # 07 (PART - II)

CAPACITOR

EXERCISE –I

1. $k = \frac{f}{x} = \frac{5000}{0.2} = 2{,}5000\ N/m$

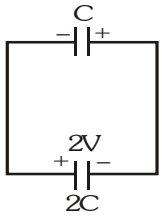
$$\frac{U_{SPR}}{U_{CAP}} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}CV^2} = \frac{25000 \times 0.2 \times 0.2}{10 \times 10^{-6} \times 10^8} = 1$$

2. $P = \frac{\Delta U}{\Delta t} = \frac{\frac{1}{2}CV^2}{\Delta t} = \frac{\frac{1}{2} \times 40 \times 10^{-6} \times 9 \times 10^6}{2 \times 10^{-3}} = 90kW$

3. $V_0\ (C+CV) = CV + (2C)\ (2V)$

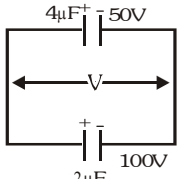
$V_0 = V$ (Final pot. diff.)

$\therefore\ U_{final} = \frac{1}{2}(C+2C)\ V^2 = \frac{3CV^2}{2}$



4. $(4+2)\ V = (4 \quad 50) + (2 \quad 100)$

$V = \frac{400}{6} = \frac{200}{3} V$



$U_{initial} = \left(\frac{1}{2} \times 4 \times (50)^2 + \frac{1}{2} \times 2 \times (100)^2\right) \times 10^{-6}$

$= (5000 + 10000) \quad 10^{-6} = 1.5 \quad 10^{-2}\ J$

$U_{final} = \frac{1}{2}(4+2) \times 10^{-6} \times \frac{200}{3} \times \frac{200}{3}$

$= 1.33 \quad 10^{-2}\ J$

5. Before sharing (सम्पर्क में आने से पहले) $U_i = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

After sharing (सम्पर्क में आने के बाद) $U_f = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)}$

$$\Delta U = U_f - U_i = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)} - \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$$

$$= -\frac{(Q_1C_2 - Q_2C_1)^2}{2C_1C_2(C_1+C_2)}$$

–ve sign indicates there is decrease in energy

(ऋणात्मक चिन्ह ऊर्जा में कमी को दर्शाता है)

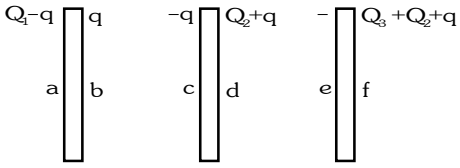
But $Q_1C_2 - Q_2C_1 \neq 0 \quad \Rightarrow Q_1 C_2 \neq Q_2C_1$

$\Rightarrow Q_1 4\pi\varepsilon_0 R_2 \neq Q_2 4\pi\varepsilon_0 R_1 \ \Rightarrow Q_1R_2 \neq Q_2R_1$

6. $C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}} \left(t = \frac{d}{2}, K = \infty\right)$

$$= \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2\epsilon_0 A}{d} = 2C_0$$

7.



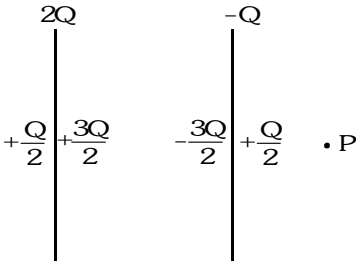
Here $Q_1 - q = Q_2 + Q_3 + q \Rightarrow\ q = \frac{Q_1 - (Q_2 + Q_3)}{2}$

Charge on a= Charge on f

(a पर आवेश = f पर आवेश)

$\Rightarrow Q_1 - q = \frac{\Sigma Q}{2} = \frac{Q_1 + Q_2 + Q_3}{2}$

8.



Force on either plate (कोई एक प्लेट पर बल)

$$= \frac{(3Q/2)^2}{2A\epsilon_0} = \frac{9Q^2}{8A\epsilon_0}$$

Force on point 'P' due to capacitor = 0

(संधारित्र के कारण बिन्दु P पर बल शून्य है)

Potential diff. between the plates

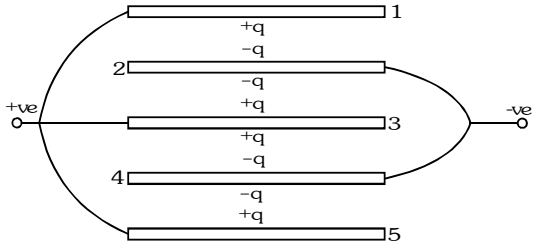
(प्लेटों के मध्य विभवान्तर) $= \frac{3Q}{2C}$

Energy stored in electric field between the plates

(प्लेटों के मध्य विद्युत क्षेत्र में संचित ऊर्जा)

$$= \frac{1}{2}C \times \left(\frac{3Q}{2C}\right)^2 = \frac{9Q^2}{8C}$$

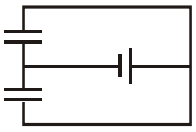
9.



Therefore (इसलिए)

$q_2 = -2q$, $q_3 = +2q$,
$q_4 = -2q$ and $q_5 = +q$

10.



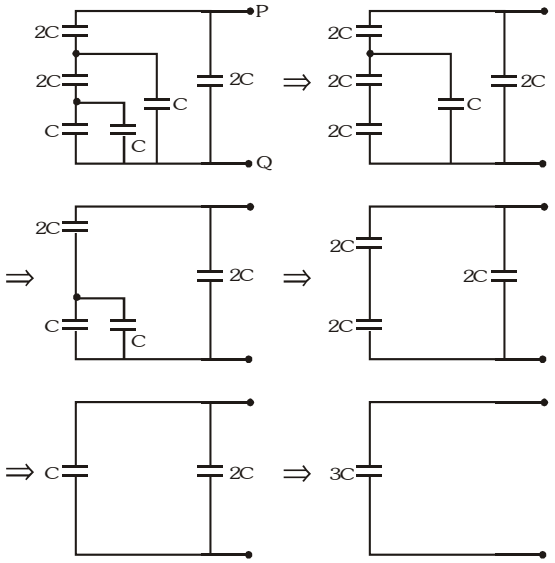
$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}\left(\frac{8.85\times10^{-12}\times0.1}{0.885\times10^{-3}}\times2\right)\times10^2 = 10^{-1}\mu J$$

11. Each capacitor has potential difference 'V' and energy $\frac{1}{2}CV^2$. After reconnecting total energy remains constant and total voltage becomes NV.

(प्रत्येक संधारित्र का विभवान्तर V व ऊर्जा $\frac{1}{2}CV^2$ है। पुनः जोड़ने के बाद कुल ऊर्जा नियत व वोल्टता NV होगी)

12.



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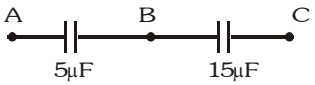


$5(V_A - V_B) = 15\ (V_B - V_C) \Rightarrow 5(2000 - V_B) = 15(V_B - 0)$
$\Rightarrow 2000 - V_B = 3V_B \Rightarrow V_B = 500V$

14. $C_{eff} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + \ldots$

$$= \frac{C}{1-1/2} = 2C = 2\mu F$$

15. For 'n' plates; effective C will be (n–1)C.
(n प्लेटों के लिये, तुल्य धारिता (n–1)C होगी)

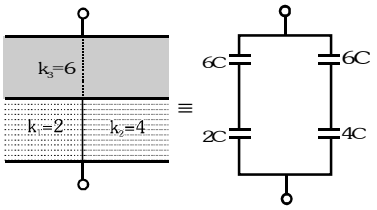
16. $CV + 2CV = KCV' + 2CV' \Rightarrow V' = \frac{3V}{K+2}$

17. $C = \frac{\epsilon_0 A}{d} = 9pF$

$$C' = \frac{\epsilon_0 A}{d - t_1 + \frac{t_1}{K_2} - t_2 + \frac{t_2}{K_2}} = \frac{\epsilon_0 A}{d - \frac{d}{3} + \frac{d}{9} - \frac{2d}{3} + \frac{d}{9}}$$

$$= \frac{9}{2}\frac{\epsilon_0 A}{d} = \frac{81}{2}pF = 40.5pF$$

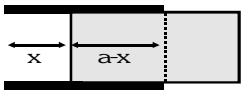
18.



where $C = \frac{\epsilon_0 A}{d}$

$$C_{eq} = \frac{6C\times2C}{8C} + \frac{6C\times4C}{10C} = 3.9\ C$$

19.



$$C = \frac{\epsilon_0 ax}{d} + \frac{K\epsilon_0 (a-x)a}{d}$$

$$C = \frac{K\epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)}{d}x \quad \text{where } x = vt$$

$\therefore$ C– t graph is linear with negative slope.
(C–t वक्र रेखीय होगा जिसका ढाल ऋणात्मक होगा)

20. $\frac{1}{2}CV^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

21. $C = 4\pi \in_0 a$

$$C' = \frac{4\pi \in_0 ab}{b-a} = \frac{4\pi \in_0 a}{1-\frac{a}{b}} = \frac{4\pi \in_0 a}{1-\left(\frac{n-1}{n}\right)} = n(4\pi \in_0 a)C$$

22.

Breaking voltage (भंजन वोल्टता)	20 V	80 V
Safe Voltage (सुरक्षित वोल्टता)	20 V	10 V

∴ Charge on each capacitor = 20 8 = 160 μC
(प्रत्येक संधारित्र पर आवेश)

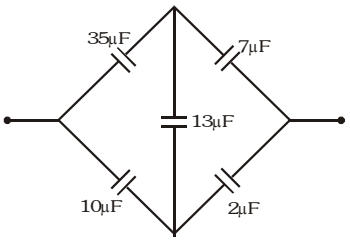
23.

Breaking voltage (भंजन वोल्टता)	6 kV	4 kV
Safe Voltage (सुरक्षित वोल्टता)	6 kV	3 kV

∴ Total voltage (कुल वोल्टता) = 9 kV

24. Capacitance between 1 and 3 and between 2 and 4 are symmetrical.
(1 व 3 व 2 व 4 के मध्य धारिता सममित होगी)

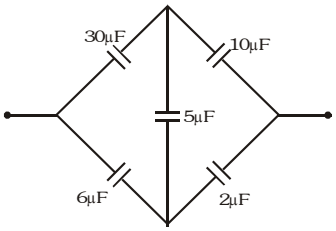
25.



The system is a balanced Wheatstone bridge.
(निकाय व्हीटस्टोन सेतु से सन्तुलित है)

$$\therefore C_{eff} = \left(\frac{35\times 7}{35+7} + \frac{10\times 2}{10+2}\right) = \frac{15}{2}\mu F$$

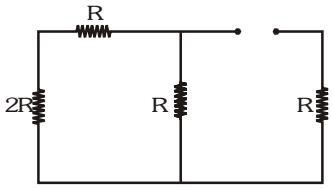
26.



The system is a balanced Wheatstone bridge.
(निकाय व्हीट स्टोन सेतु से संतुलित है)

$$\therefore C_{eff} = \left(\frac{10\times 30}{10+30} + \frac{6\times 2}{6+2}\right) = 9\mu F$$

27.



To find the time constant of a RC circuit, Short circuit the battery
(RC परिपथ का समय नियतांक ज्ञात करने के लिये बैटरी को लघुपथित करते हैं)

$$R_{eff} = \frac{7R}{4} \qquad \therefore \tau = \frac{7RC}{4}$$

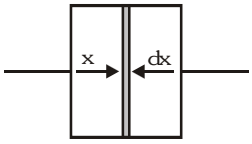
28. There is no closed path for flow of current. Hence no current flows. Hence heat developed is zero.
(यहां धारा के प्रवाह के लिये कोई बंद पथ नहीं है। अत: कोई धारा का प्रवाह नहीं होगा। इसलिये उत्पन्न ऊष्मा का मान शून्य होगा)

29. $V_A = 3\left(\frac{q}{C}\right) = 3 \quad 2.5 = 7.5$ volt

EXERCISE –II

1. $E = \frac{V_0}{d} \Rightarrow E_F < E_D$ Also $\sigma_A > \sigma_B$

2.



$$\int \frac{1}{dC} = \int \frac{dy}{K \in_0 A} = \int_0^d \frac{dy}{\lambda \in_0 A \sec\left(\frac{\pi y}{2d}\right)}$$

$$\Rightarrow C = \frac{\lambda \in_0 A\pi}{2d}$$

3. Both A and B are always in parallel.
(A व B दोनों हमेशा समान्तर क्रम में होंगे)

4. $V = V_0.e^{-t/RC}$

$$\left|\frac{dV}{dt}\right| = \frac{V_0}{RC}e^{-t/RC} = \text{slope (ढाल)}$$

At t = 0, for $R = R_A$;slope is least in curve–3.
(t=0 पर, $R=R_A$ के लिये ढाल वक्र 3 में अधिकतम होगा)

5. $q = q_0e^{-t/\tau} \quad \therefore i = \frac{dq}{dt} = \frac{q_0}{\tau}e^{-t/\tau} = i_0e^{-t/\tau}$

$$\therefore q_0 = i_0\tau$$

Initial stored energy (संचित प्रारम्भिक ऊर्जा)

$$= \frac{1}{2}CV^2 = \frac{1}{2}(CV)V$$

$$= \frac{1}{2}(i_0\tau)(i_0R) = \frac{1}{2}i_0^2R\tau$$

6. As B is in parallel with C and the potential develops slowly. Hence during charging more heat is produced in A than in B. In steady state, same current passes through A and B.
(क्योंकि B, C के साथ समान्तर क्रम में है तथा विभव धीरे–धीरे विकसित होता है। अत: आवेशन के दौरान A में B की तुलना में अधिक ऊष्मा उत्पन्न होती है। स्थायी अवस्था में A से B की ओर समान धारा प्रवाहित होती है)

$$\therefore V_{capacitor} = \frac{E}{2} \qquad \therefore E_{capacitor} = \frac{1}{2}C\left(\frac{E}{2}\right)^2 = \frac{CE^2}{8}$$

7. $q = q_0e^{-t/\tau}$

$$\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{\tau}e^{-t/\tau} = \frac{CV_0}{RC}e^{-t/\tau} = \frac{V_0}{R}e^{-t/\tau}$$

At t=0; $i_1 = \frac{V_0}{R_1}$; $i_2 = \frac{V_0}{R_2}$

$\because R_1 = R_2 \qquad \therefore i_1 = i_2$

As τ is less for C_1 and hence it looses charges faster than C_2.
(चूंकि C_1 के लिये τ कम होता है अत: C_1 में आवेश की हानि C_2 में आवेश की हानि की तुलना में शीघ्र हो जाती है)

8.

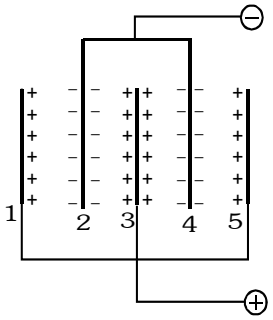
$C_{eff} = 1/4\ \mu F$

$\therefore$ Total no. of rows of capacitor $= \frac{C_{net}}{C_{eff}} = \frac{3}{1/4} = 12$

(संधारित्र की पंक्तियों की कुल संख्या)

$\therefore$ Total no. of capacitors needed = 12 4 = 48
(आवश्यक संधारित्रों की कुल संख्या)

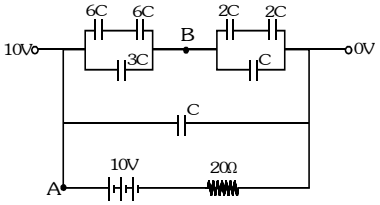
9.



Charge on plate ≠ 1 (प्लेट 1 पर आवेश) $= \frac{\in_0 AV}{d}$

Charge on plate ≠ 4 प्लेट 4 पर आवेश) $= -\frac{2 \in_0 AV}{d}$

10.



$(V_A - V_B)\ 6C = (V_B - 0)\ 2C \Rightarrow V_B = 7.5\ V$
$\therefore V_A - V_B = 10 - 7.5 = 2.5\ V$

11. Force on plate (प्लेट पर बल)

$$= \frac{\sigma^2 A}{2 \in_0} = \frac{Q^2}{2A \in_0} = Kx = mg$$

$$\therefore Q = \sqrt{2mgA \in_0}$$

12. $C_{eff} = C_{EF} = \frac{\in_0 A}{d} \therefore E_{net} = \frac{1}{2}CV^2 = \frac{\in_0 AV^2}{2d}$

13. $i = 10e^{-t/RC} \Rightarrow 2.5 = 10\ e^{-2/RC}$

$\Rightarrow$ RC = $\tau = \frac{1}{\ell n2}$ & C = $\frac{1}{10\ell n2}$

For capacitor (संधारित्र के लिये)

$\frac{V_0}{R} = 10 \Rightarrow V_0$ = 10R = 100 volt

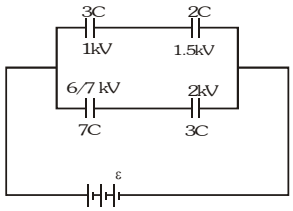
Total heat developed = Total initial energy stored in capacitor. (उत्पन्न कुल ऊष्मा = संधारित्र में संचित कुल प्रारम्भिक ऊर्जा) = $\frac{1}{2}CV^2 = \frac{500}{\ell n2}$

Thermal power in resistor(प्रतिरोध में ऊष्मीय शक्ति)

$P = i^2R = 100Re^{-2t/RC}$

$\therefore$ Time–constant (समय नियतांक)= $\frac{RC}{2} = \frac{1}{2\ell n2}$

14.



Safe voltages in each arm are mentioned.
(प्रत्येक भुजा में सुरक्षित वोल्टता दर्शाई गई है)
$\because$ (1+1.5) < (6/7 +2) $\therefore$ E_{safe} = 1+1.5 = 2.5 kV

15. Time constant (समय नियतांक)

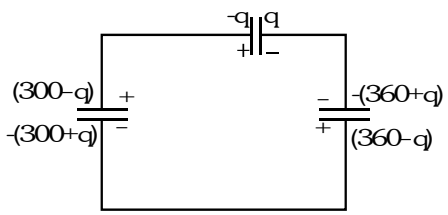
= $CR_{eff} = (100 \times 10^{-6})\left(\frac{10^3}{2}\right)$ s = 50 m/s

16. $i_1 = \frac{V}{R}e^{-t/RC_1}$, $i_2 = \frac{V}{R}e^{-t/RC_2}$

$\therefore \frac{i_1}{i_2} = e^{t/R\left(\frac{1}{C_2}-\frac{1}{C_1}\right)} = e^{+\frac{t}{2RC_2}}$

$\Rightarrow i_1/i_2$ increases with time, t.
(i_1/i_2 समय t के साथ बढ़ता है)

17.



$\Rightarrow \frac{300-q}{2} - \frac{q}{1.5} + \frac{360-q}{3} = 0 \Rightarrow q$ = 180

$\therefore$ $q_{1.5\mu F}$ = 180 μC, $q_{3\mu F}$ = 540 μC, $q_{2\mu F}$ = 480μC

18. $i = \frac{i_0}{2} = i_0e^{-t/RC} \Rightarrow \frac{1}{2} = e^{-\ell n4/RC}$

$\Rightarrow$ RC=2 $\Rightarrow$ (2+r) $\frac{1}{2}$ =2 $\Rightarrow$ r = 2Ω

19. At t=0, $V_C = 0 \Rightarrow i_{R_3} = 0$

$Q_{max} = C\left[\frac{\varepsilon}{\frac{R_1R_2}{R_1+R_2}+R_3}\right] = \frac{10C}{1+1} = 5\times1 = 5\mu C$

$\therefore \left(I_{R_3}\right)_{max} = \frac{V_C}{R_3} = \frac{5}{1} = 5A$

Since R_1 and R_2 are in parallel hence current ratio of R_1 and R_2 will remain same.
(क्योंकि R_1 व R_2 समान्तर क्रम में है अत: R_1 व R_2 की धाराओं का अनुपात समान होगा)

20. $q = q_0e^{-t/RC} \Rightarrow I = \frac{q_0}{RC}e^{-t/RC}$

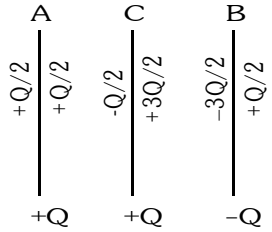
$\Rightarrow \ell nI = \ell n\left(\frac{q_0}{RC}\right) - \frac{t}{RC} = \ell n\left(\frac{V_0}{R}\right) - \frac{t}{RC}$

As I_{max} does not change $\therefore$ R = constant

$\left|\frac{d(\ell nI)}{dt}\right| = \left|0 - \frac{1}{RC}\right| \Rightarrow \left[\frac{d(\ell nI)}{dt}\right]_1 > \left[\frac{d(\ell nI)}{dt}\right]_2$

$\therefore$ $C_2 > C_1 \Rightarrow$ C is increased

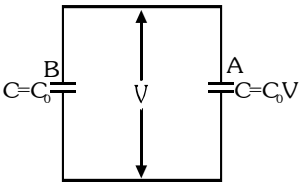
21.



Initial $V'_{AB} = \frac{Q}{C} = \frac{Qd}{\epsilon_0 A}$

Final $V_{AB} = \frac{Q/2}{\left(\frac{2\epsilon_0 A}{d}\right)} + \frac{(3Q/2)}{\left(\frac{2\epsilon_0 A}{d}\right)} = \frac{Qd}{\epsilon_0 A} = V_{AB}$

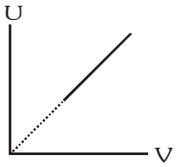
22.



$(C_0 + C_0V)\ V = 30\ C_0$
$\Rightarrow V^2 + V - 30 = 0 \Rightarrow V$ = 5 volt
$\therefore$ $V_A = V_B$ = 5 volt
$Q_A = 5^2C_0$ = 25 C_0; $Q_B = 5C_0$

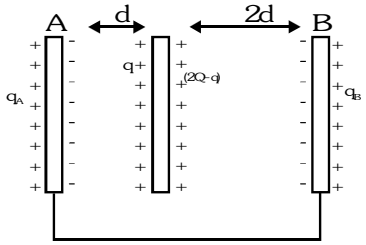
23. $C = \frac{\epsilon_0 ax}{d} + \frac{K\epsilon_0 (a-x)a}{d}$

$= \frac{K\epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)vt}{d}$



$V = \frac{Q}{C}$ and $U = \frac{QV}{2}$ $\therefore \frac{U}{V} = \frac{Q}{2}$

24.



$\Delta V = \frac{qd}{\epsilon_0 A} = \frac{(2Q-q)(2d)}{\epsilon_0 A} \Rightarrow q = \frac{4Q}{3}$

Total charge on inner faces of A and B = –2Q

(A व B की आन्तरिक भुजाओं पर कुल आवेश)

Rest charge will equally appear on their outer faces

(विराम आवेश इसकी बाहरी सतहों पर समान होगा)

$= \frac{Q-(-2Q)}{2} = \frac{3Q}{2}$

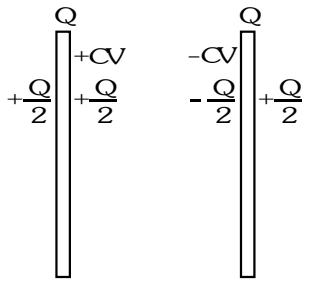
Final charge on plate A (प्लेट A पर अन्तिम आवेश)

$= \frac{3Q}{2} - \frac{4Q}{3} = \frac{Q}{6}$

$\therefore$ Charge flown through wire (तार से आवेश प्रवाह)

$= Q - \frac{Q}{6} = \frac{5Q}{6}$

25. Final charge distribution (अन्तिम आवेश वितरण)



Therefore potential difference across the capacitor

(इसलिए संधारित्र के सिरों पर विभवान्तर)

$= \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$

26. $Q = \frac{C}{2}E$

$Q' = \frac{KCC}{KC+C}E = \frac{KC}{K+1}E$

$\therefore Q'-Q = \frac{KCE}{K+1} - \frac{CE}{2} = \frac{(K-1)CE}{2(K+1)}$

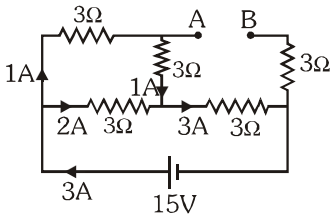
This charge is supplied by battery.

(यह आवेश बैटरी द्वारा दिया जाता है)

27. $C_{eq} = \frac{KC}{K+1}$, $C'_{eq} = \frac{C}{2} \Rightarrow \frac{Q'_2}{Q_2} = \frac{K+1}{2K}$

28. At t= ∞, capacitor gets open circuited

(t= ∞ पर संधारित्र एक खुला परिपथ होता है)



$\therefore I = \frac{15}{5} = 3A \Rightarrow V_A - 3 \quad 1 - 3 \quad 3 = V_B$

$\Rightarrow V_A - V_B = 12$ V

29. At t=0, $V_{capacitors} = 0$

$\Rightarrow I_2 = I_3 = 0$ and $I_1 = \frac{6}{2} = 3A$

At $t \to \infty$, $I_1 = I_3 = \frac{6}{2+8} = 0.6$ A, $I_2 = 0$

30. In steady state (स्थायी अवस्था में)

$I_{upper\ arm} = I_{lower\ arm} = \frac{120}{6} = 20A$

For the right most loop (दांये लूप के लिये)

$3I - 3I + \frac{q}{C_2} = 0 \Rightarrow q = 0$

For the left most loop (बांये लूप के लिये)

$20 \quad 1 + \frac{q}{C_1} - 20 \quad 2 = 0$

$\Rightarrow q = (40-20)\ C_1 = 20C_1 = 40\ \mu C$

31. Charge on 3μF capacitor (3μF संधारित्र पर आवेश)

$= 6 \quad 7 = 42\ \mu C$

$\therefore V_{3\mu F} = \frac{42}{3} = 14$ volt

$\therefore V_{3.9\ \mu F} = 14 + 6 = 20$ volt

Charge on 3.9 μF capacitor = $20 \quad 3.9 = 78\ \mu C$

(3.9 μF संधारित्र पर आवेश)

$\therefore$ Total charge (कुल आवेश)= $78 + 42 = 120\ \mu C$

$$\therefore V_{12\mu F} = \frac{120}{12} = 10V$$

$$\therefore \varepsilon = 20 + 10 = 30\ V$$

32. Energy (ऊर्जा)= $\frac{Q^2}{2C} = \frac{Q^2 d}{2 \in_0 A}$

As d decreases, E decreases

(चूंकि d घटता है, E घटेगा)

33. $Q = CV = \frac{\in_0 AV}{d}$

$$E = \frac{V'}{d} = \frac{V/K}{d} = \frac{V}{Kd}$$

$$W = \frac{1}{2}Q^2\left(\frac{1}{C} - \frac{1}{C'}\right) = \frac{CV^2}{2}\left(1 - \frac{1}{K}\right)$$

34. $\varepsilon = \frac{Q_0}{C_1} \qquad \therefore Q_1 = Q_0; \quad Q_2 = \left(\frac{Q_0}{C_1}\right)C_2$

$$V_1 = V_2 = \varepsilon = \frac{Q_0}{C_1}; \quad U_1 = \frac{1}{2}C_1\left(\frac{Q_0}{C_1}\right)^2 = \frac{Q_0^2}{2C_1}$$

$$U_2 = \frac{1}{2}C_2\left(\frac{Q_0}{C_1}\right)^2 = \frac{Q_0^2 C_2}{2C_1^2}$$

35. S–open ; $V_{inner} = V_{outer}$

S –closed ; $V_{inner} = 0$

$$\Rightarrow \frac{KQ}{3R} + \frac{Kq}{R} = 0 \Rightarrow q = -Q/3$$

$C_{initial} = 4\pi \in_0 (3R)$

$$C_{final} = 4\pi \in_0 (3R) + \frac{4\pi \in_0 (3R)(R)}{(3R - R)}$$

$\therefore C_{final} > C_{initial}$

36. $W_{ext} = -\Delta U = U_i - U_f$

$$= \frac{1}{2} \times 2\mu F \times 400 - \frac{1}{2} \times 1\mu F \times 400 = 200\mu J$$

37. $U_{initial} = \frac{1}{2}CV^2; \qquad U_{final} = \frac{1}{2}CV^2 \quad \therefore \Delta U = 0$

$\therefore$ Heat = work done by battery

(ऊष्मा = बैटरी द्वारा किया गया कार्य)

$= [CV-(-CV)]V = 2CV^2$

38. $eV = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$

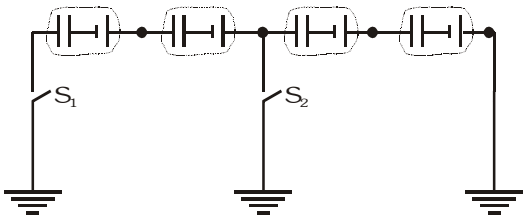
$\Rightarrow 1.6 \quad 10^{-19} \quad 20 = \frac{1}{2} \quad 9.11 \quad 10^{-31} \quad (v^2 - 0)$

$\Rightarrow v = 2.65 \quad 10^6$ m/s

39. V decreases continuously from left to right except in conductor where it is constant.

(चालक में बांये से दांये जाने पर V निरन्तर घटता है, जहां यह नियत होता है)

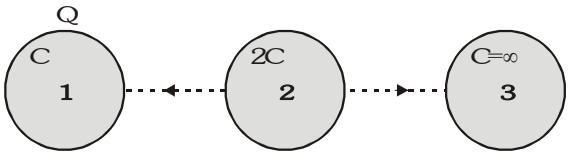
40.



Potential difference across each capacitor and cell combination is zero.

(प्रत्येक संधारित्र तथा सेल संयोजन में सिरों पर विभवान्तर शून्य होगा)

41.



Initial charge on 1 = Q when C_1 & C_2 touches

(1 पर प्रारम्भिक आवेश =Q जब C_1 व C_2 स्पर्श करते हैं)

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{C}{2C} = \frac{1}{2} \Rightarrow Q_1 = \frac{Q}{3}, Q_2 = \frac{2Q}{3}$$

Now when Q_2 & Q_3 is touched

(अब जब Q_2 व Q_3 को स्पर्श कराते हैं)

$$\Rightarrow \frac{Q_2}{Q_3} = \frac{C_2}{C_3} = \frac{2C}{\infty} = 0 \Rightarrow Q_2 = 0$$

Again when Q_1 & Q_2 is touched

(पुनः जब Q_1 व Q_2 को स्पर्श कराते हैं)

$$Q_2 = 2\frac{(Q/3)}{3} \Rightarrow Q_1 = \frac{(Q/3)}{3} = \frac{Q}{9}$$

Similarly we can say after N times it becomes

(इसी प्रकार हम कह सकते हैं कि N बार दोहराने पर यह)

$$Q_1 = \frac{Q}{3^N}$$

42. $\Delta Q = 2CV - (-CV) = 3CV$

$W_B = \Delta Q(2V) = 6CV^2$

$$\Delta U = U_f - U_i = \frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2 = \frac{3CV^2}{2}$$

$$\therefore \text{Heat (ऊर्जा)} = W_B - \Delta U = \frac{9CV^2}{2}$$

$$U_f = \frac{1}{2}C(2V)^2 = 2CV^2$$

$$\therefore \frac{\text{Heat}}{U_f} = \frac{9}{4} = 2.25$$

EXERCISE –III

Fill in the blanks

1. Net charge on capacitor is zero. Hence total flux through a closed surface enclosing the capacitor is zero.
(संधारित्र पर नेट आवेश शून्य है। अतः संधारित्र को परिबद्ध करने वाली बंद सतह से निर्गमित विद्युत फ्लक्स शून्य है)

2. $C_{Maximum}$ = All capacitors are in parallel
(सभी संधारित्र समान्तर क्रम में है)
$= 3C = 18\mu F$
$C_{Minimum}$ = All capacitor are in series
(सभी संधारित्र श्रेणी क्रम में है)
$= C/3 = 2\mu F$

3. $V_1 = V_2 \Rightarrow \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$

4. Charge holding capacity increases, hence capacity increases. (आवेश धारण करने की क्षमता बढ़ती है अतः धारिता बढ़ती है)

5. Air capacitor and dielectric capacitors are in series.
(वायु संधारित्र तथा परावैद्युतांक संधारित्र श्रेणीक्रम में है)

$$\therefore C = \frac{C_1C_2}{C_1 + C_2} = \frac{\left(\frac{2\epsilon_0 A}{d}\right) \times \left(\frac{2K\epsilon_0 A}{d}\right)}{\left(\frac{2\epsilon_0 A}{d}\right)(1+K)} = \frac{2KC}{1+K}$$

Match the column

1. Initial charge (प्रारम्भिक आवेश) $q_1 = \frac{CE}{2}$

Final charge (अन्तिम आवेश) $q_2 = CE$

Initial stored energy (संचित प्रारम्भिक ऊर्जा)

$$U_1 = \frac{1}{2}C(E/2)^2 + \frac{1}{2}C(E/2)^2 = \frac{CE^2}{4}$$

Final stored shergy (संचित अंतिम ऊर्जा) $U_2 = \frac{CE^2}{2}$

Charge supplied by battery (बैटरी द्वारा प्रदान आवेश)

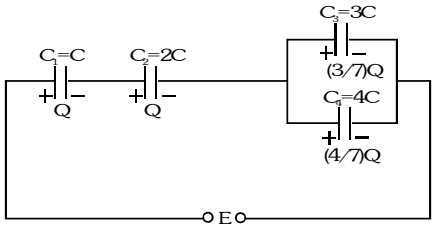
$$\Delta Q = q_2 - q_1 = CE - \frac{CE}{2} = \frac{CE}{2}$$

Work done by battery $W_B = \Delta QE = \frac{CE^2}{2}$
(बैटरी द्वारा किया गया कार्य)

Heat developed in the system(निकाय में उत्पन्न ऊष्मा)

$$H = W_B - \Delta U = \frac{CE^2}{2} - \left(\frac{CE^2}{2} - \frac{CE^2}{4}\right) = \frac{CE^2}{4}$$

2.



At C_1 = $V_1 = \frac{Q}{C}$ and $U_1 = \frac{Q^2}{2C}$

At C_2 = $V_2 = \frac{Q}{2C}$ and $U_2 = \frac{Q^2}{4C}$

At C_3 = $V_3 = \frac{Q}{7C}$ and $U_3 = \frac{3Q^2}{98C}$

At C_4 = $V_4 = \frac{Q}{7C}$ and $U_4 = \frac{4Q^2}{98C}$

Therefore $V_{max} = V_1$ and $V_{min} = V_3 = V_4$
and $U_{max} = U_1$ and $U_{min} = U_3$

Comperehension -1

1. In steady state (स्थायी अवस्था में)

$$I_{circuit} = \frac{V}{R_1 + R_2} = \frac{18}{3+6} = 2A$$

$V_{R_2} = V_{C_2} = IR_2 = 2 \quad 6 = 12$ V

$Q_{C_2} = C_2 V_{C_2} = 12 \quad 4 = 48$ μC

2. $Q_{initial} = Q_{C_1} + Q_{C_2} = IR_1C_1 + IR_2C_2$
$= 3 \quad 2 \quad 2 + 3 \quad 4 \quad 4 = 12 + 48 = 60\mu C$
$Q_{final} = V(C_1 + C_2) = 18(2+4) = 108$ μC
$\therefore \Delta Q = 108 - 60 = 48$ μC (through S_1)

3. $U_{initial} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$

$$= \frac{1}{2} \times 2 \times 6^2 + \frac{1}{2} \times 4 \times 12^2 = 324\ \mu J$$

$U_{final} = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(2+4)18^2 = 972\mu J$
$\Delta Q = Q_f - Q_i = 48$ μC
$W_{Battery} = \Delta Q.V = 48 \quad 18 = 864$ μJ
$\therefore$ Heat = $W_B - \Delta U = 864 - (972 - 324) = 216$ μJ

Comprehension-2

1. Time Constant (समय नियतांक)
$\tau = R_1C = 8 \quad 6 = 48$ μs

2. $V_{t = 2\tau} = V_0(1 - e^{-t/\tau}) = 12(1 - e^{-2\tau/\tau})$

$$= 12\left(1 - \frac{1}{7.4}\right) = 10.4 \text{ V}$$

3. $(V_{R_1})_{t = 2\tau} = V_0 - V_{capacitor} = 12 - 10.4 = 1.6$ V

4. $V_{R_2} = V_0 = 12$V

Comprehension -3

1. $V_b = \varepsilon_0 (1 - e^{-t/RC})$
$\Rightarrow 110 = 120 (1 - e^{-t/RC})$
$\Rightarrow e^{-t/RC} = 1/12$
$\Rightarrow t/RC = \ell n/12 = 2.5$
$\Rightarrow t = RC \quad 2.5 = 10^6 \quad 10^{-6} \quad 2.5 = 5/2$ sec

2. $\tau_0 = 10^{-6} \quad 10 = 10$ μs

3. Flash duration (फ्लेश अन्तराल)= $3\tau_0 = 30$ μs

4. Energy in flash (फ्लेश में ऊर्जा)

$$= \frac{1}{2} CV^2 = \frac{1}{2} \quad 1 \quad 10^{-6} \quad 110 \quad 110 = 6.1 \text{ mJ}$$

Comprehension-4

1. $q_{1max} = q_{2max}$
C_1 and C_2 may be different and hence E_1 and E_2 may be different.(C_1 व C_2 भिन्न हो सकते हैं तथा E_1 व E_2 भिन्न हो सकते हैं)

2. $\tau_2 > \tau_1 \Rightarrow R_2C_2 > R_1C_1 \Rightarrow \frac{R_1}{R_2} < \frac{C_2}{C_1}$

Comprehension-5

1. $\frac{C_A}{C_B} = \frac{\epsilon_0 A/d}{K \epsilon_0 A/d} = 1 : K$

2. $\frac{V_A}{V_B} = \frac{Q/C_A}{Q/C_B} = \frac{C_B}{C_A} = K : 1$

3. $(V_A)_{initial} = \frac{V}{2}$; $(V_A)_{final} = \frac{E}{C} \frac{(KC)}{(K+1)} = \frac{KE}{K+1}$

$\therefore \frac{(V_A)_{Initial}}{(V_A)_{Final}} = \frac{K+1}{2K}$

4. $(V_B)_{Initial} = \frac{V}{2}$; $(V_B)_{Final} = \frac{Q}{C_B} = \frac{E(KC)}{(K+1)} \times \frac{1}{KC} = \frac{E}{K+1}$

$\therefore \frac{(V_B)_{Initial}}{(V_B)_{Final}} = (K+1) : 2$

5. $(U_A)_{final} = \frac{Q^2}{2C_A}$; $(U_B)_{final} = \frac{Q^2}{2C_B}$ $\therefore \left(\frac{U_A}{U_B}\right)_{Final} = K:1$

EXERCISE –IV(A)

1. $CV = \frac{qt}{t} \Rightarrow 400 \quad 10^{-6} \quad 100 = 100\ t$

$\Rightarrow t = 400\ s$

2. Equivalent capacity between A and B
(A व B के मध्य तुल्य धारिता)

$$C = \frac{9}{3} + 3 = 6\mu F$$

(i) Stored charge (संचित आवेश)

$Q = CV = 6 \quad 10^{-6} \quad 4 = 24\ \mu C$

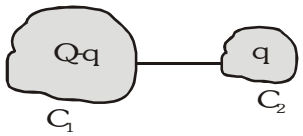
(ii) Stored energy (संचित ऊर्जा)

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \quad 6 \quad 10^{-6} \quad 16 = 48\ \mu J$$

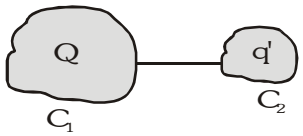
3. Electric field (विद्युत आवेश)

$$E = \frac{V_A - V_B}{d} = \frac{(10,000 - 0)}{(2 \times 10^{-3})} = 5 \quad 10^6\ V/m$$

4.



$$\frac{Q-q}{C_1} = \frac{q}{C_2} \ldots(i)$$



$$\frac{Q}{C_1} = \frac{q'}{C_2} \ldots(ii)$$

Eq. (i) ÷ (ii) : $q' = \frac{Qq}{Q-q}$

5. Common potential (उभयनिष्ठ विभव)

$$V_{cm} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{2 \times 200 + 3 \times 400}{2+3} = 320V$$

Charge on $C_1Q_1 = C_1 V_{cm} = 2 \quad 320\ \mu C = 640\ \mu C$
Charge on $C_2Q_2 = C_2 V_{cm} = 3 \quad 320\ \mu C = 960\ \mu C$

6. (i) On connecting with the second capacitor the charge distributes equally
(दूसरे संधारित्र के साथ जोड़ने पर आवेश समान रूप से वितरित होगा)

$$\therefore V_{CM} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{0.1 \times 10}{0.1 + 0.1} = 5V$$

Total stored energy (संचित कुल ऊर्जा)

$$U_f = \frac{1}{2} C_1V_{CM}^2 + \frac{1}{2} C_2V_{CM}^2$$

$$= \frac{1}{2} \quad 0.1 \quad 10^{-6} \quad (5)^2 + \frac{1}{2} \quad 0.1 \quad 10^{-6} \quad (5)^2$$

$= 2.5\ \mu J$

(ii) Initial stored energy in first capacitor
(पहले संधारित्र में संचित प्रारम्भिक ऊर्जा)

$$U_i = \frac{1}{2} C_1V_1^2 = \frac{1}{2} \quad 0.1 \quad 10^{-6} \quad 10^2 = 5.0\ \mu J$$

$$\Rightarrow \frac{U_f}{U_i} = \frac{2.5}{5.0} = \frac{1}{2}$$

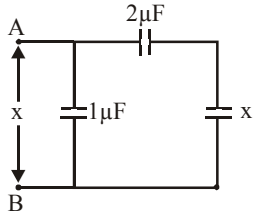
7. $\because C = \frac{\epsilon_0 A}{d}$; $q = \left(\frac{\epsilon_0 A}{d}\right) V$

Slope (ढाल) = $\frac{\epsilon_0 A}{d}$ $\therefore C_2 > C_1 > C_3$

8. By using KCL
$C_1 (V_A - V_0) + C_2 (V_B - V_0) + C_3 (V_C - V_0) = 0 \Rightarrow V_0$

$$= \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}$$

9. $x = \frac{2x}{2+x} + 1$



(Let $C_{eq} = x$)

$$x = \frac{2x + 2 + x}{2 + x}$$

$\Rightarrow x(2 + x) = 3x + 2$
$\Rightarrow 2x + x^2 = 3x + 2 \Rightarrow x^2 - x - 2 = 0$
Use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = 2$$

and -1
$x = 2$, $C_{eq} = 2\mu F$

10. $\frac{C_A}{C_B} = \frac{\left(\frac{K_1 \epsilon_0 A}{d/4}\right)}{\left(\frac{K_2 \epsilon_0 A}{3d/4}\right)} = \frac{3K_1}{K_2} = 3 \quad 3 = 9$

Net capacity

$$C = \frac{C_AC_B}{C_A + C_B} = \frac{(9C_B)(C_B)}{9C_B + C_B} = \frac{9}{10} C_B$$

$$= \frac{9}{10}\left[\frac{K_2 \epsilon_0 A}{(3d/4)}\right] = \frac{6K_2 \epsilon_0 A}{5d} = \frac{1.2K_2 \epsilon_0 A}{d}$$

11. $\because E = \frac{V}{d}$

$$\therefore d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}\ m$$

Now $C = \frac{\epsilon_0 \epsilon_r A}{d}$

$$\Rightarrow A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{88.5 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 10} = 10^{-3}\ m^2$$

12. $C_X = \frac{\varepsilon_0 A}{d}$, $C_Y = \frac{5\varepsilon_0 A}{d} \Rightarrow C_Y = 5C_X$

(i) C_X and C_Y are in series, so charge on each
(C_X व C_Y श्रेणीक्रम में है, अत: प्रत्येक पर आवेश)

$q = C_X V_X = C_Y V_Y \quad \Rightarrow \quad \frac{V_X}{V_Y} = 5$

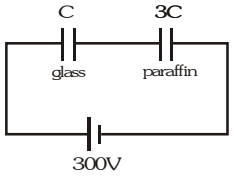
$\because V_X + V_Y = 12 \quad \therefore \quad 6V_Y = 12$

$\Rightarrow V_Y = \frac{12}{6} = 2$ volt and $V_X = 10$ volt

(ii) Energy stored in capacitor (संधारित्र में संचित ऊर्जा)

$$U = \frac{q^2}{2C} \Rightarrow \frac{U_X}{U_Y} = \left(\frac{q^2}{2C_X}\right)\left(\frac{2C_Y}{q^2}\right) = \frac{C_Y}{C_X} = 5$$

13. $CV_1 = 3CV_2$ ....(i)



$V_1 + V_2 = 300$ ...(ii)

$\Rightarrow V_1 = 75V; \quad V_2 = 225$ V

(i) $\therefore E_1 = \frac{V_1}{d_1} = \frac{75 \times 100}{0.5} = 1.5 \quad 10^4$ V/m

$E_2 = \frac{V_2}{d_2} = \frac{225 \times 100}{0.5} = 4.5 \quad 10^4$ V/m

(ii) $V_1 = 75$ V; $\quad V_2 = 225$ V

(iii) $Q = \left(\frac{C_1 C_2}{C_1 + C_2}\right) V = \frac{3}{4} C, V = \frac{3}{4}\left(\frac{2 \in_0 A}{d}\right) 300$

$\Rightarrow \frac{Q}{A} = \frac{6 \times 300 \times 8.89 \times 10^{-12}}{4 \times 0.5 \times 10^{-2}} = 8 \quad 10^{-7}$ C/m^2

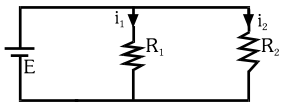
14. When S_{W1} is closed and S_{W2} is open then capacitor B is charged upto 10V.
(जब S_{W1} बंद तथा S_{W2} खुली हुई है तो संधारित्र B, 10V तक आवेशित होगा)

Now S_{W1} is open and S_{W2} is closed then
(अब S_{W1} खुली तथा S_{W2} बंद है तो)

$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{3 \times 10 + 2 \times 0}{3 + 2} = 6V$$

$Q_A = 2 \quad 10^{-6} \; V_{cm} = 12$ μC
$Q_B = 3 \quad 10^{-6} \; V_{cm} = 18$ μC

15. (i) At t = 0, capacitor has zero resistance, i.e., R_1 and R_2 are in parallel.
(t = 0 पर संधारित्र का प्रतिरोध शून्य है अर्थात् R_1 तथा R_2 समान्तर क्रम में है)
The simple circuit is shown in figure
(सरलीकृत परिपथ चित्र में दर्शाये अनुसार है)



$i_1 = \frac{E}{R_1}$ and $i_2 = \frac{E}{R_2}$

(ii) At steady state (t=∞), capacitor has infinite resistance. (स्थायी अवस्था पर (t=∞) संधारित्र का प्रतिरोध अनन्त होगा)

Hence, $i_1 = \frac{E}{R_1}$, $i_2 = 0$

(iii) Final potential difference across capacitor is E. (संधारित्र के सिरों पर अन्तिम विभवान्तर E है)

$\therefore$ Final energy stored (संधारित्र अन्तिम ऊर्जा)

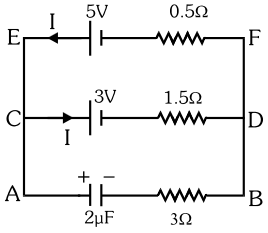
$$U = \frac{1}{2}CE^2$$

(iv) When switch is opened, capacitor will discharge through two resistance as R_1 and R_2 (both in series).
(जब स्विच खुला है तो संधारित्र दो प्रतिरोधों R_1 व R_2 से निरावेशित होगा (दोनों श्रेणीक्रम में)
Hence, $\tau_c = C(R_1 + R_2)$

(v) When switch is closed, capacitor will charged through resistance R_2 .
(जब स्विच बंद है तो संधारित्र प्रतिरोध R_2 से आवेशित होगा)

So $\tau = R_2C$

16. (a) In steady state no current in capacitor's branch. (स्थायी अवस्था में संधारित्र में कोई धारा प्रवाहित नहीं होगी)



So current (अत: धारा) $I = \frac{2}{0.5 + 1.5} = 1$A

voltage across capacitor

(संधारित्र के सिरों पर वोल्टता)

$V_C = 3 + 1.5 \; 1 = 4.5$ V

$\Rightarrow Q = CV_C = 2 \quad 10^{-6} \quad 4.5$

$= 9 \quad 10^{-6}$ C

17. For the circuit ACDA and the cell :
(परिपथ ACDA तथा सेल के लिये)
$6 - I_1(5) - 6 = 0 \Rightarrow I_1 = 0, \quad \therefore I = 0$
For the loop BCD (लूप BCD के लिये): $V_{2\mu F} = 6V$
For the loop ABD (लूप ABD के लिये): $V_{7\mu F} = 6V$
$\therefore Q_{7\mu F} = 6 \quad 7 = 42\ \mu C$

18. Total heat dissipated (उत्पन्न कुल ऊष्मा)

$$H = \frac{1}{2}CV^2 = \frac{1}{2} \quad 5 \quad 10^{-6} \quad 200 \quad 200 = 0.1\ J$$

H_1 = Heat developed across $R_1 = \int I^2 R_1 dt$
(R_1 के सिरों पर उत्पन्न ऊष्मा)

H_2 = Heat developed across $R_2 = \int I^2 R_2 dt$
(R_2 के सिरों पर उत्पन्न ऊष्मा)

$$\Rightarrow H_1 = \frac{(H_1 + H_2)R_1}{(R_1 + R_2)} = \frac{H\ R_1}{(R_1 + R_2)}$$

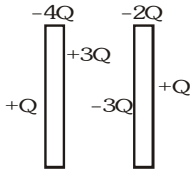
$$= \frac{0.1 \times 500}{(500 + 330)} = 60\ mJ$$

19. $R_{eff} = \frac{2 \times 3}{3 + 2} + 2.8 = 4\Omega$

$$I = \frac{V}{R_{eff}} = \frac{6}{4} = 1.5\ A$$

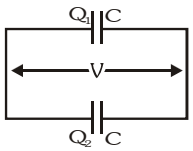
$$\therefore I_{2\Omega} = I\left(\frac{3}{2+3}\right) = \frac{1.5 \times 3}{5} = 0.9\ A$$

20.



Initial effective charge = 3Q
(तुल्य प्रारम्भिक आवेश)

$$CV + CV = Q_1 + Q_2$$
$$= 3Q + 0$$
$$= 3Q$$



$$\therefore V = \frac{3Q}{2C}$$

21. $$\frac{1}{C_{arm}} = \frac{1}{C}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right) = \frac{1}{C\left(1 - \frac{1}{2}\right)} = \frac{2}{C}$$

$$\therefore C_{effective} = 2C_{arm} = \frac{2C}{2} = C$$

22. $E_{final} = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2 d}{2 \epsilon_0 A}$; $E_{initial} = 0$

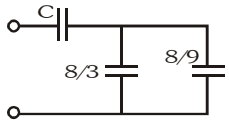
$$\therefore Heat = -(E_{initial} - E_{final}) = \frac{Q^2 d}{2 \epsilon_0 A}$$

23. $V_{2\ initial} = \frac{20}{2} = 10V$

$V_{5\ initial} = \frac{50}{5} = 10V$

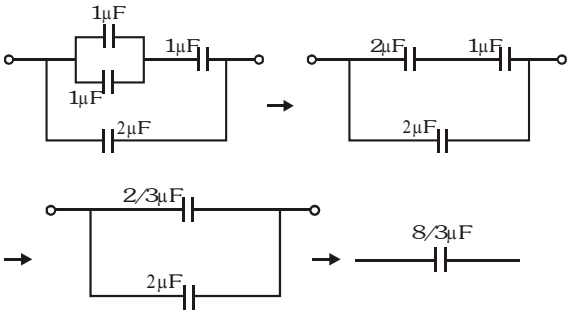
There is no potential difference.
(यहां कोई विभवान्तर नहीं है)
Hence no charge flows.
(अत: आवेश का प्रवाह नहीं होगा)
Heat produce is zero.
(उत्पन्न ऊष्मा शून्य होगी)

24.

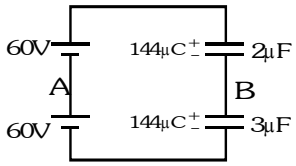


$$\Rightarrow \frac{1}{C} + \frac{9}{32} = 1 \Rightarrow C = \frac{32}{23}\mu F$$

25.

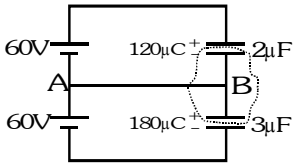


26.



$C_{eff} = \frac{2 \times 3}{2 + 3} = 1.2\mu F$

$\therefore Q_2 = Q_3 = +144\mu C$



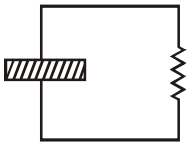
Q_{total} on the middle plates (मध्य प्लेट पर)

$$= +180 + (-120) = +60\ \mu C$$

This charge flows from A to B.
(यह आवेश A से B की ओर प्रवाहित होगा)

EXERCISE –IV(B)

1. $\frac{q}{C} - iR = 0 \Rightarrow \frac{q}{C} + \frac{dq}{dt}R = 0 \Rightarrow q = q_0 e^{-t/RC}$



equivalent circuit $\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC}$

(तुल्य परिपथ)

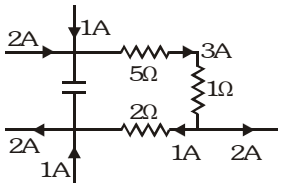
Where $R = \frac{L}{SA}$, $C = \frac{k \in_0 A}{4}$

$\therefore RC = \frac{k \in_0}{S} = \frac{5 \times 8.85 \times 10^{-12}}{7.4 \times 10^{-12}} = \frac{5 \times 8.85}{7.4}$

$\therefore i = \frac{q_0}{R_C} e^{-t/RC} = \frac{8.85 \times 10^{-3}}{\left(\frac{5 \times 8.85}{7.4}\right)} e^{-12/6}$

$= \frac{7.4}{5} \times \frac{1}{7.4}$ mA = 0.2mA

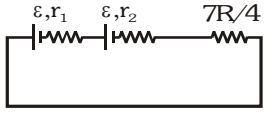
2.



$V_C = (5 + 1) \quad 3 + 2 \quad 1 = 20V$

$U_{cap} = \frac{1}{2} CV^2 = \frac{1}{2} \quad 4 \quad 20^2 = 0.8$ mJ

3.



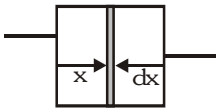
$I = \frac{2\varepsilon}{r_1 + r_2 + \frac{7R}{4}}$

Pot. diff. across (ε, r_1) cell : $\varepsilon - Ir_1 = 0$

(सेल के सिरों पर विभवान्तर)

$\Rightarrow \varepsilon = Ir_1 \Rightarrow \varepsilon = \frac{2\varepsilon r_1}{r_1 + r_2 + \frac{7R}{4}} \Rightarrow \frac{4(r_1 - r_2)}{7} = R$

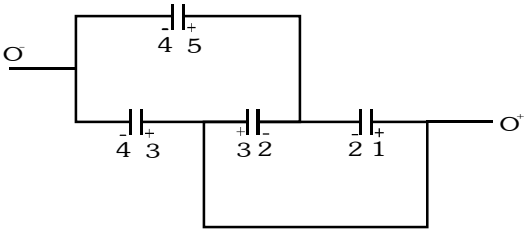
4.



$\int \frac{1}{dC} = \int \frac{dx}{KS \in_0} = \int_0^d \frac{dx}{KS \in_0 \left(1 + \sin\frac{\pi x}{d}\right)}$

$\Rightarrow C = \frac{K_1 S \in_0 \pi}{2d} \left[\int_0^d \frac{dx}{\left(1 + \sin\frac{\pi x}{d}\right)} = \frac{2d}{\pi}\right]$

5. $C_{eq} = \frac{2C}{3} + C = \frac{5C}{3}$



$Q_3 = \frac{4}{3} \in_0 \frac{AV_0}{d}$ & $Q_5 = \frac{2}{3} \frac{\in_0 AV_0}{d}$

6. $Q_{total} = C_1 V = \left[C_1 + \frac{C_2 C_3}{C_2 + C_3}\right] V_0$

$\Rightarrow V_0 = \frac{C_1 (C_2 + C_3) V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$

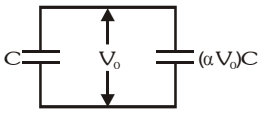
$\therefore$ Charge on C_1,

$q_1 = C_1 V_0 = \frac{C_1^2 V (C_2 + C_3)}{C_1 C_2 + C_2 C_3 + C_3 C_1}$

Charge on C_2 and C_3

$q_2 = q_3 = \left(\frac{C_2 C_3}{C_2 + C_3}\right) V_0 = \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$

7.



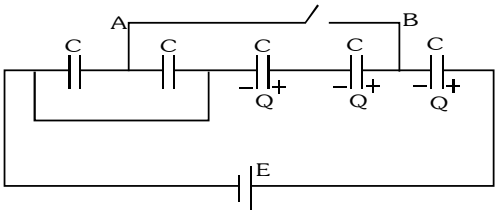
Total charge remains constant(कुल आवेश नियत होगा)

$156\ C = (\alpha V_0) C V_0 + C V_0$

$\Rightarrow V_0^2 + V_0 - 156 = 0$ ($\alpha = 1$)

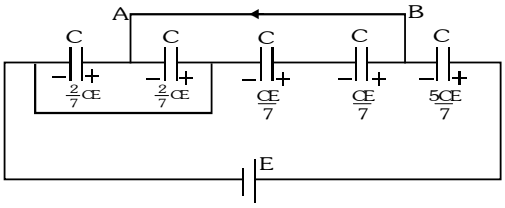
$\Rightarrow (V_0 + 13)(V_0 - 12) = 0 \Rightarrow V_0 = 12$ volt

8. **Initial condition**



$Q = \frac{CE}{3}$

Final condition



Charge flown from B to A = $\frac{4}{7}CE$

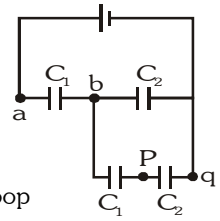
9. Extra weight needed (आवश्यक अतिरिक्त भार)

$$= \left(\frac{6}{\epsilon_0}\right)^2 \times \frac{\epsilon_0 A}{2} = E^2 \quad \frac{\epsilon_0 A}{2} = \left(\frac{V}{d}\right)^2 \frac{\epsilon_0 A}{2}$$

$$\Rightarrow mg = \left(\frac{5000}{5\times10^{-3}}\right)^2 \times \frac{8.85\times10^{-12}\times100}{2\times100\times100}$$

$\Rightarrow$ m = 4.52 10^{-3} kg

11. $Q_{pq} = 2C_2 = 6C_1 = Q_{bp}$



$$\therefore V_{bp} = \frac{6C_1}{C_1} = 6V$$

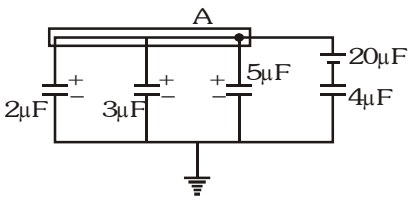
$\therefore V_{bq} = 6 + 2 = 8V$

Total charge flown into right loop
(दांये लूप में से कुल आवेश प्रवाह)

$= C_2V_{bq} + C_1V_{bp}$
$= 3C_1 \quad 8 + C_1 \quad 6 = 30C_1$

$$\therefore V_{ab} = \frac{Q_{total}}{C_{ab}} = \frac{30C_1}{C_1} = 30 \text{ volt}$$

12. Applying junction law at A :
(A पर संधि नियम लगाने पर)

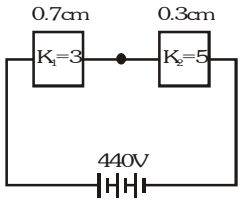


$2(V_A-5) + 3(V_A - 20) + 5(V_A - 10) + 4(V_A - 20)=0$

$$\Rightarrow V_A = \frac{100}{7} = 14.28 \text{ volt}$$

$\therefore Q_{2\mu F} = 28.56\ \mu C$, $Q_{3\mu F} = 42.84\ \mu C$,
$Q_{5\mu F} = 71.40\ \mu C$, $Q_{4\mu F} = 22.88\ \mu C$

13. $V_1C_1 = V_2C_2$ and $V_1 + V_2 = 440$



$$\Rightarrow V_2 = \frac{V_1C_1}{C_2} \Rightarrow V_1 + \frac{V_1C_1}{C_2} = 440$$

$$\Rightarrow V_1 = \frac{440C_2}{C_1 + C_2} = \frac{440}{\frac{C_1}{C_2}+1}$$

$$\Rightarrow V_1 = \frac{440}{\left(\frac{K_1/d_1}{K_2/d_2}\right)+1} = \frac{440}{\frac{9}{35}+1} = \frac{440+35}{44} = 350V$$

$$\therefore E_1 = \frac{V_1}{d} = \frac{350\times100}{0.7} = 5 \quad 10^4 \text{ V/m}$$

$$E_2 = \frac{V_2}{d} = \frac{90}{0.3} \quad 100 = 3 \quad 10^4 \text{ V/m}$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}C_1V_1^2}{\frac{1}{2}C_2V_1^2} = \frac{35}{9}$$

14. Work done by battery (बैटरी द्वारा किया गया कार्य)
$= \Delta QV = (3CV)V = 3CV^2$

Energy stored in capacitors = $\frac{1}{2}(3C)V^2$
(संधारित्र में संचित ऊर्जा)

(i) $\therefore$ Heat developed = $W_B - \Delta U = \frac{1}{2}(3C)V^2$
(उत्पन्न ऊष्मा)

(ii) Work done by external agent = – (K–1)
(बाह्य कारक द्वारा किया गया कार्य)

(iii) Final voltage after 'dielectric is removed = V'
(परावैद्युतांक पट्टिका को हटाने के बाद अन्तिम वोल्टता)

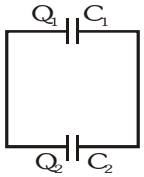
$$3CV' = (K+2)CV \Rightarrow V' = V\left(\frac{K+2}{3}\right)$$

$$W_{agent} = U_i - U_f$$

$$= \frac{1}{2}(3C)V^2\left(\frac{K+2}{3}\right)^2 - \frac{1}{2}(K+2)CV^2$$

$$= \frac{(K+2)(K-1)CV^2}{6}$$

15. $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ....(i)



$Q_1 + Q_2 = 2Q$ ....(ii)

$$C_1 = \frac{\epsilon_0 A}{d-x} \text{ and } C_2 = \frac{\epsilon_0 A}{d+x}$$

$$\Rightarrow Q_2 = \frac{Q(d-x)}{d} \text{ and } Q_1 = \frac{Q(d+x)}{d}$$

$$\Rightarrow \frac{dQ_2}{dt} = -\frac{Q}{2d}\left(\frac{dx}{dt}\right) \ \& \ \frac{dQ_1}{dt} = \frac{Q}{2d}\left(\frac{dx}{dt}\right)$$

$$\therefore I = \frac{dQ_1}{dt} - \frac{dQ_2}{dt} = \frac{Q}{d}\left(\frac{dx}{dt}\right) = \frac{200}{0.1}\times0.001 = 2\mu A$$

16. $U_1 = \frac{Q^2}{2C_1}$, $C_1 = 4\pi \in_0 \left[\frac{ab}{b-a} + b\right]$

$U_2 = \frac{Q^2}{2C_2}$, $C_2 = 4\pi\in_0 b$

$\therefore \Delta U = U_1 - U_2 = 9$ J

17. $C_{initial} = \frac{2C \times C}{2C + C} = \frac{2C}{3}$; $C_{final} = C$

(i) $\therefore \Delta Q = \Delta C \quad V$

$$= \left(C - \frac{2C}{3}\right)V = \frac{CV}{3} = \frac{2 \times 30}{3} = 20\mu C$$

(ii) $H = W_B - \Delta U = \Delta QV - \left(\frac{1}{2}CV^2 - \frac{1}{2}\frac{2CV^2}{3}\right)$

$= 600 - (900 - 600) = 300\ \mu J = 0.3$ mJ

(iii) Energy supplied by the battery

$= \Delta QV = 600\ \mu J = 0.6$ mJ

(बैटरी द्वारा दी गई ऊर्जा)

(iv) Initial charge on each capacitor

(प्रत्येक संधारित्र पर प्रारम्भिक आवेश)

$= \frac{2C}{3}V = 40\mu C$

Final charge on right capacitor = 60 μC

(दांये संधारित्र पर अन्तिम आवेश)

Final charge on left capacitors = 0

(बांये संधारित्र पर अन्तिम आवेश)

$\therefore$ Total charge from through switch, S = 60μC

(स्विच S से प्रवाहित कुल आवेश)

EXERCISE-V-A

1. $C_{eff_{(Parallel)}} = nC$

If connected across V volts then energy stored

(यदि V वोल्ट से जोड़ा जाये तो संचित ऊर्जा)

$= \frac{1}{2}(nC)V^2$

2. Capacitance of an isolated sphere is

(विलगित गोले की धारिता)

$C=(4\pi\epsilon_0)$(Radius) (त्रिज्या)

$$C = \frac{1}{9\times10^9} \times 1 = 0.11 \times 10^{-9} = 1.1 \times 10^{-10} F$$

4. Work done $= \frac{1}{2}\frac{Q^2}{C} = = \frac{1}{2}\frac{\left(8\times10^{-18}\right)^2}{100\times10^{-6}}$

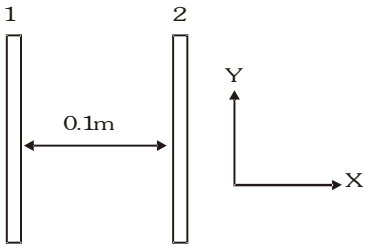
$$= \frac{1}{2} \times \frac{64 \times 10^{-36}}{10^{-4}} = 32 \quad 10^{-32}\ J$$

5. $\frac{1}{2}CV^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

6. Two plates stacked together form a single capacitor of capacitance C. n plates stacked together form (n–1) number of capacitors of effective capacitance (n–1)C.

(दो पट्टिकाओं को सम्बद्ध करके C धारिता का एक संधारित्र बनाया जाता है। n पट्टिकाओं को एक साथ सम्बद्ध करके (n–1)C तुल्य धारिता के (n–1) संधारित्र बनाये जाते हैं)

7.



Applying law of conservation of energy

(ऊर्जा संरक्षण के नियम द्वारा)

We get $\frac{1}{2}mv^2 = eV$

[Here, v = speed of electron, $V=V_2-V_1$=potential difference]

[यहां v= इलेक्ट्रॉन की चाल, $V=V_2=V_1$ = विभवान्तर]

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times20}{9\times10^{-31}}}$$

On solving, we get $v=2.65 \quad 10^6$ m/s

8. Energy stored in a capacitor when it is charged by a potential difference of V_0 volt $=\frac{1}{2}QV_0$

(संधारित्र में संचित ऊर्जा जब इसे V_0 वोल्ट विभवान्तर द्वारा आवेशित किया जाता है $=\frac{1}{2}QV_0$)

Total work done by battery in sending a charge of Q through emf V_0=QV_0

(विद्युत वाहक बल V_0 से प्रवाहित आवेश Q को भेजने में बैटरी द्वारा किया गया कुल कार्य =QV_0)

$$\text{hence } \frac{\text{energy stored in capacitor}}{\text{work done by battery}}=\frac{\frac{1}{2}QV_0}{QV_0}=\frac{1}{2}$$

$$\left[\frac{\text{संधारित्र में संचित ऊर्जा}}{\text{बैटरी द्वारा किया गया कार्य}}=\frac{\frac{1}{2}QV_0}{QV_0}=\frac{1}{2}\right]$$

9. Net work done by the system in the process is zero, as in removing the dielectric, work done is equal and opposite to the work done is re-inserting the dielectric.

(इस प्रक्रिया में निकाय द्वारा किया गया कार्य शून्य है क्योंकि परावैद्युत पट्टिका को हटाने में किया गया कार्य परावैद्युतांक पट्टिका को पुन: प्रवेशित करने में किये गये कार्य के बराबर होता है)

10. $C=\frac{\delta A}{d}=9Pf\ ;\ C_{eq}=\frac{C_1C_2}{C_1+C_2}$

$$\Rightarrow \frac{\left(\frac{3\varepsilon_0AK_1}{d}\right)\left(\frac{3\varepsilon_0AK_2}{2d}\right)}{\frac{3\varepsilon_0AK_1}{d}+\frac{3\varepsilon_0AK_1}{2d}}\Rightarrow \frac{d1}{2}F=40.5pF$$

11. $U=\frac{1}{2}CV^2\ ;\frac{U_0}{2}=\frac{1}{2}CV_0^2e^{-2t_1/RC}$

$\frac{1}{2}=e^{-2t_1/RC}$ $\left(U_0=\frac{1}{2}CV_0^2\right)$

$\frac{2t_1}{RC}=\ell n2$

$t_1=\frac{RC\ell n2}{2}$ ...(i) and $\frac{q_0}{4}=q_0e^{-t_2/RC}$

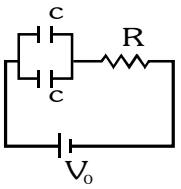
$\frac{t_2}{RC}=2\ell n2\ ;\ t_2=2RC\ \ell n2$...(ii)

from equation (i) and (ii) $\frac{t_1}{t_2}=\frac{1}{4}$

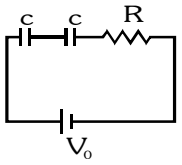
12. $V=V_0\left(1-e^{-t/RC}\right)\Rightarrow 120=200\left(1-e^{-\frac{5}{RC}}\right)$

$\Rightarrow R=2.7\quad 10^6\ \Omega$

13. Parallel



Series



$\frac{v_0}{2}=v_0\left(1-e^{-\frac{t_p}{R\times 2C}}\right)$ ...(i)

$\frac{v_0}{2}=v_0\left(1-e^{-\frac{t_s}{R\times\frac{C}{2}}}\right)$ ...(2)

from (i) and (ii) $e^{-\frac{t_p}{2Rc}}=e^{-\frac{2t_s}{Rc}}$

$t_s=\frac{t_p}{4}=\frac{10}{4}=2.5$ sec

14. t = 0.37% of V_0

= 0.37 25 = 9.25 volt

where is in between 100 and 150 sec.

15. Common voltage (उभयनिष्ठ वोल्टता)= $\frac{C_1v_1-C_2V_2}{C_1+C_2}$

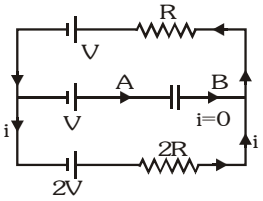
(positive plate of one capacitor is connected with negative plate of second capacitor)

(एक संधारित्र की धनात्मक प्लेट को दूसरे संधारित्र की ऋणात्मक प्लेट से जोड़ा जाता है)

$\Rightarrow 120\ C_1=200\ C_2\Rightarrow 3C_1=5C_2$

EXERCISE –V-B

1. In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,
(स्थायी अवस्था की स्थिति में, संधारित्र C में कोई धारा प्रवाहित नहीं होगी। बाहरी परिपथ में धारा होगी)



$i = \dfrac{2V - V}{2R + R} = \dfrac{V}{3R}$

Potential difference between A and B :
(A व B के मध्य विभवान्तर)

$V_A - V + V + iR = V_B$

$\therefore V_B - V_A = iR = \left(\dfrac{V}{3R}\right)R = \dfrac{V}{3}$

2. Charging current (आवेशित धारा) $I = \dfrac{E}{R}e^{-\frac{t}{RC}}$

Taking log both sides (दोनों पक्षों का log लेने पर)

$$\log I = \log\left(\frac{E}{R}\right) - \frac{t}{RC}$$

When R is doubled, slope of curve increase. Also at t=0, the current will be less. Graph Q represents the best.
(जब R दुगुना है तो वक्र का ढाल बढ़ेगा। t= 0 पर धारा कम होगी। वक्र Q अच्छी तरफ प्रदर्शित करता है)

3. Given : $V_C = 3V_R = 3(V - V_C)$
Here, V is the applied potential.
(यहां V आरोपित विभव है)

$\therefore V_C = \dfrac{3}{4}V \Rightarrow V(1-e^{-t/RC}) = \dfrac{3}{4}V \quad \therefore e^{-t/RC} = \dfrac{1}{4}$

Here $\tau_c = cR = 10s$
Substituting this value of τ_c in equation and solving
(τ_c का मान समीकरण में रखने पर)
We get : t = 13.86 s

4. $\tau = CR$

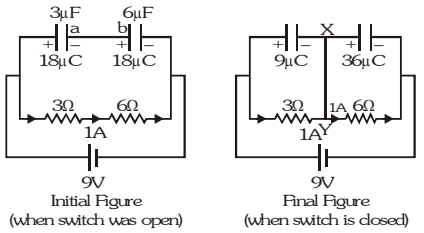
$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\ \mu s$

$\tau_2 = \left(\dfrac{C_1C_2}{C_1 + C_2}\right)\left(\dfrac{R_1R_2}{R_1 + R_2}\right) = \dfrac{8}{6} \times \dfrac{2}{3} = \dfrac{8}{9}\mu s$

$\tau_3 = (C_1 + C_2)\left(\dfrac{R_1R_2}{R_1 + R_2}\right) = (6)\left(\dfrac{2}{3}\right) = 4\mu s$

5. From Y to X charge flows to plates a and b.
(प्लेट a व b से आवेश प्रवाहित Y से X की ओर होगा)

$(q_a + q_b)_i = 0,\ (q_a + q_b)_f = 27\mu C$



$\therefore$ 27μC charge flows from Y to X.
(Y से X की ओर आवेश प्रवाह 27μC होगा)

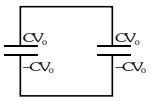
6. Time constant (समय नियतांक)= RC

Where $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{\left(\frac{2d}{3} + Vt\right)}{\epsilon_0} + \dfrac{\left(\frac{d}{3} - Vt\right)}{2\epsilon_0}$

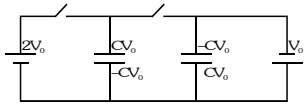
$$\Rightarrow C = \frac{6\epsilon_0}{5d + 3Vt}$$

MCQ's

1. Before S_3 is pressed (S_3 को दबाने से पहले)

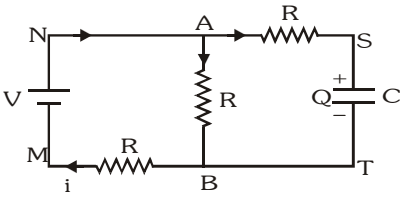


After S_3 is pressed (S_3 को दबाने के बाद)



Subjective

1. Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t.
(माना किसी समय t पर संधारित्र C में आवेश Q है तथा धारा चित्रानुसार है, चूंकि आवेश Q समय t के साथ बढ़ता है)



$\therefore i_1 = \dfrac{dQ}{dt}$

(i) Applying Kirchhoff's second law in loop MNABM
(लूप MNABM में किरचॉफ के द्वितीय नियम से)

$V=(i - i_1)R + iR \Rightarrow V = 2iR - i_1R$ ...(i)

Simillarly, applying Kirchhoff's second law in loop MNSTM
(लूप MNSTM में किरचॉफ के द्वितीय नियम से)

we have $V = i_1R + \frac{Q}{C} + iR$ ....(ii)

Eliminating i from equations (i) and (ii), we get

$$V = 3i_1R + \frac{2Q}{C} \Rightarrow 3i_1R = V - \frac{2Q}{C}$$

$$\Rightarrow i_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right) \Rightarrow \frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R} \Rightarrow \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}(1 - e^{-2t/3RC})$$

(ii) $i_1 = \frac{dQ}{dt} = \frac{V}{3R}e^{-2t/3RC}$

From equation (i)

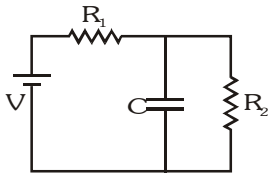
$$i = \frac{V + i_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

∴ Current through AB (AB से प्रवाहित धारा)

$$i_2 = i - i_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC} \Rightarrow i_2 = \frac{V}{2R} \text{ as } t \to \infty$$

2. Q_0 is the steady state charge stored in the capacitor.
(Q_0 स्थायी अवस्था में संधारित्र में संचित आवेश है)



Q_0 = C [PD across capacitor in steady state]

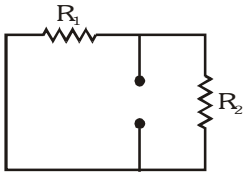
[स्थायी अवस्था में संधारित्र के सिरों पर विभवान्तर]

=C[steady state current through R_2] (R_2)

[स्थायी अवस्था में R_2 में धारा] (R_2)

$$= C\left(\frac{V}{R_1 + R_2}\right)R_2$$

$$\therefore Q_0 = \frac{CV\,R_2}{R_1 + R_2} \; \alpha \text{ is } \frac{1}{\tau_C} \Rightarrow \frac{1}{C\,R_{net}}$$



Here, R_{net} is equivalent resistance across capacitor after short circuiting the battery.

(यहां R_{net} बैटरी को लघुपथित करने पर संधारित्र के सिरों पर तुल्य प्रतिरोध है)

$$R_{net} = \frac{R_1R_2}{R_1 + R_2}$$

(As R_1 and R_2 are in parallel)

(चूंकि R_1 व R_2 समान्तर है)

$$\alpha = \frac{1}{C\left(\frac{R_1R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{C\,R_1R_2}$$