EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If ABCDEF is a regular hexagon and if $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\lambda\overrightarrow{AD}$, then λ is -

(A) 0 (B) 1 (C) 2 (D) 3

2. If $\vec{a}+\vec{b}$ is along the angle bisector of $\vec{a}$ & $\vec{b}$ then -

(A) $\vec{a}$ & $\vec{b}$ are perpendicular (B) $|\vec{a}|=|\vec{b}|$

(C) angle between $\vec{a}$ & $\vec{b}$ is 60 (D) $|\vec{a}|\neq|\vec{b}|$

3. Given the points A(–2, 3, –4), B(3, 2, 5), C(1, –1, 2) & D(3, 2, –4). The projection of the vector $\overrightarrow{AB}$ on the vector $\overrightarrow{CD}$ is -

(A) $\frac{22}{3}$ (B) $-\frac{21}{4}$ (C) $-\frac{47}{7}$ (D) –47

4. The vectors $\overrightarrow{AB}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{BC}=-\hat{i}+2\hat{k}$ are the adjacent sides of a parallelogram ABCD then the angle between the diagonals is -

(A) $\cos^{-1}\left(\sqrt{\frac{1}{85}}\right)$ (B) $\pi-\cos^{-1}\left(\sqrt{\frac{49}{85}}\right)$ (C) $\cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ (D) $\cos^{-1}\left(\sqrt{\frac{3}{10}}\right)$

5. The values of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are -

(A) –2 and 1 (B) 2 and –1 (C) 2 and 1 (D) –2 and –1

6. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=\hat{i}+2\hat{j}-\hat{k}$, then the value of $\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}=$

(A) 2 (B) 4 (C) 16 (D) 64

7. The area of the triangle whose vertices are A (1, –1, 2) ; B (2, 1, –1) ; C (3, –1, 2) is -

(A) $\sqrt{13}$ (B) $2\sqrt{13}$ (C) 13 (D) none

8. Let $\vec{a}=\hat{i}+\hat{j}$ & $\vec{b}=2\hat{i}-\hat{k}$. The point of intersection of the lines $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$ & $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ is -

(A) $-\hat{i}+\hat{j}+\hat{k}$ (B) $3\hat{i}-\hat{j}+\hat{k}$ (C) $3\hat{i}+\hat{j}-\hat{k}$ (D) $\hat{i}-\hat{j}-\hat{k}$

9. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors and λ is a real number then $\left[\lambda(\vec{a}+\vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}\right]=\left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}\right]$ for -

(A) exactly two values of λ (B) exactly three values of λ

(C) no value of λ (D) exactly one value of λ

10. Volume of the tetrahedron whose vertices are represented by the position vectors, A(0, 1, 2) ; B(3, 0, 1) ; C(4, 3, 6) & D(2, 3, 2) is -

(A) 3 (B) 6 (C) 36 (D) none

11. The sine of angle formed by the lateral face ADC and plane of the base ABC of the tetrahedron ABCD where A ≡ (3, –2, 1) ; B ≡ (3, 1, 5); C ≡ (4, 0, 3) and D ≡ (1, 0, 0) is -

(A) $\frac{2}{\sqrt{29}}$ (B) $\frac{5}{\sqrt{29}}$ (C) $\frac{3\sqrt{3}}{\sqrt{29}}$ (D) $\frac{-2}{\sqrt{29}}$

12. Given the vertices A (2, 3, 1), B (4, 1, –2), C (6, 3, 7) & D (–5, –4, 8) of a tetrahedron. The length of the altitude drawn from the vertex D is -

(A) 7 (B) 9 (C) 11 (D) none

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that $\vec{a}$ and $\vec{b}$ are non-collinear & satisfies $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between the vectors $\vec{b}$ and $\vec{c}$ then sinθ equals -

(A) $\frac{2}{3}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{1}{3}$ (D) $\frac{2\sqrt{2}}{3}$

14. The value of $\tilde{i} \times (\vec{r} \times \tilde{i}) + \tilde{j} \times (\vec{r} \times \tilde{j}) + \tilde{k} \times (\vec{r} \times \tilde{k})$ is -

(A) $\vec{r}$ (B) $2\vec{r}$ (C) $3\vec{r}$ (D) $4\vec{r}$

15. A, B, C, D be four points in a space and if, $|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}| = \lambda$ (area of triangle ABC) then the value of λ is -

(A) 4 (B) 2 (C) 1 (D) none of these

16. If the volume of the parallelopiped whose conterminous edges are represented by $-12\tilde{i} + \lambda\tilde{k},\ 3\tilde{j} - \tilde{k},\ 2\tilde{i} + \tilde{j} - 15\tilde{k}$ is 546, then λ equals-

(A) 3 (B) 2 (C) –3 (D) –2

17. Let $\vec{a} = 2\tilde{i} + 3\tilde{j} - \tilde{k}$ and $\vec{b} = \tilde{i} - 2\tilde{j} + 3\tilde{k}$. Then the value of λ for which the vector $\vec{c} = \lambda\tilde{i} + \tilde{j} + (2\lambda - 1)\tilde{k}$ is parallel to the plane containing $\vec{a}$ and $\vec{b}$, is-

(A) 1 (B) 0 (C) –1 (D) 2

18. If $\vec{a} + 5\vec{b} = \vec{c}$ and $\vec{a} - 7\vec{b} = 2\vec{c}$, then-

(A) $\vec{a}$ and $\vec{c}$ are like but $\vec{b}$ and $\vec{c}$ are unlike vectors

(B) $\vec{a}$ and $\vec{b}$ are unlike vectors and so also $\vec{a}$ and $\vec{c}$

(C) $\vec{b}$ and $\vec{c}$ are like but $\vec{a}$ and $\vec{b}$ are unlike vectors

(D) $\vec{a}$ and $\vec{c}$ are unlike vectors and so also $\vec{b}$ and $\vec{c}$

19. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar and $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors to $\vec{a}, \vec{b}$ and $\vec{c}$ respectively, then $(\ell\vec{a} + m\vec{b} + n\vec{c}).(\ell\vec{p} + m\vec{q} + n\vec{r})$ is equal to : (where ℓ, m, n are scalars)

(A) $\ell^2 + m^2 + n^2$ (B) ℓm + mn + nℓ (C) 0 (D) none of these

20. If $\vec{x}$ & $\vec{y}$ are two non collinear vectors and a, b, c represent the sides of a ΔABC satisfying $(a-b)\vec{x} + (b-c)\vec{y} + (c-a)(\vec{x} \times \vec{y}) = 0$ then ΔABC is -

(A) an acute angle triangle (B) an obtuse angle triangle

(C) a right angle triangle (D) a scalene triangle

21. If $\vec{A}, \vec{B}$ and $\vec{C}$ are three non-coplanar vectors then $(\vec{A} + \vec{B} + \vec{C}).[(\vec{A} + \vec{B})\ \ (\vec{A} + \vec{C})]$ equals -

(A) 0 (B) $[\vec{A}\ \vec{B}\ \vec{C}]$ (C) $2[\vec{A}\ \vec{B}\ \vec{C}]$ (D) $-[\vec{A}\ \vec{B}\ \vec{C}]$

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

22. ABCD is a parallelogram. E and F be the middle points of the sides AB and BC, then -

(A) DE trisect AC

(B) DF trisect AC

(C) DE divide AC in ratio 2 : 3

(D) DF divide AC in ratio 3 : 2

23. $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude then angle between $\vec{a}+\vec{b}+\vec{c}$ and $\vec{a}$ is -

(A) $\cos^{-1}\left(\frac{1}{3}\right)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(C) $\pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(D) $\tan^{-1}\sqrt{2}$

24. If $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}.\vec{b}\neq 0$, $\vec{b}.\vec{c}\neq 0$ then $\vec{a}$ and $\vec{c}$ are -

(A) perpendicular

(B) parallel

(C) non collinear

(D) linearly dependent

25. If $\vec{a}, \vec{b}$ & $\vec{c}$ are non coplanar unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between -

(A) $\vec{a}$ & $\vec{b}$ is $\frac{3\pi}{4}$

(B) $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{4}$

(C) $\vec{a}$ & $\vec{c}$ is $\frac{3\pi}{4}$

(D) $\vec{a}$ & $\vec{c}$ is $\frac{\pi}{4}$

26. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$ are position vectors of 6 points A, B, C, D, E & F respectively such that $3\vec{a}+4\vec{b} = 6\vec{c}+\vec{d} = 4\vec{e}+3\vec{f} = \vec{x}$, then -

(A) $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$

(B) line AB, CD and EF are concurrent

(C) $\frac{\vec{x}}{7}$ is position vector of the point dividing CD in ratio 1 : 6

(D) A, B, C, D, E & F are coplanar

27. Read the following statement carefully and identify the true statement -

(a) Two lines parallel to a third line are parallel.

(b) Two lines perpendicular to a third line are parallel.

(c) Two lines parallel to a plane are parallel.

(d) Two lines perpendicular to a plane are parallel.

(e) Two lines either intersect or are parallel.

(A) a & b

(B) a & d

(C) d & e

(D) a

28. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is -

(A) unit vector

(B) makes an angle $\pi/3$ with vector $2\hat{i}-4\hat{j}+3\hat{k}$

(C) parallel to the vector $-\hat{i}+\hat{j}-(1/2)\hat{k}$

(D) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

29. If a vector $\vec{r}$ of magnitude $3\sqrt{6}$ is collinear with the bisector of the angle between the vectors $\vec{a} = 7\hat{i}-4\hat{j}-4\hat{k}$ & $\vec{b} = -2\hat{i}-\hat{j}+2\hat{k}$, then $\vec{r} =$

(A) $\hat{i}-7\hat{j}+2\hat{k}$

(B) $\hat{i}+7\hat{j}-2\hat{k}$

(C) $\frac{13\hat{i}-\hat{j}-10\hat{k}}{\sqrt{5}}$

(D) $\hat{i}-7\hat{j}-2\hat{k}$

30. A parallelopiped is formed by planes drawn through the points (1, 2, 3) and (9, 8, 5) parallel to the coordinate planes then which of the following is the length of an edge of this rectangular parallelopiped -

(A) 2 (B) 4 (C) 6 (D) 8

31. If $A(\bar{a})$; $B(\bar{b})$; $C(\bar{c})$ and $D(\bar{d})$ are four points such that $\bar{a} = -2\tilde{i} + 4\tilde{j} + 3\tilde{k}$; $\bar{b} = 2\tilde{i} - 8\tilde{j}$; $\bar{c} = \tilde{i} - 3\tilde{j} + 5\tilde{k}$; $\bar{d} = 4\tilde{i} + \tilde{j} - 7\tilde{k}$, d is the shortest distance between the lines AB and CD, then

(A) d = 0, hence AB and CD intersect

(B) $d = \dfrac{[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{BD}]}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$

(C) AB and CD are skew lines and $d = \dfrac{23}{13}$

(D) $d = \dfrac{[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{AC}]}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	C	D	C	C	A	C	C	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	D	B	A	C	B	A	A	A
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	D	A,B	B,D	B,D	A,D	B,C	B,D	A,C,D	A,C	A,C,D
Que.	31									
Ans.	B,C,D									

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is magnitude $\sqrt{2/3}$ is -

(A) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (B) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (C) $-2\hat{i} - 5\hat{j} + \hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

2. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{r}_1 = \vec{a} - \vec{b} + \vec{c}$, $\vec{r}_2 = \vec{b} + \vec{c} - \vec{a}$, $\vec{r}_3 = \vec{c} + \vec{a} + \vec{b}$, $\vec{r} = 2\vec{a} - 3\vec{b} + 4\vec{c}$. If $\vec{r} = \lambda_1\vec{r}_1 + \lambda_2\vec{r}_2 + \lambda_3\vec{r}_3$, then -

(A) $\lambda_1 = 7$ (B) $\lambda_1 + \lambda_3 = 3$ (C) $\lambda_1 + \lambda_2 + \lambda_3 = 4$ (D) $\lambda_3 + \lambda_2 = 2$

3. Taken on side $\overrightarrow{AC}$ of a triangle ABC, a point M such that $\overrightarrow{AM} = \frac{1}{3}\overrightarrow{AC}$. A point N is taken on the side $\overrightarrow{CB}$ such that $\overrightarrow{BN} = \overrightarrow{CB}$ then, for the point of intersection X of $\overrightarrow{AB}$ & $\overrightarrow{MN}$ which of the following holds good ?

(A) $\overrightarrow{XB} = \frac{1}{3}\overrightarrow{AB}$ (B) $\overrightarrow{AX} = \frac{1}{2}\overrightarrow{AB}$ (C) $\overrightarrow{XN} = \frac{3}{4}\overrightarrow{MN}$ (D) $\overrightarrow{XM} = 3\overrightarrow{XN}$

4. Vector $\vec{A}$ has components A_1, A_2, A_3 along the three axes. If the co-ordinates system is rotated by 90 about z-axis, then the new components along the axes are -

(A) $A_1, -A_2, A_3$ (B) $-A_1, -A_2, A_3$ (C) $A_2, -A_1, A_3$ (D) $-A_2, -A_1, A_3$

5. Let $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{p} \times ((\vec{x} - \vec{q}) \times \vec{p}) + \vec{q} \times ((\vec{x} - \vec{r}) \times \vec{q}) + \vec{r} \times ((\vec{x} - \vec{p}) \times \vec{r}) = 0$. Then $\vec{x}$ is given by -

(A) $\frac{1}{2}(\vec{p} + \vec{q} - 2\vec{r})$ (B) $\frac{1}{3}(\vec{p} + \vec{q} + \vec{r})$ (C) $\frac{1}{2}(\vec{p} + \vec{q} + \vec{r})$ (D) $\frac{1}{3}(2\vec{p} + \vec{q} - \vec{r})$

6. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k}), \frac{1}{5}(-4\hat{i} - 3\hat{k}), \hat{j}$ is -

(A) $5\hat{i} + \hat{j} + 5\hat{k}$ (B) $-5\hat{i} + \hat{j} + 5\hat{k}$ (C) $5\hat{i} - \hat{j} - 5\hat{k}$ (D) $5\hat{i} + \hat{j} - 5\hat{k}$

7. The triple product $\left(\vec{d} + \vec{a}\right)\left[\vec{a} \times \left(\vec{b} \times \left(\vec{c} \times \vec{d}\right)\right)\right]$ simplifies to -

(A) $(\vec{b}.\ \vec{d})[\vec{d}\ \vec{a}\ \vec{c}]$ (B) $(\vec{b}.\vec{c})[\vec{a}\ \vec{b}\ \vec{d}]$ (C) $(\vec{b}.\vec{a})[\vec{a}\ \vec{b}\ \vec{d}]$ (D) none

8. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and ℓ,m,n are distinct real numbers, then $\left[(\ell\vec{a} + m\vec{b} + n\vec{c})\ (\ell\vec{b} + m\vec{c} + n\vec{a})\ (\ell\vec{c} + m\vec{a} + n\vec{b})\right] = 0$ implies -

(A) $\ell m + mn + n\ell = 0$ (B) $\ell + m + n = 0$ (C) $\ell^2 + m^2 + n^2 = 0$ (D) $\ell^3 + m^3 + n^3 = 0$

9. If unit vectors $\hat{i}$ & $\hat{j}$ are at right angles to each other and $\vec{p} = 3\hat{i} + 4\hat{j}$, $\vec{q} = 5\hat{i}$, $4\vec{r} = \vec{p} + \vec{q}$ and $2\vec{s} = \vec{p} - \vec{q}$, then -

(A) $|\vec{r} + k\vec{s}| = |\vec{r} - k\vec{s}|$ for all real k (B) $\vec{r}$ is perpendicular to $\vec{s}$

(C) $\vec{r} + \vec{s}$ is perpendicular to $\vec{r} - \vec{s}$ (D) $|\vec{r}| = |\vec{s}| = |\vec{p}| = |\vec{q}|$

10. The three vectors $\hat{i} + \hat{j},\ \hat{j} + \hat{k},\ \hat{k} + \hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume :

(A) $\frac{1}{3}$ (B) 4 (C) $\frac{3\sqrt{3}}{4}$ (D) $\frac{4}{3\sqrt{3}}$

11. If a, b, c are different real numbers and $a\hat{i}+b\hat{j}+c\hat{k}$; $b\hat{i}+c\hat{j}+a\hat{k}$ & $c\hat{i}+a\hat{j}+b\hat{k}$ are position vectors of three non-collinear points A, B & C then -

(A) centroid of triangle ABC is $\frac{a+b+c}{3}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(B) $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

12. Identify the statement (s) which is/are incorrect ?

(A) $\vec{a}\times\left[\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right]=\left(\vec{a}\times\vec{b}\right)\left(\vec{a}^2\right)$

(B) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar vectors and $\vec{v}.\vec{a}=\vec{v}.\vec{b}=\vec{v}.\vec{c}=0$ then $\vec{v}$ must be a null vector

(C) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{c}$ and $\vec{d}$ then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=0$

(D) If $\vec{a},\vec{b},\vec{c}$ and $\vec{a}\,',\vec{b}\,',\vec{c}\,'$ are reciprocal system of vectors then $\vec{a}.\vec{b}\,'+\vec{b}.\vec{c}\,'+\vec{c}.\vec{a}\,'=3$

13. Given a parallelogram OACB. The lengths of the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ & $\overrightarrow{AB}$ are a, b & c respectively. The scalar product of the vectors $\overrightarrow{OC}$ & $\overrightarrow{OB}$ is -

(A) $\frac{a^2-3b^2+c^2}{2}$ (B) $\frac{3a^2+b^2-c^2}{2}$ (C) $\frac{3a^2-b^2+c^2}{2}$ (D) $\frac{a^2+3b^2-c^2}{2}$

14. Consider ΔABC with $A\equiv(\vec{a})$, $B\equiv(\vec{b})$ and $C=(\vec{c})$. If $\vec{b}.(\vec{a}+\vec{c})=\vec{b}.\vec{b}+\vec{a}.\vec{c}$; $|\vec{b}-\vec{a}|=3$; $|\vec{c}-\vec{b}|=4$, then the angle between the medians $\overrightarrow{AM}$ and $\overrightarrow{BD}$ is -

(A) $\pi-\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$ (B) $\pi-\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$ (C) $\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$ (D) $\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$

15. If the non zero vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other then the solution of the equation, $\vec{r}\times\vec{a}=\vec{b}$ is -

(A) $\vec{r}=x\vec{a}+\frac{1}{\vec{a}.\vec{a}}\left(\vec{a}\times\vec{b}\right)$ (B) $\vec{r}=x\vec{b}-\frac{1}{\vec{b}.\vec{b}}\left(\vec{a}\times\vec{b}\right)$ (C) $\vec{r}=x\left(\vec{a}\times\vec{b}\right)$ (D) none of these

16. $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non coplanar vectors and $\vec{r}$ be any arbitrary vector, then

$(\vec{a}\quad\vec{b})\ (\vec{r}\quad\vec{c})+(\vec{b}\quad\vec{c})\ (\vec{r}\quad\vec{a})+(\vec{c}\quad\vec{a})\ (\vec{r}\quad\vec{b})$ is equal to-

(A) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (B) $2[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (C) $3[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (D) none of these

17. $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors. $\vec{r}$ is a vector satisfying $\vec{r}.\vec{a}=0$, $\vec{r}.\vec{b}=1$ and $[\vec{r}\ \vec{a}\ \vec{b}]=1$, then $\vec{r}$ is -

(A) $\vec{a}+(\vec{a}\quad\vec{b})$ (B) $\vec{b}+(\vec{a}\quad\vec{b})$ (C) $\vec{a}+\vec{b}(\vec{a}\quad\vec{b})$ (D) $\vec{a}-\vec{b}+(\vec{a}\quad\vec{b})$'

BRAIN TEASERS **ANSWER KEY** EXERCISE - 02

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,C	B,C	B,C	C	C	B,C	A	B	A,B,C	D
Que.	11	12	13	14	15	16	17			
Ans.	A,B,C,D	A,C,D	D	A	A	B	B			

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. There exists infinitely many vectors of given magnitude which are perpendicular to a given plane.
2. There exists infinitely many vectors of given magnitude which are perpendicular to a given line.
3. Given that $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left[\,\vec{b}\ \ \vec{c}\ \ \vec{d}\,\right] = 0$ & $\vec{r} = \lambda\left(\vec{a}\times\vec{b}\right)$ then $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = \vec{r}.\vec{d} = 0$.
4. The point (1, 2, 3) lies on the line $\vec{r} = (2\tilde{i} + 3\tilde{j} + 4\tilde{k}) + \lambda(\tilde{i} + \tilde{j} + \tilde{k})$.
5. The area of a parallelogram whose two adjacent edges are two diagonals of a given parallelogram is double the area of given parallelogram.
6. If $\vec{A},\vec{B},\vec{C}$ are three non-coplanar vectors, then $\dfrac{\vec{A}.(\vec{B}\times\vec{C})}{(\vec{C}\times\vec{A}).\vec{B}} + \dfrac{\vec{B}.(\vec{A}\times\vec{C})}{\vec{C}.(\vec{A}\times\vec{B})} = 0$ [JEE 1985]
7. $[2\vec{a}+3\vec{b}\ \ 3\vec{a}+4\vec{b}\ \ 4\vec{a}+5\vec{b}] = 0$

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	ABC is a triangle. If P is a point inside the ΔABC such that areas of the triangle PBC, PCA and PAB, all are equal, then with respect to the ΔABC, P is its	(p)	centroid
(B)	If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the three non-collinear points A, B and C respectively such that the vector $\vec{V} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is a null vector, then with respect to the ΔABC, P is its	(q)	orthocentre
(C)	If P is a point inside the ΔABC such that the vector $\vec{R} = (BC)(\overrightarrow{PA}) + (CA)(\overrightarrow{PB}) + (AB)(\overrightarrow{PC})$ is a null vector, then with respect to the ΔABC, P is its	(r)	incentre
(D)	If P is a point in the plane of the triangle ABC such that the scalar product $\overrightarrow{PA}.\overrightarrow{CB}$ and $\overrightarrow{PB}.\overrightarrow{AC}$ vanishes, then with respect to the ΔABC, P is its	(s)	circumcentre

2. Let a, b, c be vectors then -

	Column-I		Column-II
(A)	$[\vec{a} + \vec{b},\ \vec{b} + \vec{c},\ \vec{c} + \vec{a}]$	(p)	$\lvert\vec{b}\rvert^2\,[\vec{a}\ \ \vec{c}\ \ \vec{b}]$
(B)	$[(\vec{a}\quad\vec{b})\quad(\vec{a}\quad\vec{c})].\vec{b}$	(q)	$(\vec{a}.\vec{b})[\vec{a}\ \ \vec{b}\ \ \vec{c}]$
(C)	$[\vec{a}\quad\vec{b},\ \vec{b}\quad\vec{c},\ \vec{c}\quad\vec{a}]$	(r)	$2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$
(D)	$\vec{b}.\ \{(\vec{a}\quad\vec{b})\quad(\vec{c}\quad\vec{b})\}$	(s)	$[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$

3.

	Column-I		Column-II
(A)	Let $\vec{a} = \tilde{i} + \tilde{j}$ & $\vec{b} = 2\tilde{i} - \tilde{k}$. If the point of intersection of the lines $\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ & $\vec{r}\times\vec{b} = \vec{a}\times\vec{b}$ is 'P', then $\ell^2(OP)$ (where O is the origin) is	(p)	0
(B)	If $\vec{a} = \tilde{i} + 2\tilde{j} + 3\tilde{k}, \vec{b} = 2\tilde{i} - \tilde{j} + \tilde{k}$ and $\vec{c} = 3\tilde{i} + 2\tilde{j} + \tilde{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to $x\vec{a} + y\vec{b} + z\vec{c}$, then x + y + z is equal to	(q)	5
(C)	The number of values of x for which the angle between the vectors $\vec{a} = x^9\tilde{i} + (x^3-1)\tilde{j} + 2\tilde{k}$ & $\vec{b} = (x^3-1)\tilde{i} + x\tilde{j} + \frac{1}{2}\tilde{k}$ is obtuse	(r)	7
(D)	Let $P_1 \equiv 2x - y + z = 7$ & $P_2 \equiv x + y + z = 2$. If P be a point that lies on P_1, P_2 and XOY plane, Q be the point that lies on P_1, P_2 and YOZ plane and R be the point that lies on P_1, P_2 & XOZ plane, then [Area of triangle PQR] (where [.] is greatest integer function)	(s)	11

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

1. **Statement-I** : The volume of a parallelopiped whose co-terminous edges are the three face diagonals of a given parallelopiped is double the volume of given parallelopied.

Because

Statement-II : For any vectors $\vec{a},\vec{b},\vec{c}$ we have $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$

(A) A (B) B (C) C (D) D

2. **Statement-I** : Let $A(\vec{a})$ & $B(\vec{b})$ be two points in space. Let $P(\vec{r})$ be a variable point which moves in space such that $\overrightarrow{PA}.\overrightarrow{PB} \le 0$, such a variable point traces a three-dimensional figure whose volume is given by $\frac{\pi}{6}\left\{\vec{a}^2 + \vec{b}^2 - 2\vec{a}.\vec{b}\right\}.|\vec{a}-\vec{b}|$

Because

Statement-II : Diameter of sphere subtends acute angle at any point inside the sphere & its volume is given by $\frac{4}{3}\pi r^3$, where 'r' is the radius of sphere.

(A) A (B) B (C) C (D) D

3. **Statement-I** : Let $\vec{a},\vec{b},\vec{c}$ be there non-coplanar vectors. Let $\vec{p}_1$ be perpendicular to plane of $\vec{a}\,\&\,\vec{b}$, $\vec{p}_2$ perpendicular to plane $\vec{b}$ & $\vec{c}$, $\vec{p}_3$ perpendicular to plane of $\vec{c}$ & $\vec{a}$ then $\vec{p}_1, \vec{p}_2$ & $\vec{p}_3$ are non-coplanar.

Because

Statement-II : $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$

(A) A (B) B (C) C (D) D

4. **Statement-I** : If $\vec{r} = \vec{a} + \lambda\vec{b}$ & $\vec{r} = \vec{p} + \mu\vec{d}$ be two lines such that $\vec{b} = t\vec{d}$ & $\vec{a} - \vec{p} = s\vec{b}$ where λ, μ t & s be non-zero scalars then the two lines have unique point of intersection.

Because

Statement-II : Two non-parallel coplanar lines have unique point of intersection.

(A) A (B) B (C) C (D) D

5. **Statement-I** : If $\vec{a} = \hat{i}, \vec{b} = \hat{j}$ and $\vec{c} = \hat{i} + \hat{j}$, then $\vec{a}$ and $\vec{b}$ are linearly independent but $\vec{a}, \vec{b}$ and $\vec{c}$ are linearly dependent.

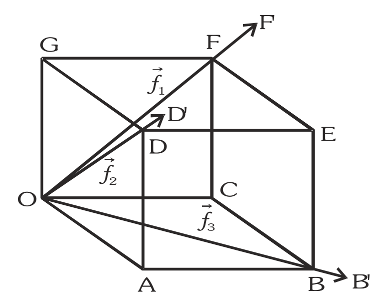
Because

Statement-II : If $\vec{a}$ and $\vec{b}$ are linearly dependent and $\vec{c}$ is any vector, then $\vec{a}$, $\vec{b}$ and $\vec{c}$ are linearly dependent.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1 :



Three forces $\vec{f}_1, \vec{f}_2$ & $\vec{f}_3$ of magnitude 2, 4 and 6 units respectively act along three face diagonals of a cube as shown in figure. Let P_1 be a parallelopiped whose three co-terminus edges be three vectors $\vec{f}_1, \vec{f}_2$ & $\vec{f}_3$. Let the joining of mid-points of each pair of opposite edges of parallelopiped P_1 meet in point X.

On the basis of above information, answer the following questions :

1. The magnitude of the resultant of the three forces is -

(A) 5 (B) 10 (C) 15 (D) none of these

2. The volume of the parallelopiped P_1 is -

(A) $48\sqrt{2}$ (B) $96\sqrt{2}$ (C) $24\sqrt{2}$ (D) $50\sqrt{2}$

3. $\ell(OX)$ is equal to -

(A) 5 (B) 1.5 (C) 2 (D) 2.5

Comprehension # 2 :

Consider three vectors $\vec{p} = \hat{i} + \hat{j} + \hat{k}$, $\vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $r = \hat{i} + \hat{j} + 3\hat{k}$ and let $\vec{s}$ be a unit vector.

On the basis of above information, answer the following questions :

1. $\vec{p}$, $\vec{q}$ and $\vec{r}$ are -

(A) linearly dependent

(B) can form the sides of a possible triangle

(C) such that the vector $(\vec{q} - \vec{r})$ is orthogonal to $\vec{p}$

(D) such that each one of these can be expressed as a linear combination of the other two

2. If $(\vec{p} \times \vec{q}) \times \vec{r} = u\vec{p} + v\vec{q} + w\vec{r}$, then (u+v+w) equals to -

(A) 8 (B) 2 (C) –2 (D) 4

3. The magnitude of the vector $(\vec{p}.\vec{s})(\vec{q} \times \vec{r}) + (\vec{q}.\vec{s})(\vec{r} \times \vec{p}) + (\vec{r}.\vec{s})(\vec{p} \times \vec{q})$ is -

(A) 4 (B) 8 (C) –2 (D) 2

Comprehension # 3 :

Three points A(1, 1, 4), B(0, 0, 5) & C(2, –1, 0) forms a plane. P is a point lying on the line $\vec{r} = \hat{i} + 3\hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k})$.

The perpendicular distance of point P from plane ABC is $\frac{2\sqrt{6}}{3}$.

'Q' is a point inside the tetrahedron PABC such that resultant of vectors $\overrightarrow{AQ}$, $\overrightarrow{BQ}$, $\overrightarrow{CQ}$ & $\overrightarrow{PQ}$ is a null vector.

On the basis of above information, answer the following questions :

1. Co-ordinates of point 'P' is -

(A) (2, 4, 1) (B) (1, 3, 0) (C) (4, 6, 3) (D) (7, 9, 6)

2. Volume of tetrahedron PABC is -

(A) $\frac{4\sqrt{81}}{9}$ (B) $\frac{2\sqrt{81}}{9}$ (C) $\frac{\sqrt{81}}{9}$ (D) $\frac{6\sqrt{81}}{9}$

3. Co-ordinates of point 'Q' is -

(A) $\left(\frac{5}{4}, 1, \frac{5}{2}\right)$ (B) (5, 1, 5) (C) $\left(\frac{5}{2}, 1, \frac{5}{4}\right)$ (D) $\left(\frac{5}{4}, 5, \frac{5}{2}\right)$

MISCELLANEOUS TYPE QUESTION — **ANSWER KEY** — EXERCISE -3

- ***True / False***

1. F 2. T 3. F 4. T 5. T 6. T 7. T

- ***Match the Column***

1. (A) → (p), (B) → (p), (C) → (r), (D) → (q) 2. (A) → (r), (B) → (q), (C) → (s), (D) → (p)

3. (A) → (s), (B) → (r), (C) → (p), (D) → (p)

- ***Assertion & Reason***

1. A 2. C 3. A 4. D 5. B

- ***Comprehension Based Questions***

Comprehension # 1 : 1. B 2. C 3. A Comprehension # 2 : 1. C 2. B 3. A

Comprehension # 3 : 1. A 2. B 3. A

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. The sides of parallelogram are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors, parallel to their diagonals.

2. If G is the centroid of a triangle ABC, then prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=0$

3. Find out whether the following pairs of lines are parallel, non-parallel & intersecting, or non-parallel & non-intersecting.

(a) $\vec{r}_1=\hat{i}+\hat{j}+2\hat{k}+\lambda(3\hat{i}-2\hat{j}+4\hat{k})$
$\vec{r}_2=2\hat{i}+\hat{j}+3\hat{k}+\mu(-6\hat{i}+4\hat{j}-8\hat{k})$

(b) $\vec{r}_1=\hat{i}-\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$
$\vec{r}_2=2\hat{i}+4\hat{j}+6\hat{k}+\mu(2\hat{i}+\hat{j}+3\hat{k})$

(c) $\vec{r}_1=\hat{i}+\hat{k}+\lambda(\hat{i}+3\hat{j}+4\hat{k})$
$\vec{r}_2=2\hat{i}+3\hat{j}+\mu(4\hat{i}-\hat{j}+\hat{k})$

4. (a) Show that the points $\vec{a}-2\vec{b}+3\vec{c};2\vec{a}+3\vec{b}-4\vec{c}$ & $-7\vec{b}+10\vec{c}$ are collinear.

(b) Prove that the points A = (1, 2, 3), B(3, 4, 7), C(–3, –2, –5) are collinear & find the ratio in which B divides AC.

5. Points X & Y are taken on the sides QR & RS, respectively of a parallelogram PQRS, so that $\overrightarrow{QX}=4\overrightarrow{XR}$ & $\overrightarrow{RY}=4\overrightarrow{YS}$. The line XY cuts the line PR at Z. Prove that $\overrightarrow{PZ}=\left(\frac{21}{25}\right)\overrightarrow{PR}$.

6. Using vectors prove that the altitudes of a triangle are concurrent.

7. Using vectors show that the mid-point of the hypotenuse of a right angled triangle is equidistant from its vertices.

8. Using vectors show that a parallelogram whose diagonals are equal is a rectangle.

9. Using vectors show that a quadrilateral whose diagonals bisect each other at right angles is a rhombus.

10. Two medians of a triangle are equal, then using vector show that the triangle is isosceles.

11. 'O' is the origin of vectors and A is a fixed point on the circle of radius 'a' with centre O. The vector $\overrightarrow{OA}$ is denoted by $\vec{a}$. A variable point 'P' lies on the tangent at A & $\overrightarrow{OP}=\vec{r}$. Show that $\vec{a}.\vec{r}=|a|^2$. Hence if P ≡ (x, y) & A ≡ (x_1, y_1) deduce the equation of tangent at A to this circle.

12. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60 . Suppose that $|\vec{u}-\hat{i}|$ is geometric mean of $|\vec{u}|$ and $|\vec{u}-2\hat{i}|$ where $\hat{i}$ is the unit vector along x-axis then $|\vec{u}|$ has the value equal to $\sqrt{a}-\sqrt{b}$ where $a, b \in N$. Find the value $(a + b)^3 + (a - b)^3$.

13. $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are the position vectors of the points A ≡ (x, y, z); B ≡ (y, –2z, 3x) ; C ≡ (2z, 3x, –y) and D ≡ (1, –1, 2) respectively. If $|\vec{a}|=2\sqrt{3};(\vec{a}\wedge\vec{b})=(\vec{a}\wedge\vec{c});(\vec{a}\wedge\vec{d})=\frac{\pi}{2}$ and $(\vec{a}\wedge\hat{j})$ is obtuse, then find x, y, z.

14. If $\vec{r}$ and $\vec{s}$ are nonzero constant vectors and the scalar b is chosen such that $|\vec{r}+b\vec{s}|$ is minimum, then show that the value of $|b\vec{s}|^2+|\vec{r}+b\vec{s}|^2$ is equal to $|\vec{r}|^2$.

15. (a) Find a unit vector â which makes an angle (π/4) with axis of z & is such that â+i+j is a unit vector.

(b) Prove that $\left(\frac{\vec{a}}{\vec{a}^2}-\frac{\vec{b}}{\vec{b}^2}\right)^2=\left(\frac{\vec{a}-\vec{b}}{|\vec{a}||\vec{b}|}\right)^2$

16. Given four non zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ and $\left(\widehat{\vec{a}\,\vec{b}}\right) = \left(\widehat{\vec{b}\,\vec{c}}\right) = \frac{\pi}{3}$, $\left(\widehat{\vec{d}\,\vec{a}}\right) = \alpha, \left(\widehat{\vec{d}\,\vec{b}}\right) = \beta$ then prove that $\left(\widehat{\vec{d}\,\vec{c}}\right) = \cos^{-1}(\cos\beta - \cos\alpha)$

17. Given three points on the xy plane O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $\left(\overline{PA}.\overline{PB}\right) + 3\left(\overline{OA}.\overline{OB}\right) = 0$. If the maximum and minimum values of $\left|\overline{PA}\right|\left|\overline{PB}\right|$ are M and m respectively then find the values of $M^2 + m^2$.

18. If O is origin of reference, point A($\vec{a}$); B($\vec{b}$); C($\vec{c}$); D($\vec{a}+\vec{b}$); E($\vec{b}+\vec{c}$); F($\vec{c}+\vec{a}$); G($\vec{a}+\vec{b}+\vec{c}$) where $\vec{a} = a_1\tilde{i} + a_2\tilde{j} + a_3\tilde{k}$; $\vec{b} = b_1\tilde{i} + b_2\tilde{j} + b_3\tilde{k}$ and $\vec{c} = c_1\tilde{i} + c_2\tilde{j} + c_3\tilde{k}$, then prove that these points are vertices of a cube having length of its edge equal to unity provided the matrix.

$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orthogonal. Also find the length XY such that X is the point of intersection of CM and GP;

Y is the point of intersection of OQ and DN where P, Q, M, N are respectively the midpoint of sides CF, BD, GF and OB

19. Let $\vec{A} = 2\vec{i} + \vec{k}$, $\vec{B} = \vec{i} + \vec{j} + \vec{k}$, and $\vec{C} = 4\vec{i} - 3\vec{j} + 7\vec{k}$ Determine a vector $\vec{R}$, satisfying $\vec{R}\quad\vec{B} = \vec{C}\quad\vec{B}$ and $\vec{R}\quad\vec{A} = 0$

20. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are position vectors of the vertices of a cyclic quadrilateral ABCD prove that :

$$\frac{\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{d} + \vec{d}\times\vec{a}\right|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})} + \frac{\left|\vec{b}\times\vec{c} + \vec{c}\times\vec{d} + \vec{d}\times\vec{b}\right|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})} = 0$$

21. Let $\vec{a} = \sqrt{3}\,i - j$ and $\vec{b} = \frac{1}{2}\tilde{i} + \frac{\sqrt{3}}{2}\tilde{j}$ and $\vec{x} = \vec{a} + (q^2 - 3)\vec{b}$, $\vec{y} = -p\vec{a} + q\vec{b}$. If $\vec{x} \perp \vec{y}$, then express p as a function of q, say p = f(q), ($p \neq 0$ and $q \neq 0$) and find the intervals of monotonicity of f(q).

22. If $\vec{a} = \tilde{i} + \tilde{j} - \tilde{k}$, $\vec{b} = -\tilde{i} + 2\tilde{j} + 2\tilde{k}$ & $\vec{c} = -\tilde{i} + 2\tilde{j} - \tilde{k}$, find a unit vectors normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$.

[REE 2000]

23. Prove that $\left|\vec{a}\times\vec{b}\right| = \sqrt{-\vec{b}.\left[\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right]}$

24. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and $\vec{d}$ is a unit vector, then find the value of, $|\,(\vec{a}.\vec{d})(\vec{b}\times\vec{c}) + (\vec{b}.\vec{d})(\vec{c}\times\vec{a}) + (\vec{c}.\vec{d})(\vec{a}\times\vec{b})\,|$ independent of $\vec{d}$. **[REE 99]**

25. Find the vector $\vec{r}$ which is perpendicular to $\vec{a} = \tilde{i} - 2\tilde{j} + 5\tilde{k}$ and $\vec{b} = 2\tilde{i} + 3\tilde{j} - \tilde{k}$ and $\vec{r}.(2\tilde{i} + \tilde{j} + \tilde{k}) + 8 = 0$.

26. Two vertices of a triangle are at $-\hat{i}+3\hat{j}$ and $2\hat{i}+5\hat{j}$ and its orthocentre is at $\hat{i}+2\hat{j}$. Find the position vector of third vertex. **[REE 2001]**

27. Find the point R in which the line AB cuts the plane CDE where

$\vec{a}$ = i + 2j + k , $\vec{b}$ = 2i + j + 2k $\vec{c}$ = – 4j + 4k , $\vec{d}$ = 2i – 2j + 2k & $\vec{e}$ = 4i + j + 2k.

28. Solve for $\vec{x}$: $\vec{x}$ $\vec{a}$ + $(\vec{x}.\vec{b})\vec{a} = \vec{c}$, where $\vec{a}$ and $\vec{c}$ are non zero non collinear and $\vec{a}.\vec{b} \neq 0$

CONCEPTUAL SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE - 4(A)

1. $\frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$, $\frac{-1}{\sqrt{69}}\hat{i}-\frac{2}{\sqrt{69}}\hat{j}+\frac{8}{\sqrt{69}}\hat{k}$ **3.** (a) parallel (b) the lines intersect at the point p.v. $-2\hat{i}+2\hat{j}$ (c) lines are skew **4.** (b) Externally in ratio 1 : 3 **11.** $xx_1 + yy_1 = a^2$ **12.** 28

13. x = 2, y = –2, z = –2 **15.** (a) $\frac{-1}{2}i-\frac{1}{2}j+\frac{1}{\sqrt{2}}k$ **17.** 34 **18.** $\frac{\sqrt{11}}{3}$ **19.** $-\hat{i} - 8\hat{j} + 2\hat{k}$

21. $p = \frac{q(q^2-3)}{4}$; decreasing in q ∈ (–1, 1), q ≠ 0 **22.** $\pm\hat{i}$ **24.** $[\vec{a}\,\vec{b}\,\vec{c}]$ **25.** $\vec{r} = -13\hat{i}+11\hat{j}+7\hat{k}$

26. $\frac{5}{7}\hat{i}+\frac{17}{7}\hat{j}+\lambda\hat{k}$ where λ ∈ R **27.** p.v. of $\vec{R}$ = r = 3i + 3k **28.** $\frac{1}{\vec{a}.\vec{b}}\left[(\vec{b}\times\vec{c})+\frac{\vec{a}.\vec{c}}{\vec{a}^2}\left(\vec{a}-(\vec{b}\times\vec{a})\right)\right]$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. The position vectors of the points A, B, C are respectively (1, 1, 1) ; (1, –1, 2) ; (0, 2, –1). Find a unit vector parallel to the plane determined by ABC & perpendicular to the vector (1, 0, 1).

2. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then show that the value of the scalar triple product $\left[n\vec{a}+\vec{b} \;\; n\vec{b}+\vec{c} \;\; n\vec{c}+\vec{a}\right]$ is $(n^3 + 1)\begin{vmatrix} \vec{a}.\hat{i} & \vec{a}.\hat{j} & \vec{a}.\hat{k} \\ \vec{b}.\hat{i} & \vec{b}.\hat{j} & \vec{b}.\hat{k} \\ \vec{c}.\hat{i} & \vec{c}.\hat{j} & \vec{c}.\hat{k} \end{vmatrix}$

3. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a}+\vec{b} = \mu\vec{p}$, $\vec{b}.\vec{q} = 0$ & $\left(\vec{b}\right)^2 = 1$, where μ is a scalar then prove that $\left|\left(\vec{a}.\vec{q}\right)\vec{p} - \left(\vec{p}.\vec{q}\right)\vec{a}\right| = \left|\vec{p}.\vec{q}\right|$

4. ABCD is a tetrahedron with pv's of its angular point as A(–5, 22, 5); B(1, 2, 3); C(4, 3, 2) and D(–1, 2, –3). If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelograms is $\sqrt{S}$ then find the values of S.

5. Given four points P_1, P_2, P_3 and P_4 on the coordinate plane with origin O which satisfy the condition

$$\overrightarrow{OP_{n-1}} + \overrightarrow{OP_{n+1}} = \frac{3}{2}\overrightarrow{OP_n}, \; n = 2, 3$$

(a) If P_1, P_2 lie on the curve xy = 1, then prove that P_3 does not lie on this curve.

(b) If P_1, P_2, P_3 lie on the circle $x^2 + y^2 = 1$, then prove that P_4 lies on this circle.

6. Find a vector $\vec{v}$ which is coplanar with the vectors $\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ and is orthogonal to the vector $2\hat{i}+\hat{j}+\hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{i}-\hat{j}+\hat{k}$ is equal to $6\sqrt{3}$.

7. If $p\vec{x} + (\vec{x} \times \vec{a}) = \vec{b}$; $(p \neq 0)$ prove that $\vec{x} = \dfrac{p^2\vec{b} + (\vec{b}.\vec{a})\vec{a} - p(\vec{b} \times \vec{a})}{p(p^2 + \vec{a}^2)}$.

8. Solve the following equation for the vector $\vec{p}$; $\vec{p} \times \vec{a} + \left(\vec{p}.\vec{b}\right)\vec{c} = \vec{b} \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non zero non coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left[\vec{p} \times \vec{a} + \dfrac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\vec{a}.\vec{c}}\vec{c}\right]$ is perpendicular to $\vec{b}-\vec{c}$.

9. Solve the simultaneous vector equations for the vectors $\vec{x}$ and $\vec{y}$. $\vec{x}+\vec{c}\times\vec{y}=\vec{a}$ and $\vec{y}+\vec{c}\times\vec{x}=\vec{b}$ where $\vec{c}$ is a non zero vector.

10. Let $\begin{vmatrix} (a_1-a)^2 & (a_1-b)^2 & (a_1-c)^2 \\ (b_1-a)^2 & (b_1-b)^2 & (b_1-c)^2 \\ (c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2 \end{vmatrix} = 0$ and if the vectors $\vec{\alpha} = \tilde{i} + a\tilde{j} + a^2\tilde{k}$; $\vec{\beta} = \tilde{i} + b\tilde{j} + b^2\tilde{k}$; $\vec{\gamma} = \tilde{i} + c\tilde{j} + c^2\tilde{k}$ are non coplanar, show that the vectors $\vec{\alpha}_1 = \tilde{i} + a_1\tilde{j} + a_1^2\tilde{k}$; $\vec{\beta}_1 = \tilde{i} + b_1\tilde{j} + b_1^2\tilde{k}$ and $\vec{\gamma}_1 = \tilde{i} + c_1\tilde{j} + c_1^2\tilde{k}$ are coplanar.

11. The vector $\overrightarrow{OP} = \tilde{i} + 2\tilde{j} + 2\tilde{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

12. If $\vec{x}\times\vec{y} = \vec{a}$, $\vec{y}\times\vec{z} = \vec{b}$, $\vec{x}.\vec{b} = \gamma$, $\vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$, then find $\vec{x}$, $\vec{y}$ & $\vec{z}$ in terms of $\vec{a}$, $\vec{b}$ & γ. **[REE 98]**

13. Find the value of λ such that a, b, c are all non-zero and

$(-4\tilde{i}+5\tilde{j})a + (3\tilde{i}-3\tilde{j}+\tilde{k})b + (\tilde{i}+\tilde{j}+3\tilde{k})c = \lambda\ (a\tilde{i}+b\tilde{j}+c\tilde{k})$ [REE 2001]

BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(B)

1. $\pm\dfrac{1}{3\sqrt{3}}(i+5j-k)$ 4. 110 5. 9 6. $9(-\tilde{j}+\tilde{k})$

8. $\left\{\vec{p} = \dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{(\vec{a}.\vec{c})(\vec{a}.\vec{b})}\left(\vec{a}+\vec{c}\times\vec{b}\right) + \dfrac{(\vec{b}.\vec{c})\,\vec{b}}{(\vec{a}.\vec{b})} - \dfrac{(\vec{b}.\vec{b})\,\vec{c}}{(\vec{a}.\vec{b})}\right\}$

9. $\vec{x} = \dfrac{\vec{a}+(\vec{c}.\vec{a})\vec{c}+\vec{b}\times\vec{c}}{1+\vec{c}^2}$, $y = \dfrac{\vec{b}+(\vec{c}.\vec{b})\vec{c}+\vec{a}\times\vec{c}}{1+\vec{c}^2}$

11. $\dfrac{4}{\sqrt{2}}\tilde{i} - \dfrac{1}{\sqrt{2}}\tilde{j} - \dfrac{1}{\sqrt{2}}\tilde{k}$

12. $x = \dfrac{\dfrac{\vec{a}\times\vec{b}}{\gamma} - \vec{a}\times\dfrac{\vec{a}\times\vec{b}}{\gamma}}{\left(\dfrac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$; $y = \dfrac{\vec{a}\times\vec{b}}{\gamma}$; $z = \dfrac{\dfrac{\vec{a}\times\vec{b}}{\gamma} + \vec{b}\times\dfrac{\vec{a}\times\vec{b}}{\gamma}}{\left(\dfrac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$

13. $\lambda = -2 \pm \sqrt{29}$

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If **a**, **b**, **c** are three non zero vectors out of which two are not collinear. If **a** + 2**b** and **c** ; **b** + 3**c** and **a** are collinear then **a** + 2**b** + 6**c** is- [AIEEE-2002]

(1) Parallel to **c** (2) Parallel to **a** (3) Parallel to **b** (4) 0

2. If $[\vec{a}\ \ \vec{b}\ \ \vec{c}\,] = 4$ then $[\vec{a}\ \ \ \vec{b}\ \ \vec{b}\ \ \ \vec{c}\ \ \vec{c}\ \ \ \vec{a}\,] =$ [AIEEE-2002]

(1) 4 (2) 2 (3) 8 (4) 16

3. If **c** = 2λ (**a** **b**) + 3μ(**b** **a**) ; **a** **b** ≠ 0, **c**.(**a** **b**)=0 then- [AIEEE-2002]

(1) $\lambda = 3\mu$ (2) $2\lambda = 3\mu$ (3) $\lambda + \mu = 0$ (4) None of these

4. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$, then orthogonal projection of $\vec{a}$ on $\vec{b}$ is- [AIEEE-2002]

(1) $3\hat{i} - 3\hat{j} + \hat{k}$ (2) $\dfrac{9(5\hat{i}-3\hat{j}+\hat{k})}{35}$ (3) $\dfrac{(5\hat{i}-3\hat{j}+\hat{k})}{35}$ (4) $9(5\hat{i} - 3\hat{j} + \hat{k})$

5. A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is- [AIEEE-2002]

(1) $\dfrac{4\hat{i}+3\hat{j}-\hat{k}}{\sqrt{26}}$ (2) $\dfrac{2\hat{i}-6\hat{j}-3\hat{k}}{7}$ (3) $\dfrac{3\hat{i}-2\hat{j}+6\hat{k}}{7}$ (4) $\dfrac{2\hat{i}-3\hat{j}-6\hat{k}}{7}$

6. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, then $|\,\vec{w}.\hat{n}\,|$ is equal to- [AIEEE-2003]

(1) 3 (2) 0 (3) 1 (4) 2

7. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is- [AIEEE-2003]

(1) 50 units (2) 20 units (3) 30 units (4) 40 units

8. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is- [AIEEE-2003]

(1) $\sqrt{288}$ (2) $\sqrt{18}$ (3) $\sqrt{72}$ (4) $\sqrt{33}$

9. $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\,\vec{a}\,| = 1$, $|\,\vec{b}\,| = 2$, $|\,\vec{c}\,| = 3$, then $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is equal to- [AIEEE-2003]

(1) 1 (2) 0 (3) –7 (4) 7

10. Consider point A, B, C and D with postion vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then ABCD is a- [AIEEE-2003]

(1) parallelogram but not a rhombus (2) square
(3) rhombus (4) None of these

11. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}).(\vec{u} - \vec{v})\ \ (\vec{v} - \vec{w})$ equals- [AIEEE-2003]

(1) $3\vec{u}.(\vec{v}\ \ \ \vec{w})$ (2) 0 (3) $\vec{u}.(\vec{v}\ \ \ \vec{w})$ (4) $\vec{u}.(\vec{w}\ \ \ \vec{v})$

12. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals- [AIEEE-2004]

(1) $\lambda\vec{a}$ (2) $\lambda\vec{b}$ (3) $\lambda\vec{c}$ (4) 0

13. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. Then work done in standard units by the forces is given by-[AIEEE-2004]

(1) 40 (2) 30 (3) 25 (4) 15

14. If $\vec a$, $\vec b$, $\vec c$ are non-coplanar vectors and λ is a real number, then the vectors $\vec a + 2\vec b + 3\vec c$, $\lambda\vec b + 4\vec c$ and $(2\lambda - 1)\vec c$ are non-coplanar for- **[AIEEE-2004]**

(1) all values of λ (2) all except one value of λ

(3) all except two values of λ (4) no value of λ

15. Let $\vec u$, $\vec v$, $\vec w$ be such that $|\vec u| = 1$, $|\vec v| = 2$, $|\vec w| = 3$. If the projection of $\vec v$ along $\vec u$ is equal to projection of $\vec w$ along $\vec u$ and $\vec v$ and $\vec w$ are perpendicular to each other then $|\vec u - \vec v + \vec w|$ equals- **[AIEEE-2004]**

(1) 2 (2) $\sqrt{7}$ (3) $\sqrt{14}$ (4) 14

16. Let $\vec a$, $\vec b$ and $\vec c$ be non-zero vectors such that $(\vec a \quad \vec b) \quad \vec c = \frac{1}{3}|\vec b||\vec c|\vec a$. If θ is the acute angle between the vectors $\vec b$ and $\vec c$, then $\sin\theta$ equals- **[AIEEE-2004]**

(1) $\frac{1}{3}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{2\sqrt{2}}{3}$

17. If C is the mid point of AB and P is any point outside AB, then- **[AIEEE-2005]**

(1) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ (2) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

(3) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec 0$ (4) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec 0$

18. For any vector $\vec a$, the value of $(\vec a \quad \hat i)^2 + (\vec a \quad \hat j)^2 + (\vec a \quad \hat k)^2$ is equal to- **[AIEEE-2005]**

(1) $3\vec a^{\,2}$ (2) $\vec a^{\,2}$ (3) $2\vec a^{\,2}$ (4) $4\vec a^{\,2}$

19. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat i + a\hat j + c\hat k$, $\hat i + \hat k$ and $c\hat i + c\hat j + b\hat k$ lie in a plane, then c is- **[AIEEE-2005]**

(1) the Geometric Mean of a and b (2) the Arithmetic Mean of a and b

(3) equal to zero (4) the Harmonic Mean of a and b

20. If $\vec a$, $\vec b$, $\vec c$ are non-coplanar vectors and λ is a real number then $[\lambda(\vec a + \vec b) \quad \lambda^2\vec b \quad \lambda\vec c] = [\vec a \quad \vec b + \vec c \quad \vec d]$ for- **[AIEEE-2005]**

(1) exactly one value of λ (2) no value of λ

(3) exactly three values of λ (4) exactly two values of λ

21. Let $\vec a = \hat i - \hat k$, $\vec b = x\hat i + \hat j + (1 - x)\hat k$ and $\vec c = y\hat i + x\hat j + (1 + x - y)\hat k$. Then $[\vec a, \vec b, \vec c]$ depends on- **[AIEEE-2005]**

(1) only y (2) only x (3) both x and y (4) neither x nor y

22. If $(\vec a \quad \vec b) \quad \vec c = \vec a \quad (\vec b \quad \vec c)$, where $\vec a$, $\vec b$ and $\vec c$ are any three vectors such that $\vec a.\vec b \neq 0$, $\vec b.\vec c \neq 0$, then $\vec a$ and $\vec c$ are- **[AIEEE-2006]**

(1) inclined at an angle of $\pi/6$ between them (2) perpendicular

(3) parallel (4) inclined at an angle of $\pi/3$ between them

23. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}$, $\overrightarrow{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A onto BC. the magnitude of the resultant is- **[AIEEE-2006]**

(1) $\frac{(AB)(AC)}{AB+AC}$ (2) $\frac{1}{AB} + \frac{1}{AC}$ (3) $\frac{1}{AD}$ (4) $\frac{AB^2+AC^2}{(AB)^2(AC)^2}$

24. The values of a, for which the points A, B, C with position vectors $2\hat i - \hat j + \hat k$, $\hat i - 3\hat j - 5\hat k$ and $a\hat i - 3\hat j + \hat k$ respectively are the vertices of a right-angled triangle with $C = \frac{\pi}{2}$ are- **[AIEEE-2006]**

(1) –2 and –1 (2) –2 and 1 (3) 2 and –1 (4) 2 and 1

25. If $\vec{u}$ and $\vec{v}$ are unit vectors and θ is the acute angle between them, then $2\vec{u} \quad 3\vec{v}$ is a unit vector for- [AIEEE-2007]

(1) Exactly two values of θ (2) More than two values of θ

(3) No value of θ (4) Exactly one value of θ

26. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals- [AIEEE-2007]

(1) 0 (2) 1 (3) –4 (4) –2

27. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$, lies in the plane the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisect the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ? [AIEEE-2008]

(1) $\alpha = 2, \beta = 2$ (2) $\alpha = 1, \beta = 2$ (3) $\alpha = 2, \beta = -1$ (4) $\alpha = 1, \beta = 1$

28. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is- [AIEEE-2008]

(1) 0 (2) $\pi/4$ (3) $\pi/2$ (4) π

29. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality

$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$ holds for :- [AIEEE-2009]

(1) More than two but not all values of (p, q) (2) All values of (p, q)

(3) Exactly one value of (p, q) (4) Exactly two values of (p, q)

30. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is : [AIEEE-2010]

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$ (3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

31. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$ [AIEEE-2010]

(1) (–3, 2) (2) (2, –3) (3) (–2, 3) (4) (3, –2)

32. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to :- [AIEEE-2011]

(1) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (2) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (3) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (4) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

33. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is :- [AIEEE-2011]

(1) 5 (2) 3 (3) – 5 (4) – 3

34. If the vectors $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}$ $(p \neq q \neq r \neq 1)$ are coplanar, then the value of pqr – (p + q + r) is :- [AIEEE-2011]

(1) –2 (2) 2 (3) 0 (4) –1

35. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is colliner with $\vec{a}$, then $\vec{a} + 3\vec{b} + 6\vec{c}$ is : [AIEEE-2011]

(1) $\vec{a} + \vec{c}$ (2) $\vec{a}$ (3) $\vec{c}$ (4) $\vec{0}$

36. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c} = \vec{a} + 2\vec{b}$ and $\vec{d} = 5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is : [AIEEE-2012]

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

37. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[AIEEE-2012]**

(1) $\vec{r} = -3\vec{q} + \dfrac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (2) $\vec{r} = 3\vec{q} - \dfrac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \left(\dfrac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (4) $\vec{r} = \vec{q} - \left(\dfrac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

38. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is : **[JEE (Main)-2013]**

(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{45}$

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	4	4	2	2	3	1	4	4	3	4	3	4	1	3	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans	4	1	3	1	2	4	3	3	4	4	4	4	4	3	1
Que.	31	32	33	34	35	36	37	38							
Ans	1	2	3	1	4	4	3	3							

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. Select the correct alternative :

(a) If the vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ form the sides BC, CA & AB respectively of a triangle ABC, then

(A) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$ (B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(C) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$ (D) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

(b) Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\vec{0}$. Let P_1& P_2 be planes determined by the pairs of vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$, $\vec{d}$ respectively. Then the angle between P_1 and P_2 is :

(A) 0 (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

(c) If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}\right]=$

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

[JEE 2000 (Screening) 1+1+1M out of 35]

2. Let ABC and PQR be any two triangles in the same plane. Assume that the perpendicular from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent.

[JEE 2000 (Mains) 10M out of 100]

3. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ does not exceed

(A) 4 (B) 9 (C) 8 (D) 6

(b) Let $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $[\vec{a},\ \vec{b},\ \vec{c}]$ depends on

(A) only x (B) only y (C) neither x nor y (D) both x and y

[JEE '2001 (Screening) 1+1M out of 35]

4. Show by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.

[JEE 2001 (Mains) 5M out of 100]

5. Find 3-dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying $\vec{v}_1.\vec{v}_1=4$, $\vec{v}_1.\vec{v}_2=-2$, $\vec{v}_1.\vec{v}_3=6$, $\vec{v}_2.\vec{v}_2=2$, $\vec{v}_2.\vec{v}_3=-5$, $\vec{v}_3.\vec{v}_3=29$. **[JEE 2001 (Mains) 5M out of 100]**

6. Let $\vec{A}(t)=f_1(t)\hat{i}+f_2(t)\hat{j}$ and $\vec{B}(t)=g_1(t)\hat{i}+g_2(t)\hat{j}$, $t\in[0,1]$, where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are non-zero vectors for all t and $\vec{A}(0)=2\hat{i}+3\hat{j}$, $\vec{A}(1)=6\hat{i}+2\hat{j}$, $\vec{B}(0)=3\hat{i}+2\hat{j}$ and $\vec{B}(1)=2\hat{i}+6\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ b are parallel for some t.

[JEE 2001 (Mains) 5M out of 100]

7. (a) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is - **[JEE 2002 (Screening), 3M]**

(A) 45 (B) 60 (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

(b) Let $\vec{V}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{W}=\hat{i}+3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is - **[JEE 2002 (Screening), 3M]**

(A) -1 (B) $\sqrt{10}+\sqrt{6}$ (C) $\sqrt{59}$ (D) $\sqrt{60}$

8. Let v be the volume of the parallelopiped formed by the vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If a_r, b_r, c_r, where r = 1, 2, 3, are non-negative real numbers and $\sum_{r=1}^{3}\left(a_r+b_r+c_r\right)=3L$, show that $V\le L^3$. **[JEE 2002 (Mains)]**

9. The value of a for which the volume of parallelopiped formed by the vectors $\hat{i}+a\hat{j}+\hat{k}$ and $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ as coterminous edge is minimum is - **[JEE 03 (Screening), 3M]**

(A) -3 (B) 3 (C) $1/\sqrt{3}$ (D) none of these

10. If $\vec{u}$, $\vec{v}$, $\vec{w}$ are three non-coplanar unit vectors and α, β , γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}$, $\vec{y}$, $\vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $[\vec{x}\times\vec{y} \;\; \vec{y}\times\vec{z} \;\; \vec{z}\times\vec{x}] = \frac{1}{16}\,[\vec{u} \;\; \vec{v} \;\; \vec{w}]^2 \;\sec^2\frac{\alpha}{2}\;\sec^2\frac{\beta}{2}\;\sec^2\frac{\gamma}{2}$ **[JEE 03 (Mains) 4M]**

11. **(a)** If for vectors $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{b} = 1$, $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$, $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ then vector $\vec{b}$ is –

(A) $\hat{i}-\hat{j}+\hat{k}$ (B) $2\hat{j}-\hat{k}$ (C) $\hat{i}$ (D) $2\hat{i}$

(b) A given unit vector is orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$ and coplanar with $\hat{i}-\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ then the vector is -

(A) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (B) $\frac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (C) $\frac{2\hat{i}-5\hat{k}}{\sqrt{29}}$ (D) $\frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

[JEE 04 (screening) 3+3M]

12. $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four distinct vectors satisfying the conditions $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$, then prove that $\vec{a}.\vec{b}+\vec{c}.\vec{d} \neq \vec{a}.\vec{c}+\vec{b}.\vec{d}$ **[JEE 04 (Mains) 2M]**

13. If $\vec{a},\vec{b},\vec{c}$ are three non-zero, non-coplanar vectors and $\vec{b}_1 = \vec{b}-\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2 = \vec{b}+\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{c}_1 = \vec{c}-\frac{\vec{c}\,\vec{a}}{|\vec{a}|^2}\vec{a}+\frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_2 = \vec{c}-\frac{\vec{c}\,\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{b}_1.\vec{c}}{|\vec{b}_1|^2}\vec{b}_1$, $\vec{c}_3 = \vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}+\frac{\vec{b}_1.\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$ then the set of orthogonal vectors is -

(A) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (B) $(\vec{a}, \vec{b}_1, \vec{c}_2)$ (C) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (D) $(\vec{a}, \vec{b}_2, \vec{c}_2)$

[JEE 05 (screening) 3M]

14. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.

[JEE 05 (Mains) 4M out of 60]

15. **(a)** Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ has the magnitude equal to $\frac{1}{\sqrt{3}}$ is -

(A) $4\hat{i}-\hat{j}+4\hat{k}$ (B) $3\hat{i}+\hat{j}-3\hat{k}$ (C) $2\hat{i}+\hat{j}-2\hat{k}$ (D) $4\hat{i}+\hat{j}-4\hat{k}$

[JEE 06, 3M]

(b) Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is -

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

[JEE 06, 5M]

16. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is :- **[JEE 07, 3M]**

(A) zero (B) one (C) two (D) three

17. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. Which one of the following is correct? **[JEE 07, 3M]**

(A) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} = \vec{0}$ (B) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \neq \vec{0}$

(C) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{a}\times\vec{c} \neq \vec{0}$ (D) $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ are mutually perpendicular

18. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement-1 : $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$. **because**

Statement-2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$. **[JEE 07, 3M]**

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

19. The edges of a parallelopiped are of unit length and are parallel to non–coplanar unit vectors $\vec{a}, \vec{b}, \vec{c}$ such that $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \frac{1}{2}$. Then, the volume of the parallelopiped is :- **[JEE 08, 3M, –1M]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

20. Let two non-collinear unit vectors $\vec{a}$ and $\vec{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\vec{a}\cos t + \vec{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\vec{u}$ be the unit vector along $\overrightarrow{OP}$. Then - **[JEE 08, 3M, –1M]**

(A) $\vec{u} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$ and $M = (1 + \vec{a}\cdot\vec{b})^{1/2}$ (B) $\vec{u} = \frac{\vec{a}-\vec{b}}{|\vec{a}-\vec{b}|}$ and $M = (1 + \vec{a}\cdot\vec{b})^{1/2}$

(C) $\vec{u} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$ and $M = (1 + 2\vec{a}\cdot\vec{b})^{1/2}$ (D) $\vec{u} = \frac{\vec{a}-\vec{b}}{|\vec{a}-\vec{b}|}$ and $M = (1 + 2\vec{a}\cdot\vec{b})^{1/2}$

21. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 1$ and $\vec{a}\cdot\vec{c} = \frac{1}{2}$, then :- **[JEE 2009, 3M, –1M]**

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
(C) $\vec{b}, \vec{d}$ are non-parallel (D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

22. Match the statements / expressions given in **Column I** with the values given in **Column II** **[JEE 2009, 8M]**

Column–I		Column–II	
(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(P)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$ where [y] denotes the largest integer less than or equal to y	(Q)	$\frac{\pi}{4}$
(C)	Volume of the parallelepiped with its edges	(R)	$\frac{\pi}{3}$
	represented by the vectors $\hat{i}+\hat{j}$, $\hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(S)	$\frac{\pi}{2}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c} = \vec{0}$	(T)	π

23. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a **[JEE 10, 3M, –1M]**

(A) parallelogram, which is neither a rhombus nor a rectangle
(B) square
(C) rectangle, but not a square
(D) rhombus, but not a square

24. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is **[JEE 10, 3M, –1M]**

25. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by -

[JEE 10, 5M, –1M]

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

26. **(a)** Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$ (C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

(b) The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are

(A) $\hat{j} - \hat{k}$ (B) $-\hat{i} + \hat{j}$ (C) $\hat{i} - \hat{j}$ (D) $-\hat{j} + \hat{k}$

(c) Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is **[JEE 2011, 3+4+4]**

27. **(a)** If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

(b) If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

[JEE 2012, 4+3]

28. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **[JEE-Advanced 2013, 2M]**

(A) 5 (B) 20 (C) 10 (D) 30

29. Consider the set of eight vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE-Advanced 2013, 4, (–1)]**

30. Match List-I with List-II and select the correct answer using the code given below the lists.

	List-I		List-II
P.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2\left(\vec{a}\times\vec{b}\right), 3\left(\vec{b}\times\vec{c}\right)$ and $\left(\vec{c}\times\vec{a}\right)$ is	1.	100
Q.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3\left(\vec{a}+\vec{b}\right), \left(\vec{b}+\vec{c}\right)$ and $2\left(\vec{c}+\vec{a}\right)$ is	2.	30
R.	Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $\left(2\vec{a}+3\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ is	3.	24
S.	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $\left(\vec{a}+\vec{b}\right)$ and $\vec{a}$ is	4.	60

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

[JEE-Advanced 2013, 3, (–1)]

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [B]

1. (a) B ; (b) A ; (c) A **3.** (a) B (b) C **5.** $\vec{v}_1 = 2\tilde{i}$, $\vec{v}_2 = -\tilde{i} \pm j$, $\vec{v}_3 = 3\tilde{i} \pm 2\tilde{j} \pm 4\tilde{k}$ **12.** (a) B ; (b) C

9. D **11.** (a) C (b) A **13.** B **14.** $\hat{w} = \hat{v} - 2(\hat{a}.\hat{v})\hat{a}$ **15.** (a) A ; (b) B,D

16. C **17.** B **18.** C **19.** A **20.** A **21.** C

22. (A) $\rightarrow$ (Q, S), (B) $\rightarrow$ (P, R, S, T), (C) $\rightarrow$ (T), (D) $\rightarrow$ (R) **23.** A **24.** 5 **25.** B

26. (a) C; (b) A,D; (c) 9 **27.** (a) 3; (b) C **28.** C **29.** 5 **30.** C