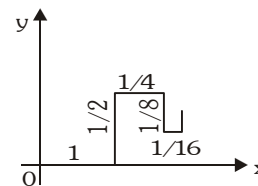


## EXERCISE - 01

## CHECK YOUR GRASP

### SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

- The maximum value of the sum of the A.P. 50, 48, 46, 44, ..... is -  
(A) 325 (B) 648 (C) 650 (D) 652
- Let  $T_r$  be the  $r^{\text{th}}$  term of an A.P. for  $r = 1, 2, 3, \dots$ . If for some positive integers  $m, n$  we have  $T_m = \frac{1}{n}$  &  $T_n = \frac{1}{m}$ , then  $T_{mn}$  equals -  
(A)  $\frac{1}{mn}$  (B)  $\frac{1}{m} + \frac{1}{n}$  (C) 1 (D) 0
- The interior angles of a convex polygon are in AP. The smallest angle is  $120^\circ$  & the common difference is  $5^\circ$ . Find the number of sides of the polygon -  
(A) 9 (B) 16 (C) 12 (D) none of these
- The first term of an infinitely decreasing G.P. is unity and its sum is  $S$ . The sum of the squares of the terms of the progression is -  
(A)  $\frac{S}{2S-1}$  (B)  $\frac{S^2}{2S-1}$  (C)  $\frac{S}{2-S}$  (D)  $S^2$
- A particle begins at the origin and moves successively in the following manner as shown, 1 unit to the right,  $\frac{1}{2}$  unit up,  $\frac{1}{4}$  unit to the right,  $\frac{1}{8}$  unit down,  $\frac{1}{16}$  unit to the right etc. The length of each move is half the length of the previous move and movement continues in the 'zigzag' manner indefinitely. The co-ordinates of the point to which the 'zigzag' converges is -  
(A)  $(\frac{4}{3}, \frac{2}{3})$  (B)  $(\frac{4}{3}, \frac{2}{5})$  (C)  $(\frac{3}{2}, \frac{2}{3})$  (D)  $(2, \frac{2}{5})$
- Let  $a_n$  be the  $n^{\text{th}}$  term of a G.P. of positive numbers. Let  $\sum_{n=1}^{100} a_{2n} = \alpha$  &  $\sum_{n=1}^{100} a_{2n-1} = \beta$  such that  $\alpha \neq \beta$ . Then the common ratio of the G.P. is -  
(A)  $\frac{\alpha}{\beta}$  (B)  $\frac{\beta}{\alpha}$  (C)  $\sqrt{\frac{\alpha}{\beta}}$  (D)  $\sqrt{\frac{\beta}{\alpha}}$
- If  $p, q, r$  in harmonic progression and  $p$  &  $r$  be different having same sign then the roots of the equation  $px^2 + qx + r = 0$  are -  
(A) real and equal (B) real and distinct (C) irrational (D) imaginary
- If  $x > 1, y > 1, z > 1$  are in G.P., then  $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$  are in -  
(A) A.P. (B) H.P. (C) G.P. (D) none of above
- If  $\ln(a+c), \ln(c-a), \ln(a-2b+c)$  are in A.P., then :  
(A)  $a, b, c$  are in A.P. (B)  $a^2, b^2, c^2$  are in A.P.  
(C)  $a, b, c$  are in G.P. (D)  $a, b, c$  are in H.P.



10. If the  $(m+1)^{\text{th}}$ ,  $(n+1)^{\text{th}}$  &  $(r+1)^{\text{th}}$  terms of an AP are in GP &  $m, n, r$  are in HP, then the ratio of the common difference to the first term of the AP is -
- (A)  $\frac{1}{n}$  (B)  $\frac{2}{n}$  (C)  $-\frac{2}{n}$  (D) none of these
11. The sum of roots of the equation  $ax^2 + bx + c = 0$  is equal to the sum of squares of their reciprocals. Then  $bc^2$ ,  $ca^2$  and  $ab^2$  are in -
- (A) AP (B) GP (C) HP (D) none of these
12. The quadratic equation whose roots are the A.M. and H.M. between the roots of the equation,  $2x^2 - 3x + 5 = 0$  is -
- (A)  $4x^2 - 25x + 10 = 0$  (B)  $12x^2 - 49x + 30 = 0$   
 (C)  $14x^2 - 12x + 35 = 0$  (D)  $2x^2 + 3x + 5 = 0$
13. If the sum of the first  $n$  natural numbers is  $1/5$  times the sum of their squares, then the value of  $n$  is -
- (A) 5 (B) 6 (C) 7 (D) 8
14. Suppose  $p$  is the first of  $n(n > 1)$  AM's between two positive numbers  $a$  and  $b$ , then value of  $p$  is -
- (A)  $\frac{na+b}{n+1}$  (B)  $\frac{na-b}{n+1}$  (C)  $\frac{nb+a}{n+1}$  (D)  $\frac{nb-a}{n+1}$
15. If  $\frac{1}{a} + \frac{1}{a-2b} + \frac{1}{c} + \frac{1}{c-2b} = 0$  and  $a, b, c$  are not in A.P., then -
- (A)  $a, b, c$  are in G.P. (B)  $a, \frac{b}{2}, c$  are in A.P. (C)  $a, \frac{b}{2}, c$  are in H.P. (D)  $a, 2b, c$  are in H.P.
16. The sum to  $n$  terms of the series  $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \dots$  is -
- (A)  $\frac{3n}{n+1}$  (B)  $\frac{6n}{n+1}$  (C)  $\frac{9n}{n+1}$  (D)  $\frac{12n}{n+1}$
17. If  $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots + \text{to } \infty = \frac{\pi^4}{90}$ , then  $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots + \text{to } \infty$  is equals to -
- (A)  $\frac{\pi^4}{96}$  (B)  $\frac{\pi^4}{45}$  (C)  $\frac{89\pi^4}{90}$  (D) none of these
18. If  $\sum_{s=1}^n \left\{ \sum_{r=1}^s r \right\} = an^3 + bn^2 + cn$ , then find the value of  $a + b + c$ .
- (A) 1 (B) 0 (C) 2 (D) 3
19. If  $a, b, c$  are positive numbers in G.P. and  $\log\left(\frac{5c}{a}\right), \log\left(\frac{3b}{5c}\right)$  and  $\log\left(\frac{a}{3b}\right)$  are in A.P., then  $a, b, c$  forms the sides of a triangle which is -
- (A) equilateral (B) right angled (C) isosceles (D) none of these

**SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)**

20. If sum of  $n$  terms of a sequence is given by  $S_n = 3n^2 - 5n + 7$  &  $t_r$  represents its  $r^{\text{th}}$  term, then -
- (A)  $t_7 = 34$  (B)  $t_2 = 7$  (C)  $t_{10} = 34$  (D)  $t_8 = 40$

21. If 10 harmonic means  $H_1, H_2, H_3, \dots, H_{10}$  are inserted between 7 and  $-\frac{1}{3}$ , then -

- (A)  $H_1 = -7$                       (B)  $H_2 = \frac{3}{7}$                       (C)  $H_1 = -\frac{1}{7}$                       (D)  $H_{10} = -\frac{7}{19}$

22. If  $t_n$  be the  $n^{\text{th}}$  term of the series  $1 + 3 + 7 + 15 + \dots$ , then -

- (A)  $t_5 + 1 = 32$                       (B)  $t_7 = 2^7 + 1$                       (C)  $t_{10} = 2^{10} - 1$                       (D)  $t_{100} = 2^{50} + 1$

23. Indicate the correct alternative(s), for  $0 < \phi < \frac{\pi}{2}$ , if  $x = \sum_{n=0}^{\infty} \cos^{2n} \phi$ ,  $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$  and  $z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$ ,

then -

- (A)  $xyz = xz + y$                       (B)  $xyz = xy + z$                       (C)  $xyz = x + y + z$                       (D)  $xyz = yz + x$

BRAIN TEASERS			ANSWER KEY							EXERCISE-2
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	A	B	B	A	D	B	D	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	A	D	B	A	A	D	A,D
Que.	21	22	23							
Ans.	A,D	A,C	B,C							

**EXERCISE - 02****BRAIN TEASERS****SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)**

1. Consider an A.P. with first term 'a' and the common difference d. Let  $S_k$  denote the sum of the first K terms.

Let  $\frac{S_{kx}}{S_x}$  is independent of x, then -

- (A)  $a = d/2$  (B)  $a = d$  (C)  $a = 2d$  (D) none of these
2. Let  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 + 3ax^2 + 3bx + c = 0$ . If  $\alpha, \beta, \gamma$  are in H.P. then  $\beta$  is equal to -  
 (A)  $-c/b$  (B)  $c/b$  (C)  $-a$  (D)  $a$
3.  $\sum_{r=1}^{\infty} (2r-1) \left(\frac{9}{11}\right)^r$  is equal to -  
 (A) 45 (B) 55  
 (C) sum of first nine natural numbers (D) sum of first ten natural numbers
4. For the A.P. given by  $a_1, a_2, \dots, a_n, \dots$ , with non-zero common difference, the equations satisfied are-  
 (A)  $a_1 + 2a_2 + a_3 = 0$  (B)  $a_1 - 2a_2 + a_3 = 0$   
 (C)  $a_1 + 3a_2 - 3a_3 - a_4 = 0$  (D)  $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$
5. If  $a, a_1, a_2, \dots, a_{10}, b$  are in A.P. and  $a, g_1, g_2, \dots, g_{10}, b$  are in G.P. and  $h$  is the H.M. between  $a$  and  $b$ , then  
 $\frac{a_1 + a_2 + \dots + a_{10}}{g_1 g_{10}} + \frac{a_2 + a_3 + \dots + a_9}{g_2 g_9} + \dots + \frac{a_5 + a_6}{g_5 g_6}$  is -  
 (A)  $\frac{10}{h}$  (B)  $\frac{15}{h}$  (C)  $\frac{30}{h}$  (D)  $\frac{5}{h}$
6. The sum of the first  $n$  terms of the series  $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \dots$  is  $\frac{n(n+1)^2}{2}$ , when  $n$  is even. When  $n$  is odd, the sum is -  
 (A)  $\frac{n^2(n+1)}{2}$  (B)  $\frac{n(n+1)(2n+1)}{6}$  (C)  $\frac{n(n+1)^2}{2}$  (D)  $\frac{n^2(n+1)^2}{2}$
7. If  $(1 + 3 + 5 + \dots + a) + (1 + 3 + 5 + \dots + b) = (1 + 3 + 5 + \dots + c)$ , where each set of parentheses contains the sum of consecutive odd integers as shown such that - (i)  $a + b + c = 21$ , (ii)  $a > 6$   
 If  $G = \text{Max}\{a, b, c\}$  and  $L = \text{Min}\{a, b, c\}$ , then -  
 (A)  $G - L = 4$  (B)  $b - a = 2$  (C)  $G - L = 7$  (D)  $a - b = 2$
8. If  $a, b$  and  $c$  are distinct positive real numbers and  $a^2 + b^2 + c^2 = 1$ , then  $ab + bc + ca$  is -  
 (A) equal to 1 (B) less than 1 (C) greater than 1 (D) any real number
9. Let  $p, q, r \in \mathbb{R}^+$  and  $27 pqr \geq (p + q + r)^3$  and  $3p + 4q + 5r = 12$  then  $p^3 + q^4 + r^5$  is equal to -  
 (A) 2 (B) 6 (C) 3 (D) none of these
10. The sum of the first 100 terms common to the series 17, 21, 25, ..... and 16, 21, 26, ..... is -  
 (A) 101100 (B) 111000 (C) 110010 (D) 100101

11. If  $a, b, c$  are positive such that  $ab^2c^3 = 64$  then least value of  $\left(\frac{1}{a} + \frac{2}{b} + \frac{3}{c}\right)$  is -  
(A) 6 (B) 2 (C) 3 (D) 32
12. If  $a_1, a_2, \dots, a_n \in \mathbb{R}^+$  and  $a_1 \cdot a_2 \cdot \dots \cdot a_n = 1$  then the least value of  $(1 + a_1 + a_1^2)(1 + a_2 + a_2^2) \dots (1 + a_n + a_n^2)$  is -  
(A)  $3^n$  (B)  $n3^n$  (C)  $3^{3n}$  (D) data inadequate
13. Let  $a_1, a_2, a_3, \dots$  and  $b_1, b_2, b_3, \dots$  be arithmetic progression such that  $a_1 = 25, b_1 = 75$  and  $a_{100} + b_{100} = 100$ , then -  
(A) The common difference in progression ' $a_i$ ' is equal but opposite in sign to the common difference in progression ' $b_j$ '.  
(B)  $a_n + b_n = 100$  for any  $n$ .  
(C)  $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3), \dots$  are in A.P.  
(D)  $\sum_{r=1}^{100} (a_r + b_r) = 10^4$
14. If the AM of two positive numbers be three times their geometric mean then the ratio of the numbers is -  
(A)  $3 \pm 2\sqrt{2}$  (B)  $\sqrt{2} \pm 1$  (C)  $17 + 12\sqrt{2}$  (D)  $(3 - 2\sqrt{2})^{-2}$
15. If first and  $(2n - 1)^{\text{th}}$  terms of an A.P., G.P. and H.P. are equal and their  $n^{\text{th}}$  terms are  $a, b, c$  respectively, then -  
(A)  $a + c = 2b$  (B)  $a \geq b \geq c$  (C)  $a + c = b$  (D)  $b^2 = ac$
16. Let  $a, x, b$  be in A.P. ;  $a, y, b$  be in G.P. and  $a, z, b$  be in H.P. If  $x = y + 2$  and  $a = 5z$  then -  
(A)  $y^2 = xz$  (B)  $x > y > z$  (C)  $a = 9, b = 1$  (D)  $a = \frac{9}{4}, b = \frac{1}{4}$
17. The  $p^{\text{th}}$  term  $T_p$  of H.P. is  $q(q + p)$  and  $q^{\text{th}}$  term  $T_q$  is  $p(p + q)$  when  $p > 1, q > 1$ , then -  
(A)  $T_{p+q} = pq$  (B)  $T_{pq} = p + q$  (C)  $T_{p+q} > T_{pq}$  (D)  $T_{pq} > T_{p+q}$
18.  $a, b, c$  are three distinct real numbers, which are in G.P. and  $a + b + c = xb$ , then -  
(A)  $x < -1$  (B)  $-1 < x < 2$  (C)  $2 < x < 3$  (D)  $x > 3$
19. Let  $a_1, a_2, \dots, a_{10}$  be in A.P. &  $h_1, h_2, \dots, h_{10}$  be in H.P. . If  $a_1 = h_1 = 2$  &  $a_{10} = h_{10} = 3$  then  $a_4 h_7$  is -  
(A) 2 (B) 3 (C) 5 (D) 6

BRAIN TEASERS					ANSWER KEY			EXERCISE-2		
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	A,C	B,D	C	A	A,D	B	C	A
Que.	11	12	13	14	15	16	17	18	19	
Ans.	C	A	A,B,C,D	C,D	B,D	A,B,C	A,B,C	A,D	D	

**EXERCISE - 03****MISCELLANEOUS TYPE QUESTIONS****FILL IN THE BLANKS**

- The sum of  $n$  terms of two A.P.'s are in the ratio of  $(n + 7) : (3n + 11)$ . The ratio of their 9th term is \_\_\_\_\_.
- The sum of the first nineteen terms of an A.P.  $a_1, a_2, a_3, \dots$  if it is known that  $a_4 + a_8 + a_{12} + a_{16} = 224$ , is \_\_\_\_\_.
- If  $x \in \mathbb{R}$  and the numbers  $(5^{1+x} + 5^{1-x}), a/2, (25^x + 25^{-x})$  form an A.P. then 'a' must lie in the interval \_\_\_\_\_.
- If  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  upto  $\infty = \frac{\pi^2}{6}$ , then  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots =$  \_\_\_\_\_.
- When 9th term of an A.P. is divided by its 2nd term the quotient is 5 & when 13th term is divided by the 6th term, the quotient is 2 and remainder is 5. The first term and the common difference of the A.P. are \_\_\_\_\_ & \_\_\_\_\_ respectively.
- The sum to infinity of the series  $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots$  is equal to \_\_\_\_\_.
- If  $\sin(x - y), \sin x$  and  $\sin(x + y)$  are in H.P., then  $\sin x \cdot \sec \frac{y}{2} =$  \_\_\_\_\_.

**MATCH THE COLUMN**

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.	Column-I	Column-II
(A)	If $a_i$ 's are in A.P. and $a_1 + a_3 + a_4 + a_5 + a_7 = 20$ , $a_4$ is equal to	(p) 21
(B)	Sum of an infinite G.P. is 6 and its first term is 3. then harmonic mean of first and third terms of G.P. is	(q) 4
(C)	If roots of the equation $x^3 - ax^2 + bx + 27 = 0$ , are in G.P. with common ratio 2, then $a + b$ is equal to	(r) 24
(D)	If the roots of $x^4 - 8x^3 + ax^2 + bx + 16 = 0$ are positive real numbers then $a$ is	(s) $6/5$

2.	Column-I	Column-II
(A)	$n^{\text{th}}$ term of the series 4, 11, 22, 37, 56, 79,.....	(p) $2n^2 + n$
(B)	$ 1^2 - 2^2 + 3^2 - 4^2 \dots \dots \dots 2n \text{ terms} $ is equal to	(q) $2n^2 + n + 1$
(C)	sum to $n$ terms of the series 3, 7, 11, 15,..... is	(r) $-(n^2 + n)$
(D)	coefficient of $x^n$ in $2x(x - 1)(x - 2) \dots \dots \dots (x - n)$ is	(s) $\frac{1}{2}(n^2 + n)$

**ASSERTION & REASON**

These questions contains, Statement-I (assertion) and Statement-II (reason).

- Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
- Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.
- Statement-I is true, Statement-II is false.
- Statement-I is false, Statement-II is true.

1. **Statement-I** : If  $a, b, c$  are three distinct positive number in H.P., then  $\left(\frac{a+b}{2a-b}\right) + \left(\frac{c+b}{2c-b}\right) > 4$

**Because**

**Statement-II** : Sum of any number and it's reciprocal is always greater than or equal to 2.

(A) A (B) B (C) C (D) D

2. **Statement-I** : If  $x^2y^3 = 6(x, y > 0)$ , then the least value of  $3x + 4y$  is 10

**Because**

**Statement-II** : If  $m_1, m_2 \in \mathbb{N}, a_1, a_2 > 0$  then  $\frac{m_1a_1 + m_2a_2}{m_1 + m_2} \geq (a_1^{m_1} a_2^{m_2})^{\frac{1}{m_1 + m_2}}$  and equality holds when

$a_1 = a_2$ .

(A) A (B) B (C) C (D) D

3. **Statement-I** : For  $n \in \mathbb{N}, 2^n > 1 + n(\sqrt{2^{n-1}})$

**Because**

**Statement-II** : G.M. > H.M. and (AM) (HM) = (GM)<sup>2</sup>

(A) A (B) B (C) C (D) D

4. **Statement-I** : If  $a, b, c$  are three positive numbers in G.P., then  $\left(\frac{a+b+c}{3}\right) \cdot \left(\frac{3abc}{ab+bc+ca}\right) = (\sqrt[3]{abc})^2$

**Because**

**Statement-II** : (A.M.) (H.M.) = (G.M.)<sup>2</sup> is true for any set of positive numbers.

(A) A (B) B (C) C (D) D

5. **Statement-I** :  $n^{\text{th}}$  term ( $T_n$ ) of the sequence (1, 6, 18, 40, 75, 126,....) is  $an^3 + bn^2 + cn + d$ , and  $6a + 2b - d$  is = 4.

**Because**

**Statement-II** If the second successive differences (Differences of the differences) of a series are in A.P., then  $T_n$  is a cubic polynomial in  $n$ .

(A) A (B) B (C) C (D) D

6. **Statement-I** : The format of  $n^{\text{th}}$  term ( $T_n$ ) of the sequence ( $\ell n2, \ell n4, \ell n32, \ell n1024, \dots$ ) is  $an^2 + bn + c$ .

**Because**

**Statement-II** : If the second successive differences between the consecutive terms of the given sequence are in G.P., then  $T_n = a + bn + cr^{n-1}$ , where  $a, b, c$  are constants and  $r$  is common ratio of G.P.

(A) A (B) B (C) C (D) D

## COMPREHENSION BASED QUESTIONS

### Comprehension # 1

There are  $4n + 1$  terms in a sequence of which first  $2n + 1$  are in Arithmetic Progression and last  $2n + 1$  are in Geometric Progression the common difference of Arithmetic Progression is 2 and common ratio of Geometric Progression is  $1/2$ . The middle term of the Arithmetic Progression is equal to middle term of Geometric Progression. Let middle term of the sequence is  $T_m$  and  $T_m$  is the sum of infinite Geometric Progression whose

sum of first two terms is  $\left(\frac{5}{4}\right)^2 n$  and ratio of these terms is  $\frac{9}{16}$ .

**On the basis of above information, answer the following questions :**

- Number of terms in the given sequence is equal to -  
(A) 9 (B) 17 (C) 13 (D) none
- Middle term of the given sequence, i.e.  $T_m$  is equal to -  
(A)  $16/7$  (B)  $32/7$  (C)  $48/7$  (D)  $16/9$
- First term of given sequence is equal to -  
(A)  $-8/7, -20/7$  (B)  $-36/7$  (C)  $36/7$  (D)  $48/7$

4. Middle term of given A. P. is equal to -  
 (A)  $6/7$  (B)  $10/7$  (C)  $78/7$  (D) 11
5. Sum of the terms of given A. P. is equal to -  
 (A)  $6/7$  (B) 7 (C) 3 (D) 6

**Comprehension # 2 :**

If  $a_i > 0$ ,  $i = 1, 2, 3, \dots, n$  and  $m_1, m_2, m_3, \dots, m_n$  be positive rational numbers, then

$$\left( \frac{m_1 a_1 + m_2 a_2 + \dots + m_n a_n}{m_1 + m_2 + \dots + m_n} \right) \geq \left( a_1^{m_1} a_2^{m_2} \dots a_n^{m_n} \right)^{1/(m_1 + m_2 + \dots + m_n)} \geq \frac{(m_1 + m_2 + \dots + m_n)}{\frac{m_1}{a_1} + \frac{m_2}{a_2} + \dots + \frac{m_n}{a_n}}$$

is called weighted mean theorem

where  $A^* = \frac{m_1 a_1 + m_2 a_2 + \dots + m_n a_n}{m_1 + m_2 + \dots + m_n}$  = Weighted arithmetic mean

$$G^* = \left( a_1^{m_1} a_2^{m_2} \dots a_n^{m_n} \right)^{1/(m_1 + m_2 + \dots + m_n)} = \text{Weighted geometric mean}$$

and  $H^* = \frac{m_1 + m_2 + \dots + m_n}{\frac{m_1}{a_1} + \frac{m_2}{a_2} + \dots + \frac{m_n}{a_n}}$  = Weighted harmonic mean

i.e.,  $A^* \geq G^* \geq H^*$

Now, let  $a + b + c = 5$  ( $a, b, c > 0$ ) and  $x^2 y^3 = 243$  ( $x > 0, y > 0$ )

**On the basis of above information, answer the following questions :**

- The greatest value of  $ab^3c$  is -  
 (A) 3 (B) 9 (C) 27 (D) 81
- Which statement is correct -  
 (A)  $\frac{1}{5} \geq \frac{1}{\frac{1}{a} + \frac{3}{b} + \frac{1}{c}}$  (B)  $\frac{1}{25} \geq \frac{1}{\frac{1}{a} + \frac{9}{b} + \frac{1}{c}}$  (C)  $\frac{1}{5} \geq \frac{1}{\frac{1}{a} + \frac{9}{b} + \frac{1}{c}}$  (D)  $\frac{1}{25} \geq \frac{1}{\frac{1}{a} + \frac{6}{b} + \frac{1}{c}}$
- The least value of  $x^2 + 3y + 1$  is -  
 (A) 15 (B) greater than 15 (C) 3 (D) less than 15
- Which statement is correct -  
 (A)  $\frac{2x+3y}{5} \geq 3 \geq \frac{5}{\frac{3}{x} + \frac{2}{y}}$  (B)  $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+2y}$   
 (C)  $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+4y}$  (D)  $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{2x+3y}$

MISCELLANEOUS TYPE QUESTION	ANSWER KEY	EXERCISE-3
<ul style="list-style-type: none"> <li><b>Fill in the Blanks</b>            1. 12 : 31   2. 1064   3. <math>[12, \infty]</math>   4. <math>\pi^2/8</math>   5. <math>a = 3</math> <math>d = 4</math>   6. 2   7. <math>\pm \sqrt{2}</math></li> <li><b>Match the Column</b>            1. (A) <math>\rightarrow</math> (q), (B) <math>\rightarrow</math> (s), (C) <math>\rightarrow</math> (p), (D) <math>\rightarrow</math> (r)   2. (A) <math>\rightarrow</math> (q), (B) <math>\rightarrow</math> (p), (C) <math>\rightarrow</math> (p), (D) <math>\rightarrow</math> (r)</li> <li><b>Assertion &amp; Reason</b>            1. C   2. A   3. C   4. C   5. A   6. B</li> <li><b>Comprehension Based Questions</b>            Comprehension # 1 : 1. C   2. C   3. B   4. A   5. D            Comprehension # 2 : 1. C   2. C   3. B   4. B</li> </ul>		



## EXERCISE - 04 [A]

## CONCEPTUAL SUBJECTIVE EXERCISE

- Given that  $a^x = b^y = c^z = d^u$  &  $a, b, c, d$  are in GP, show that  $x, y, z, u$  are in HP.
- There are  $n$  AM's between 1 & 31 such that 7th mean :  $(n - 1)$ th mean = 5 : 9, then find the value of  $n$ .
- Find the sum of the series,  $7 + 77 + 777 + \dots$  to  $n$  terms.
- If the  $p^{\text{th}}, q^{\text{th}}$  &  $r^{\text{th}}$  terms of an AP are in GP. Show that the common ratio of the GP is  $\frac{q-r}{p-q}$ .
- Express the recurring decimal  $0.1\overline{576}$  as a rational number using concept of infinite geometric series.
- If one AM 'a' & two GM's  $p$  &  $q$  be inserted between any two given numbers then show that  $p^3 + q^3 = 2apq$ .
- Find three numbers  $a, b, c$  between 2 & 18 which satisfy following conditions :
  - their sum is 25
  - the numbers 2,  $a, b$  are consecutive terms of an AP &
  - the numbers  $b, c, 18$  are consecutive terms of a GP.
- Find the sum of the first  $n$  terms of the series :  $1 + 2\left(1 + \frac{1}{n}\right) + 3\left(1 + \frac{1}{n}\right)^2 + 4\left(1 + \frac{1}{n}\right)^3 + \dots$
- Let  $a_1, a_2, a_3, \dots, a_n$  be an AP. Prove that :
 
$$\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \dots + \frac{1}{a_n a_1} = \frac{2}{a_1 + a_n} \left[ \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n} \right]$$
- The harmonic mean of two numbers is 4. The arithmetic mean  $A$  & the geometric mean  $G$  satisfy the relation  $2A + G = 27$ . Find the two numbers.
- Prove that :  $(ab + xy)(ax + by) \geq 4abxy$  where  $a, b, x, y \in \mathbb{R}^+$
- If  $a, b, c \in \mathbb{R}^+$  &  $a + b + c = 1$ ; then show that  $(1 - a)(1 - b)(1 - c) \geq 8abc$
- If  $a, b, c$  are sides of a scalene triangle then show that  $(a + b + c)^3 > 27(a + b - c)(b + c - a)(c + a - b)$
- For positive number  $a, b, c$  show that  $\frac{bc}{a} + \frac{ac}{b} + \frac{ab}{c} \geq a + b + c$
- The odd positive numbers are written in the form of a triangle

1  
 3 5  
 7 9 11  
 13 15 17 19  
 .....  
 .....

..... find the sum of terms in  $n^{\text{th}}$  row.

CONCEPTUAL SUBJECTIVE EXERCISE		ANSWER KEY	EXERCISE-4(A)
2. 14	3. $S = (7/81)(10^{n+1} - 9n - 10)$	5. 35/222	7. $a = 5, b = 8, c = 12$
8. $n^2$	10. 6, 3	15. $n^3$	

**EXERCISE - 04 [B]****BRAIN STORMING SUBJECTIVE EXERCISE**

- In a A.P. & an H.P. have the same first term, the same last term & the same number of terms; prove that the product of the  $r^{\text{th}}$  term from the beginning in one series & the  $r^{\text{th}}$  term from the end in the other is independent of  $r$ .
- Sum the following series to  $n$  terms and to infinity :
  - $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \dots$
  - $\sum_{r=1}^n r(r+1)(r+2)(r+3)$
  - $\sum_{r=1}^n \frac{1}{4r^2 - 1}$
- Find the value of the sum  $\sum_{r=1}^n \sum_{s=1}^n \delta_{rs} 2^r 3^s$  where  $\delta_{rs}$  is zero if  $r \neq s$  &  $\delta_{rs}$  is one if  $r = s$ .
- Find the sum  $\sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1$ .
- If there be 'm' A.P.'s beginning with unity whose common difference is  $1, 2, 3, \dots, m$ . Show that the sum of their  $n^{\text{th}}$  terms is  $(m/2)(mn - m + n + 1)$ .
- If  $a_1, a_2, a_3, \dots, a_n$  are in H.P., then prove that  $a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = (n-1) a_1 a_n$ .
- If  $a, b, c$  are in H.P.,  $b, c, d$  are in G.P. &  $c, d, e$  are in A.P., then Show that  $e = ab/(2a - b)$ .
- The value of  $x + y + z$  is 15, if  $a, x, y, z, b$  are in A.P. while the value of  $(1/x) + (1/y) + (1/z)$  is  $5/3$  if  $a, x, y, z, b$  are in H.P. Find  $a$  &  $b$ .
- Prove that the sum of the infinite series  $\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \dots = 23$ .
- If  $a, b, c$  be in G.P. &  $\log_c a, \log_b c, \log_a b$  be in A.P., then show that the common difference of the A.P. must be  $3/2$ .
- Find the sum to  $n$  terms :
  - $\frac{1}{x+1} + \frac{2x}{(x+1)(x+2)} + \frac{3x^2}{(x+1)(x+2)(x+3)} + \dots$
  - $\frac{a_1}{1+a_1} + \frac{a_2}{(1+a_1)(1+a_2)} + \frac{a_3}{(1+a_1)(1+a_2)(1+a_3)} + \dots$
- In a G.P., the ratio of the sum of the first eleven terms to the sum of the last eleven terms is  $1/8$  and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Find the number of terms in the G.P.
- Prove that the number  $\underbrace{444\dots4}_{n \text{ digits}}, \underbrace{888\dots8}_{(n-1) \text{ digits}}9$  is a perfect square of the number  $\underbrace{666\dots6}_{(n-1) \text{ digits}}7$ .
- Find the  $n^{\text{th}}$  term and the sum to ' $n$ ' terms of the series :
  - $1 + 5 + 13 + 29 + 61 + \dots$
  - $6 + 13 + 22 + 33 + \dots$
- If  $a, b, c$  are three positive real number then prove that :  $\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}$
- If  $a, b, c$  are the sides of a triangle and  $s = \frac{a+b+c}{2}$ , then prove that  $8(s-a)(s-b)(s-c) \leq abc$ .

BRAIN STORMING SUBJECTIVE EXERCISE		ANSWER KEY	EXERCISE-4(B)
2. (a) $\frac{1}{24} - \frac{1}{6(3n+1)(3n+4)}, \frac{1}{24}$	(b) $\frac{n(n+1)(n+2)(n+3)(n+4)}{5}$	(c) $\frac{n}{2n+1}, \frac{1}{2}$	
3. $\frac{6}{5}(6^n - 1)$	4. $[n(n+1)(n+2)]/6$	8. $a = 1, b = 9$ or $b = 1, a = 9$	
11. (a) $1 - \frac{x^n}{(x+1)(x+2)\dots(x+n)}$	(b) $1 - \frac{1}{(1+a_1)(1+a_2)\dots(1+a_n)}$		
12. $n = 38$	14. (a) $2^{n+1} - 3; 2^{n+2} - 4 - 3n$	(b) $n^2 + 4n + 1; \frac{1}{6}n(n+1)(2n+13) + n$	

**EXERCISE - 05 [A]**
**JEE-[MAIN] : PREVIOUS YEAR QUESTIONS**

- If 1,  $\log_3 \sqrt{3^{1-x} + 2}$ ,  $\log_3(4 \cdot 3^x - 1)$  are in A.P. then x equals. [AIEEE 2002]

(1)  $\log_3 4$  (2)  $1 - \log_3 4$  (3)  $1 - \log_4 3$  (4)  $\log_4 3$
- Sum of infinite number of terms in G.P. is 20 and sum of their square is 100. The common ratio of G.P. is- [AIEEE 2002]

(1) 5 (2)  $3/5$  (3)  $8/5$  (4)  $1/5$
- Fifth term of a G.P. is 2, then the product of its 9 terms is- [AIEEE 2002]

(1) 256 (2) 512 (3) 1024 (4) None of these
- The sum of the series  $1^3 - 2^3 + 3^3 - \dots + 9^3 =$  [AIEEE 2002]

(1) 300 (2) 125 (3) 425 (4) 0
- Let  $T_r$  be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n,  $m \neq n$ ,  $T_m = \frac{1}{n}$  and  $T_n = \frac{1}{m}$ , then a - d equals [AIEEE 2004]

(1) 0 (2) 1 (3)  $\frac{1}{mn}$  (4)  $\frac{1}{m} + \frac{1}{n}$
- If AM and GM of two roots of a quadratic equation are 9 and 4 respectively, then this quadratic equation is- [AIEEE 2004]

(1)  $x^2 - 18x + 16 = 0$  (2)  $x^2 + 18x - 16 = 0$  (3)  $x^2 + 18x + 16 = 0$  (4)  $x^2 - 18x - 16 = 0$
- If  $a_1, a_2, a_3, \dots, a_n, \dots$  are in G.P. then the value of the determinant  $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ , is- [AIEEE 04, 05]

(1) 0 (2) 1 (3) 2 (4) -2
- If  $x = \sum_{n=0}^{\infty} a^n$ ,  $y = \sum_{n=0}^{\infty} b^n$ ,  $z = \sum_{n=0}^{\infty} c^n$  where a, b, c are in A.P. and  $|a| < 1$ ,  $|b| < 1$ ,  $|c| < 1$  then x, y, z are in- [AIEEE 2005]

(1) HP (2) Arithmetic - Geometric Progression (3) AP (4) GP
- Let  $a_1, a_2, a_3, \dots$  be terms of an A.P. If  $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}$ ,  $p \neq q$  then  $\frac{a_6}{a_{21}}$  equals- [AIEEE-2006]

(1)  $\frac{2}{7}$  (2)  $\frac{11}{41}$  (3)  $\frac{41}{11}$  (4)  $\frac{7}{2}$
- If  $a_1, a_2, \dots, a_n$  are in H.P., then the expression  $a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n$  is equal to- [AIEEE-2006]

(1)  $n a_1 a_n$  (2)  $(n-1) a_1 a_n$  (3)  $n(a_1 - a_n)$  (4)  $(n-1)(a_1 - a_n)$
- In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- [AIEEE-2007]

(1)  $\frac{1}{2} \sqrt{5}$  (2)  $\sqrt{5}$  (3)  $\frac{1}{2} (\sqrt{5} - 1)$  (4)  $\frac{1}{2} (1 - \sqrt{5})$

12. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is [AIEEE 2008]
- (1) -4 (2) -12 (3) 12 (4) 4
13. The sum to infinity of the series  $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$  is :- [AIEEE-2009]
- (1) 4 (2) 6 (3) 2 (4) 3
14. A person is to count 4500 currency notes. Let  $a_n$  denote the number of notes he counts in the  $n^{\text{th}}$  minute. If  $a_1 = a_2 = \dots = a_{10} = 150$  and  $a_{10}, a_{11}, \dots$  are in an AP with common difference -2, then the time taken by him to count all notes is :- [AIEEE-2010]
- (1) 24 minutes (2) 34 minutes (3) 125 minutes (4) 135 minutes
15. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :- [AIEEE-2011]
- (1) 20 months (2) 21 months (3) 18 months (4) 19 months
16. Let  $a_n$  be the  $n^{\text{th}}$  term of an A.P. If  $\sum_{r=1}^{100} a_{2r} = \alpha$  and  $\sum_{r=1}^{100} a_{2r-1} = \beta$ , then the common difference of the A.P. is: [AIEEE-2011]
- (1)  $\frac{\alpha - \beta}{200}$  (2)  $\alpha - \beta$  (3)  $\frac{\alpha - \beta}{100}$  (4)  $\beta - \alpha$
17. **Statement-1** : The sum of the series  $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$  is 8000.
- Statement-2** :  $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$ , for any natural number  $n$ . [AIEEE-2012]
- (1) Statement-1 is true, Statement-2 is false.  
 (2) Statement-1 is false, Statement-2 is true.  
 (3) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.  
 (4) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1.
18. If 100 times the  $100^{\text{th}}$  term of an A.P. with non-zero common difference equals the 50 times its  $50^{\text{th}}$  term, then the  $150^{\text{th}}$  term of this A.P. is : [AIEEE-2012]
- (1) zero (2) -150  
 (3) 150 times its  $50^{\text{th}}$  term (4) 150
19. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ....., is : [JEE-MAIN 2013]
- (1)  $\frac{7}{81}(179 - 10^{-20})$  (2)  $\frac{7}{9}(99 - 10^{-20})$  (3)  $\frac{7}{81}(179 + 10^{-20})$  (4)  $\frac{7}{9}(99 - 10^{-20})$

PREVIOUS YEARS QUESTIONS			ANSWER KEY		EXERCISE-5 [A]	
1. 2	2. 2	3. 2	4. 3	5. 1	6. 1	7. 1
8. 1	9. 2	10. 2	11. 3	12. 2	13. 4	14. 2
15. 2	16. 3	17. 3	18. 1	19. 3		

**EXERCISE - 05 [B]**
**JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS**

1. (a) Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is  $\frac{3}{4}$ , then - [JEE 2000, Screening, 1+1M out of 35]
- (A)  $a = \frac{7}{4}$ ,  $r = \frac{3}{7}$       (B)  $a = 2$ ,  $r = \frac{3}{8}$       (C)  $a = \frac{3}{2}$ ,  $r = \frac{1}{2}$       (D)  $a = 3$ ,  $r = \frac{1}{4}$
- (b) If a, b, c, d are positive real numbers such that  $a + b + c + d = 2$ , then  $M = (a + b)(c + d)$  satisfies the relation -
- (A)  $0 \leq M \leq 1$       (B)  $1 \leq M \leq 2$       (C)  $2 \leq M \leq 3$       (D)  $3 \leq M \leq 4$
- (c) The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. [JEE 2000, Mains, 4M out of 100]
2. (a) Let  $\alpha, \beta$  be the roots of  $x^2 - x + p = 0$  and  $\gamma, \delta$  be the roots of  $x^2 - 4x + q = 0$ . If  $\alpha, \beta, \gamma, \delta$  are in G.P., then the integer values of p and q respectively, are - [JEE 2001 Screening 1+1+1M out of 35]
- (A) -2, -32      (B) -2, 3      (C) -6, 3      (D) -6, -32
- (b) If the sum of the first 2n terms of the A.P. 2, 5, 8 ..... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ..... then n equals -
- (A) 10      (B) 12      (C) 11      (D) 13
- (c) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are
- (A) not in A.P./G.P./H.P.      (B) in A.P.  
(C) in G.P.      (D) in H.P.
- (d) Let  $a_1, a_2, \dots$  be positive real numbers in G.P.. For each n, let  $A_n, G_n, H_n$  be respectively, the arithmetic mean, geometric mean and harmonic mean of  $a_1, a_2, a_3, \dots, a_n$ . Find an expression for the G.M. of  $G_1, G_2, \dots, G_n$  in terms of  $A_1, A_2, \dots, A_n, H_1, H_2, \dots, H_n$  [JEE 2001 (Mains) ; 5M]
3. (a) Suppose a, b, c are in A.P. and  $a^2, b^2, c^2$  are in G.P. If  $a < b < c$  and  $a + b + c = \frac{3}{2}$ , then the value of a is -
- (A)  $\frac{1}{2\sqrt{2}}$       (B)  $\frac{1}{2\sqrt{3}}$       (C)  $\frac{1}{2} - \frac{1}{\sqrt{3}}$       (D)  $\frac{1}{2} - \frac{1}{\sqrt{2}}$
- [JEE 2002 ( Screening), 3M]
- (b) Let a, b be positive real numbers. If a,  $A_1, A_2, b$  are in A.P. ; a,  $G_1, G_2, b$  are in G.P. and a,  $H_1, H_2, b$  are in H.P., show that  $\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a + b)(a + 2b)}{9ab}$ . [JEE 2002, Mains, 5M out of 60]
4. If a, b, c are in A.P.,  $a^2, b^2, c^2$  are in H.P., then prove that either  $a = b = c$  or a, b,  $-\frac{c}{2}$  form a G.P. [JEE 2003, Mains, 4M out of 60]
5. If a, b, c are positive real numbers, then prove that  $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$ . [JEE 2004, 4M]
6. The first term of an infinite geometric progression is x and its sum is 5. Then - [JEE 2004]
- (A)  $0 \leq x \leq 10$       (B)  $0 < x < 10$       (C)  $-10 < x < 0$       (D)  $x > 10$

7. If total number of runs scored in  $n$  matches is  $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$  where  $n > 1$ , and the runs scored in the  $k^{\text{th}}$  match are given by  $k \cdot 2^{n+1-k}$ , where  $1 \leq k \leq n$ . Find  $n$ . [JEE-05, Mains-2M out of 60]
8. In quadratic equation  $ax^2 + bx + c = 0$ , if  $\alpha, \beta$  are roots of equation,  $\Delta = b^2 - 4ac$  and  $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$  are in G.P. then [JEE 2005 (screening)]
- (A)  $\Delta \neq 0$  (B)  $b\Delta = 0$  (C)  $c\Delta = 0$  (D)  $\Delta = 0$
9. If  $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots + (-1)^{n-1} \left(\frac{3}{4}\right)^n$  and  $b_n = 1 - a_n$  then find the minimum natural number  $n_0$  such that  $b_n > a_n \forall n \geq n_0$  [JEE 2006, 6M out of 184]

### Comprehension Based Question

#### Comprehension # 1

Let  $V_r$  denote the sum of first  $r$  terms of an arithmetic progression (A.P.) whose first term is  $r$  and the common difference is  $(2r - 1)$ .

Let  $T_r = V_{r+1} - V_r - 2$  and  $Q_r = T_{r+1} - T_r$  for  $r = 1, 2, \dots$

10. The sum  $V_1 + V_2 + \dots + V_n$  is : [JEE 2007, 4M]
- (A)  $\frac{1}{12} n(n+1)(3n^2 - n + 1)$  (B)  $\frac{1}{12} n(n+1)(3n^2 + n + 2)$
- (C)  $\frac{1}{2} n(2n^2 - n + 1)$  (D)  $\frac{1}{3} (2n^3 - 2n + 3)$
11.  $T_r$  is always : [JEE 2007, 4M]
- (A) an odd number (B) an even number
- (C) a prime number (D) a composite number
12. Which one of the following is a correct statement ? [JEE 2007, 4M]
- (A)  $Q_1, Q_2, Q_3, \dots$  are in A.P. with common difference 5
- (B)  $Q_1, Q_2, Q_3, \dots$  are in A.P. with common difference 6
- (C)  $Q_1, Q_2, Q_3, \dots$  are in A.P. with common difference 11
- (D)  $Q_1 = Q_2 = Q_3 = \dots$

#### Comprehension # 2

Let  $A_1, G_1, H_1$  denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For  $n \geq 2$ , let  $A_{n-1}$  and  $H_{n-1}$  has arithmetic, geometric and harmonic means as  $A_n, G_n, H_n$  respectively :

13. Which one of the following statements is correct ? [JEE 2007, 4M]
- (A)  $G_1 > G_2 > G_3 > \dots$
- (B)  $G_1 < G_2 < G_3 < \dots$
- (C)  $G_1 = G_2 = G_3 = \dots$
- (D)  $G_1 < G_2 < G_3 < \dots$  and  $G_4 > G_5 > G_6 > \dots$
14. Which one of the following statements is correct ? [JEE 2007, 4M]
- (A)  $A_1 > A_2 > A_3 > \dots$  (B)  $A_1 < A_2 < A_3 < \dots$
- (C)  $A_1 > A_3 > A_5 > \dots$  and  $A_2 < A_4 < A_6 < \dots$  (D)  $A_1 < A_3 < A_5 < \dots$  and  $A_2 > A_4 > A_6 > \dots$
15. Which one of the following statements is correct ? [JEE 2007, 4M]
- (A)  $H_1 > H_2 > H_3 > \dots$  (B)  $H_1 < H_2 < H_3 < \dots$
- (C)  $H_1 > H_3 > H_5 > \dots$  and  $H_2 < H_4 < H_6 > \dots$  (D)  $H_1 < H_3 < H_5 < \dots$  and  $H_2 > H_4 > H_6 > \dots$

16. Suppose four distinct positive numbers  $a_1, a_2, a_3, a_4$  are in G.P. Let  $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$  and  $b_4 = b_3 + a_4$ .  
**Statement -I** : The numbers  $b_1, b_2, b_3, b_4$  are neither in A.P. nor in G.P.  
**and**  
**Statement -II** : The numbers  $b_1, b_2, b_3, b_4$  are in H.P. [JEE 2008, 3M, -1M]  
 (A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.  
 (B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.  
 (C) Statement-I is true, Statement-II is false.  
 (D) Statement-I is false, Statement-II is true.
17. If the sum of first  $n$  terms of an A.P. is  $cn^2$ , then the sum of squares of these  $n$  terms is [JEE 2009, 3M, -1M]  
 (A)  $\frac{n(4n^2 - 1)c^2}{6}$  (B)  $\frac{n(4n^2 + 1)c^2}{3}$  (C)  $\frac{n(4n^2 - 1)c^2}{3}$  (D)  $\frac{n(4n^2 + 1)c^2}{6}$
18. Let  $S_k, k = 1, 2, \dots, 100$ , denote the sum of the infinite geometric series whose first term is  $\frac{k-1}{k!}$  and the common ratio is  $\frac{1}{k}$ . Then the value of  $\frac{100^2}{100!} + \sum_{k=1}^{100} (k^2 - 3k + 1)S_k$  is [JEE 10, 3M]
19. Let  $a_1, a_2, a_3, \dots, a_{11}$  be real numbers satisfying  
 $a_1 = 15, 27 - 2a_2 > 0$  and  $a_k = 2a_{k-1} - a_{k-2}$  for  $k = 3, 4, \dots, 11$ .  
 If  $\frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90$ , then the value of  $\frac{a_1 + a_2 + \dots + a_{11}}{11}$  is equal to [JEE 10, 3M]
20. The minimum value of the sum of real numbers  $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$  and  $a^{10}$  with  $a > 0$  is [JEE 2011, 4]
21. Let  $a_1, a_2, a_3, \dots, a_{100}$  be an arithmetic progression with  $a_1 = 3$  and  $S_p = \sum_{i=1}^p a_i, 1 \leq p \leq 100$ . For any integer  $n$  with  $1 \leq n \leq 20$ , let  $m = 5n$ . If  $\frac{S_m}{S_n}$  does not depend on  $n$ , then  $a_2$  is [JEE 2011, 4]
22. Let  $a_1, a_2, a_3, \dots$  be in harmonic progression with  $a_1 = 5$  and  $a_{20} = 25$ . The least positive integer  $n$  for which  $a_n < 0$  is [JEE 2012, 3 (-1)]  
 (A) 22 (B) 23 (C) 24 (D) 25
23. Let  $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$ . Then  $S_n$  can take value(s) [JEE-Advanced 2013, 4, (-1)]  
 (A) 1056 (B) 1088 (C) 1120 (D) 1332
24. A pack contains  $n$  cards numbered from 1 to  $n$ . Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller to the numbers on the removed cards is  $k$ , then  $k - 20 =$  [JEE-Advanced 2013, 4, (-1)]

PREVIOUS YEARS QUESTIONS			ANSWER KEY		EXERCISE-5 [B]		
1. (a) D, (b) A		2. (a) A, (b) C, (c) D, (d) $\left[ (A_1, A_2, \dots, A_n) (H_1, H_2, \dots, H_n) \right]^{\frac{1}{2n}}$					
3. (a) D	6. B	7. n = 7	8. C	9. 6	10. B	11. D	12. B
13. C	14. A	15. B	16. C	17. C	18. 3	19. 0	
20. 8	21. 9 or 3	22. D	23. A, D	24. 5			