EXERCISE - 01 CHECK YOUR GRASP

2. Let point of intersection is (x_1, y_1).

So $\sqrt{3}\,x_1 - y_1 = 4\sqrt{3}\,K$ ... (i)

$\sqrt{3}\,K\,x_1 + Ky_1 = 4\sqrt{3}$ ... (ii)

Multiply (i) and (ii), we get $3x_1^2 - y_1^2 = 48$.

6. Centre of hyperbola is (5, 0), so equation is

$$\frac{(x-5)^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a = 5$, $ae - a = 8 \Rightarrow e = \frac{13}{5}$

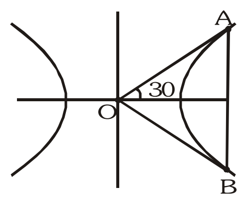
$b^2 = 144$.

So equation is $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$.

8. Let ℓ be the length of double ordinate.

Co-ordinate of point A is

$\left(\ell\frac{\sqrt{3}}{2}, \frac{\ell}{2}\right)$



so . $\frac{3\ell^2}{4a^2} - \frac{\ell^2}{4b^2} = 1$

$\Rightarrow \frac{\ell^2}{4}\left(\frac{3}{a^2} - \frac{1}{b^2}\right) = 1 \Rightarrow \frac{3}{a^2} > \frac{1}{b^2}$

$\Rightarrow \frac{b^2}{a^2} > \frac{1}{3} \Rightarrow e^2 - 1 > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3}$

10. Equation of tangents to two hyperbolas are

$y = mx \pm \sqrt{a^2m^2 - b^2}$ ...(i)

$y = mx \pm \sqrt{-b^2m^2 + a^2}$ ...(ii)

Solving (i) & (ii) we get $m = \pm 1$

$\therefore$ equation of common tangent is

$y = \pm x \pm \sqrt{a^2 - b^2}$.

13. Let the slope of common tangent be m.

Equation of tangent to parabola is

$$y = mx + \frac{2}{m} \quad \text{...(i)}$$

Equation of tangent to hyperbola is

$$y = mx \pm \sqrt{m^2 - 3} \quad \text{...(ii)}$$

By comparing (i) & (ii), we get $m = \pm 2$.

$\therefore$ Equation of common tangent is $y = \pm(2x + 1)$

i.e. $2x \pm y + 1 = 0$.

15. Let equation of asymptotes be $xy - 3x - 2y + \lambda = 0$.

Then $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow \frac{3}{2} - \frac{\lambda}{4} = 0 \quad \Rightarrow \lambda = 6$

$\therefore$ Equation of asymptotes is $xy - 3x - 2y + 6 = 0$

i.e., $(x - 2)(y - 3) = 0$.

17. Equation of normal of rectangular hyperbola $xy = c^2$ at $P(ct, c/t)$ will be

$y - \frac{c}{t} = t^2(x - ct)$

as it also passes through t_1

$\Rightarrow c\left(\frac{1}{t_1} - \frac{1}{t}\right) = ct^2(t_1 - t)$

$\Rightarrow \quad t^3 t_1 = -1$

20. Normal at θ, ϕ are

$$\begin{cases} ax\cos\theta + by\cot\theta = a^2 + b^2 \\ ax\cos\phi + by\cot\phi = a^2 + b^2 \end{cases}$$

where $\phi = \frac{\pi}{2} - \theta$ and these passes through (h, k).

$\therefore$ $ah\cos\theta + bk\cot\theta = a^2 + b^2$ ....(i)

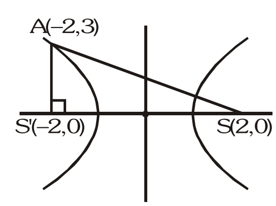
$ah\sin\theta + bk\tan\theta = a^2 + b^2$ ....(ii)

Multiply (i) by $\sin\theta$ & (ii) by $\cos\theta$ & subtract them, we get

$\Rightarrow \quad (bk + a^2 + b^2)(\sin\theta - \cos\theta) = 0$

$k = -(a^2 + b^2)/b$

23. $S \equiv (2, 0)$, $S' \equiv (-2, 0)$



Using reflection property of hyperbola,

S'A is incident ray.

Equation of incident ray S'A is $x = -2$

Equation of reflected ray SP is $3x + 4y = 6$.

Now $2ae = 4 \quad \Rightarrow \quad ae = 2$ ....(i)

Point (–2, 3) lies on hyperbola,

$\therefore \quad \frac{4}{a^2} - \frac{9}{b^2} = 1 \quad \Rightarrow \quad \frac{4}{a^2} - \frac{9}{4-a^2} = 1$

on solving it we get $a = 4$ (reject), $a = 1$ .....(ii)

$\therefore$ Using (i) & (ii), we get $e = 2$

length of latus rectum $= 2a(e^2 - 1) = 6$

2. Let mid-point of chord is (h, k).

Equation of chord is T = 0 $\Rightarrow \frac{hx}{a^2} - \frac{ky}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$

Locus is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{\alpha x}{a^2} - \frac{y\beta}{b^2}$

$\Rightarrow \frac{x(x-\alpha)}{a^2} - \frac{y(y-\beta)}{b^2} = 0$

$\Rightarrow \frac{(x-\frac{\alpha}{2})^2 - \frac{\alpha^2}{4}}{a^2} - \frac{\left(y-\frac{\beta}{2}\right)^2 - \frac{\beta^2}{4}}{b^2} = 0$

$\Rightarrow \frac{(x-\alpha/2)^2}{a^2} - \frac{(y-\beta/2)^2}{b^2} = \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2}$

4. Mid point of chord joining (x_1, y_1) & (x_2, y_2) is

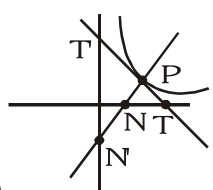
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore$ Equation of chord is

$\left(\frac{y_1+y_2}{2}\right)x + \left(\frac{x_1+x_2}{2}\right)y = 2\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_1+y_2}{2}\right)$

$\Rightarrow \frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1.$

7. Let point P be $\left(ct, \frac{c}{t}\right)$.



Equation of tangent at P is

$x + yt^2 = 2ct$

$\therefore$ T is (2ct, 0) & T' is $\left(0, \frac{2c}{t}\right)$

Now equation of normal at P is

$t^2 x - y = ct^3 - \frac{c}{t}$

$\therefore N\left(ct - \frac{c}{t^3}, 0\right)$ & $N'\left(0, \frac{c}{t} - ct^3\right)$

$\Delta = \frac{1}{2}\frac{c}{t}\left(2ct - ct + \frac{c}{t^3}\right) = \frac{1}{2}\frac{c^2}{t^4}(t^4+1)$

$\Delta' = \frac{1}{2}ct\left(\frac{c}{t} + ct^3\right) = \frac{1}{2}c^2(1+t^4)$

$\therefore \frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{2}{c^2}.$

10. Let any point on hyperbola H_1 is $(a\sec\theta, b\tan\theta)$.

Equation of chord of contact is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 2$ ...(i)

Equation of asymptotes is $y = \pm\frac{b}{a}x$ ...(ii)

From (i) & (ii) we get two intersection point

$P(2a(\sec\theta + \tan\theta), 2b(\sec\theta + \tan\theta))$

$Q(2a(\sec\theta - \tan\theta), -2b(\sec\theta - \tan\theta))$

Then area of triangle OPQ is $\Delta = 2ab$.

12. $\tan\frac{\theta}{2} = \frac{b}{a} \Rightarrow e^2 - 1 = \tan^2\frac{\theta}{2} \Rightarrow \sec\frac{\theta}{2} = e$

or $e^2 - 1 = \cot^2\frac{\theta}{2} \Rightarrow \text{cosec}\frac{\theta}{2} = e$

$\Rightarrow \sec\frac{\theta}{2} = \frac{e}{\sqrt{e^2-1}}.$

15. Given equation will represent hyperbola if

$\lambda^2 > (\lambda+2)(\lambda-1)$ $[\because h^2 > ab]$

$\Rightarrow \lambda < 2$

Also $\Delta \neq 0$

$\Rightarrow -2(\lambda^2+\lambda-2) - 4(\lambda-1) + 2\lambda^2 \neq 0$

$\Rightarrow \lambda \neq \frac{4}{3}.$

20. $\frac{x+y}{2} = t^2 + 1, \frac{x-y}{2} = t$

Eliminating t, $2(x+y) = (x-y)^2 + 4$

line $x + y = X$ & $x - y = Y$

$2X = Y^2 + 4 \Rightarrow Y^2 = 2(X-2)$

represents a parabola.

Match the column :

2. (A) Tangent to the given hyperbola at $P\left(\frac{\pi}{6}\right)$ is

$$\frac{2x}{\sqrt{3}a} - \frac{1}{\sqrt{3}}\frac{y}{b} = 1 \Rightarrow 2xb - ya = \sqrt{3}ab$$

It cuts x-axis at $\left(\frac{\sqrt{3}a}{2}, 0\right)$ & y-axis at $\left(0, -\sqrt{3}b\right)$

$\therefore$ area of triangle $= \frac{3}{4}ab$

$\Rightarrow 3a^2 = \frac{3}{4}ab \Rightarrow \frac{b}{a} = 4$

$\therefore e^2 = 17.$

(B) $e_1^2 = 1 + \frac{5\cos^2\theta}{5}$ & $e_2^{\,2} = 1 - \frac{25\cos^2\theta}{25}$

According to question $e_1^{\,2} = 3e_2^{\,2}$,

$1 + \cos^2\theta = 3 - 3\cos^2\theta \Rightarrow \cos^2\theta = \frac{1}{2}$

Smallest possible value of $\theta = \frac{\pi}{4}$.

Hence p = 24.

(C) Angle between asymptotes is

$$2\tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

$\therefore \frac{\pi}{3} = \frac{\ell\pi}{24} \Rightarrow \ell = 8.$

or $\frac{2\pi}{3} = \frac{\ell\pi}{24} \Rightarrow \ell = 16.$

(D) Equation of tangents on hyperbola at $P(x_1, y_1)$ is

$xy_1 + yx_1 = 16$

$\therefore$ It cuts the co-ordinate axes at

$A\left(\frac{16}{y_1}, 0\right)$ & $B\left(0, \frac{16}{x_1}\right)$

$\therefore \Delta = 16.$ $(\because x_1y_1 = 8)$

Assertion & Reason :

4. Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Hyperbola xy = 4 cut the circle at four points then

$x^2 + \frac{16}{x^2} + 2gx + \frac{8f}{x} + c = 0$

$x^4 + 2gx^3 + cx^2 + 8fx + 16 = 0$

$\Rightarrow x_1\, x_2\, x_3\, x_4 = 16$

$\Rightarrow$ 2. 4. 6. 1/4 = 12

$\Rightarrow$ statement I is false

statement II is true.

Comprehension # 1

1. Tangent of $xy = c^2$ at t_1 & t_2 are

$x + t_1^{\,2}\, y = 2ct_1$ ...(i)

and $x + t_2^{\,2}\, y = 2ct_2$ ...(ii)

on solving (i) & (ii) we get

$$y = \frac{2c}{t_1 + t_2} = \frac{2c}{4},\; x = \frac{2ct_1t_2}{t_1 + t_2} = \frac{4c}{4}$$

$\therefore$ point of intersection is $\left(c, \frac{c}{2}\right)$.

2. $e_1 = \sqrt{2}$, $e_2 = \sqrt{2}$

$\Rightarrow$ $(\sqrt{2}, \sqrt{2})$ is the point on the circle.

$\Rightarrow$ radius of $C_1 = 2$.

$\Rightarrow$ radius of director circle of $C_1 = 2\sqrt{2}$.

$\therefore$ $(\text{radius})^2 = 8$

3. Equation of normal of $xy = c^2$ at t_1 is

$$y - \frac{c}{t_1} = t_1^{\,2}(x - ct_1)$$

As it also passes through t_2,

$$\frac{c}{t_2} - \frac{c}{t_1} = t_1^{\,2}(ct_2 - ct_1)$$

$\Rightarrow t_1\, t_2 = -\, t_1^{-2}.$

EXERCISE - 04[A] CONCEPTUAL SUBJECTIVE EXERCISE

1. Point of intersection of lines

$7x + 13y - 87 = 0$ & $5x - 8y + 7 = 0$ is (5, 4).

Then $\frac{25}{a^2} - \frac{16}{b^2} = 1$ ...(i)

Also latus rectum LR $= \frac{2b^2}{a} = \frac{32\sqrt{2}}{5}$

$\Rightarrow \quad b^2 = \frac{16\sqrt{2}a}{5}$ ...(ii)

From (i) & (ii) $a^2 = \frac{25}{2}$, $b^2 = 16$.

3. Given hyperbola can be written as

$\frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$

so $e = \frac{5}{3}$, centre is (–1, 2)

foci = (–1 ± 5, 2) = (–6, 2) & (4, 2)

directrix is $x + 1 = \pm\frac{9}{5} \Rightarrow x = -1 \pm \frac{9}{5}$

L.R. $= \frac{32}{3}$, Length of axes is 8 and 6,

Equation of axis is $y - 2 = 0$ and $x + 1 = 0$.

5. Given conic can be written as

$\frac{(x-2)^2}{16} - \frac{(y-2)^2}{16} = -1$

so eccentricity is $\sqrt{2}$.

10. Equation of normal of given hyperbola at P is

$ax\cos\theta + by\cot\theta = a^2 + b^2$

As it cut x-axis at G, so G $(ae^2\sec\theta, 0)$

Now $SG = ae^2\sec\theta - ae$

$= e\,(ae\sec\theta - a) = e\,SP$

12. If (h, k) be mid point of any chord of hyperbola $x^2 - y^2 = a^2$, then its equation is

$hx - ky = h^2 - k^2$ ...(i)

But (i) is normal to hyperbola, then its equation is

$x\cos\theta + y\cot\theta = 2a$ ...(ii)

Comparing (i) & (ii)

$\frac{h}{\cos\theta} = \frac{-k}{\cot\theta} = \frac{h^2 - k^2}{2a}$

on solving it we get $(y^2 - x^2)^3 = 4a^2x^2y^2$

14. Let equation of asymptotes are

$2x^2 - 3xy - 2y^2 + 3x - y + 8 + \lambda = 0$

As it represents two straight lines

$\therefore -4(8+\lambda) + \frac{9}{4} - \frac{1}{2} + \frac{9}{2} - (8+\lambda)\frac{9}{4} = 0$

$\Rightarrow \lambda = -7$

So asymptotes are $2x^2 - 3xy - 2y^2 + 3x - y + 1 = 0$

$\Rightarrow 2y - x - 1 = 0$ & $2x + y + 1 = 0$

and the equation of conjugate hyperbola will be

$2x^2 - 3xy - 2y^2 + 3x - y + 8 - 14 = 0$.

EXERCISE - 04[B] BRAIN STORMING SUBJECTIVE EXERCISE

2. Equation of tangent of given hyperbola at point (h, k) is $\frac{hx}{a^2} - \frac{ky}{b^2} = 1$ ...(i)

Equation of auxillary circle is $x^2 + y^2 = a^2$ ....(ii)

from (i) & (ii)

$\left[\left(1 + \frac{ky}{b^2}\right)\frac{a^2}{h}\right]^2 + y^2 - a^2 = 0$

$\Rightarrow y^2\,(k^2a^4 + b^4h^2) + 2kb^2a^4y + b^4a^2\,(a^2 - h^2) = 0$

Now $\frac{y_1 + y_2}{y_1y_2} = -\frac{2kb^2a^4}{b^4a^2(a^2 - h^2)} = \frac{-2ka^2}{b^2a^2\left(1 - \frac{h^2}{a^2}\right)}$

$= \frac{-2k}{b^2\left(\frac{-k^2}{b^2}\right)} = \frac{2}{k}$.

6. Let mid point of chord of given hyperbola is (h, k)

Also let $\left(ct_1, \frac{c}{t_1}\right)$ & $\left(ct_2, \frac{c}{t_2}\right)$ be the end points of the chord

then $2h = c(t_1 + t_2)$ and $2k = c\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$

According to question

$c^2\,(t_1 - t_2)^2 + c^2\left(\frac{1}{t_1} - \frac{1}{t_2}\right)^2 = 4d^2$

$\Rightarrow c^2\,[(t_1 + t_2)^2 - 4t_1t_2]\left[1 + \frac{1}{(t_1t_2)^2}\right] = 4d^2$

$\Rightarrow c^2\left[\frac{4h^2}{c^2} - \frac{4h}{k}\right]\left[1 + \frac{k^2}{h^2}\right] = 4d^2$

$\Rightarrow (xy - c^2)(x^2 + y^2) = d^2xy$.

7. Let any point on circle be $(r \cos \theta, r \sin \theta)$

Then equation of chord of contact is

$$\frac{x}{a^2} r \cos \theta - \frac{y}{b^2} r \sin \theta = 1 \qquad \ldots(i)$$

Let mid point of chord of contact is (h, k)

Then equation of chord of contact is

$$\frac{hx}{a^2} - \frac{ky}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2} \qquad \ldots(ii)$$

On comparing (i) & (ii)

$$\frac{r\cos\theta}{h} = \frac{r\sin\theta}{k} = \frac{1}{\frac{h^2}{a^2} - \frac{k^2}{b^2}}$$

On solving we get required locus i.e.

$$\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2 = \frac{x^2 + y^2}{r^2}.$$

9. Equation of tangent to parabola $x^2 = 4ay$

is $\quad y - mx + am^2 = 0 \qquad \ldots(i)$

Let mid point of PQ is (x_1, y_1).

Then equation of PQ is

$$xy_1 + yx_1 = 2k^2 \qquad \ldots\ldots(ii)$$

On comparing (i) & (ii)

$$\frac{x_1}{1} = \frac{y_1}{-m} = \frac{2k^2}{am^2}$$

$$\Rightarrow \quad x_1 = \frac{2k^2}{am^2} \qquad \ldots\ldots.(iii)$$

$$y_1 = \frac{-2k^2}{am} \qquad \ldots\ldots.(iv)$$

using (iii) & (iv) eliminate m.

EXERCISE - 05 [A] — JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

5. $2ae = 4$

$ae = 2$

$a(2) = 2$

$a = 1$

$b^2 = a^2(e^2 - 1)$

$\quad = 1(4 - 1) = 3$

equation $\frac{x^2}{1} - \frac{y^2}{3} = 1$

$3x^2 - y^2 = 3$

1. Any point on $y^2 = 8x$ is $(2t^2, 4t)$ where the tangent is $yt = x + 2t^2$

Solving it with $xy = -1$, $y(yt - 2t^2) = -1$

or $ty^2 - 2t^2y + 1 = 0$

For common tangent, it should have equal roots

$\therefore$ $4t^2 - 4t = 0 \Rightarrow t = 0, 1$

$\therefore$ The common tangent is $y = x + 2$,

(when $t = 0$, it is $x = 0$ which can touch $xy = -1$ at infinity only)

2. The given equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

$\Rightarrow$ $a = \cos\alpha$, $b = \sin\alpha$

$\Rightarrow$ $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$

$\Rightarrow$ $ae = 1$

$\therefore$ foci $(\pm 1, 0)$

$\therefore$ foci remain constant with respect to α.

5. Eccentricity of ellipse = 3/5

Eccentricity of hyperbola = 5/3 and it passes through $(\pm 3, 0)$

$\Rightarrow$ its equation $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$

where $1 + \frac{b^2}{9} = \frac{25}{9}$ $\Rightarrow$ $b^2 = 16$

$\Rightarrow$ $\frac{x^2}{9} - \frac{y^2}{16} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$ and its foci are $(\pm 5, 0)$

6. Given $3x^2 + 4y^2 = 12$ an ellipse

$\therefore$ $a^2 = 4$ $b^2 = 3$

$\therefore$ $e = \sqrt{1 - \frac{3}{4}} \Rightarrow e = \frac{1}{2}$

$\therefore$ It's focus will be $(\pm 1, 0)$

Since hyperbola is confocal to given ellipse, therefore $\pm ae = \pm 1$, but $a = \sin\theta$ given

$\therefore$ $e = \frac{1}{\sin\theta}$, Now $b^2 = a^2(e^2 - 1)$

$b^2 = \sin^2\theta \frac{\cos^2\theta}{\sin^2\theta} \Rightarrow b^2 = \cos^2\theta$

Hence required equation will be,

$$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

$\Rightarrow$ $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

8. $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

either $x^2 - 5xy + 6y^2 = 0$ $\Rightarrow$ two straight lines passing through origin.

or $ax^2 + by^2 + c = 0$

(A) If $c = 0$, and a and b are of same sign then it will represent a point.

(B) If $a = b$, c is of sign opposite to a then it will represent circle.

(C) When a & b are of same sign and c is of sign opposite to a then it will represent ellipse.

(D) This is clearly incorrect.

9. The given equation is

$(x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$

$$\frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$$

$a = 2$, $b = \sqrt{2}$

hence eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$

Area $= \frac{1}{2} a(e - 1) \quad \frac{b^2}{a}$

$= \left(\sqrt{\frac{3}{2}} - 1\right)$ sq. units.

10. $x^2 - y^2 = \frac{1}{2}$ ...(i) $\rightarrow$ its $e = \sqrt{2}$

e of ellipse is $\frac{1}{\sqrt{2}}$

$\frac{x^2}{2} + \frac{y^2}{1} = b^2$ ...(ii)

add (i) & (ii) $\frac{3x^2}{2} = \frac{1}{2} + b^2$

$3x^2 = 1 + 2b^2$

$y^2 = \frac{1}{3} + \frac{2h^2}{3} - \frac{1}{6}$

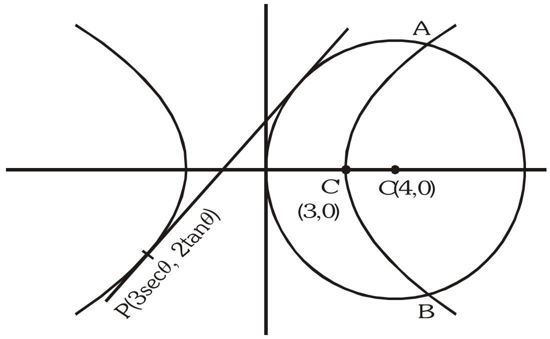
$y^2 = \frac{1}{6}(4b^2 - 1)$

$m_1 \cdot m_2 = -1 \Rightarrow \frac{1+2b^2}{3} = \frac{2(4b^2-1)}{6}$

$b^2 = 1 \Rightarrow x^2 + 2y^2 = 2.$

Paragraph for Question 11 and 12

11. Let the point on the hyperbola P(3secθ, 2tanθ)



Equation of tangent $\frac{x\sec\theta}{3}-\frac{y\tan\theta}{2}=1$

$|p| = r$

$$\frac{\left|\frac{4}{3}\sec\theta-1\right|}{\sqrt{\frac{\sec^2\theta}{9}+\frac{\tan^2\theta}{4}}}=4$$

$\Rightarrow \quad \frac{16}{9}\sec^2\theta+1-\frac{8}{3}\sec\theta=16\left(\frac{4\sec^2\theta+9\tan^2\theta}{4\times 9}\right)$

$16\sec^2\theta + 9 - 24\sec\theta = 52\sec^2\theta - 36$

$\Rightarrow \quad 36\sec^2\theta + 24\sec\theta - 45 = 0$

$\Rightarrow \quad 12\sec^2\theta + 8\sec\theta - 15 = 0$

$\Rightarrow \quad 12\sec^2\theta + 18\sec\theta - 10\sec\theta - 15 = 0$

$\Rightarrow \quad (6\sec\theta - 5)(2\sec\theta + 3) = 0$

$\sec\theta = \frac{5}{6}$ (not possible), $\sec\theta = -\frac{3}{2}$

$\tan\theta = \pm\sqrt{\frac{9}{4}-1} = \pm\frac{\sqrt{5}}{2}$

($\because$ slope is positive $\Rightarrow \tan\theta = -\frac{\sqrt{5}}{2}$)

Hence the required equation be $-\frac{3x}{2\times 3}+\frac{y\sqrt{5}}{2\times 2}=1$

$\Rightarrow \quad 2x-\sqrt{5}y+4=0$

12. Solving (a) & (b) for x, we get

$x = 6$

$y = \pm 2\sqrt{3}$

$(x - 6)^2 + y^2 - 12 = 0$

$x^2 + y^2 - 12x + 24 = 0$

Option (A) is correct

13. As directrix cut the x–axis at (± a/e, 0)

Hence, $\frac{2a}{e} + 0 = 1$ (for nearer directrix)

$\Rightarrow \quad 2a = e$ ...(i)

Now, $b^2 = a^2(e^2 - 1) = a^2(4a^2 - 1)$

$\Rightarrow \quad \frac{b^2}{a^2} = 4a^2 - 1$ ...(ii)

Given line $y = -2x + 1$ is a tangent to the hyperbola

condition of tangency is $c^2 = a^2m^2 - b^2$

$\Rightarrow \quad 1 = 4a^2 - b^2$

$\Rightarrow \quad 4a^2 - 1 = b^2$ ...(iii)

from (ii) & (iii), $a^2 = 1$

$\Rightarrow$ from (ii), $b^2 = 3$

$\Rightarrow \quad e=\sqrt{\frac{1+3}{1}}=2$

14. Given hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

ellipse is $\frac{x^2}{2^2}+\frac{y^2}{1}=1$

eccentricity of ellipse $=\sqrt{1-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

eccentricity of hyperbola $=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{4}{3}}$

$\Rightarrow \quad \frac{b^2}{a^2}=\frac{1}{3} \quad \Rightarrow \quad 3b^2 = a^2$ ....(1)

also hyperbola passes through foci of ellipse $(\pm\sqrt{3},0)$

$\frac{3}{a^2}=1 \quad \Rightarrow \quad a^2 = 3$ ....(2)

from (1) & (2)

$b^2 = 1$

equation of hyperbola is $\frac{x^2}{3}-\frac{y^2}{1}=1$

$\Rightarrow \quad x^2 - 3y^2 = 3$

eccentricity of hyperbola $=\sqrt{1+\frac{1}{3}}=\sqrt{\frac{4}{3}}$

focus of hyperbola $=\left(\pm\sqrt{3}.\frac{2}{\sqrt{3}},\ 0\right)\equiv(\pm 2,\ 0)$

15. Equation of normal at P(6, 3) on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$\frac{a^2x}{6}+\frac{b^2y}{3}=a^2e^2$

It intersects x-axis at (9, 0)

$\Rightarrow \quad a^2\frac{9}{6}=a^2e^2 \quad \Rightarrow \quad e=\sqrt{\frac{3}{2}}$

16. Let parametric coordinates be P(3secθ, 2 tanθ)

Equation of tangent at point P will be

$\frac{x\sec\theta}{3}-\frac{y\tan\theta}{2}=1$

$\because$ tangent is parallel to $2x - y = 1$

$\Rightarrow \quad \frac{2\sec\theta}{3\tan\theta}=2 \quad \Rightarrow \quad \sin\theta=\frac{1}{3}$

$\therefore$ coordinates are

$\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{2}\right)$