EXERCISE - 01

CHECK YOUR GRASP

1. Req. mean

$$= \frac{a+(a+d)+(a+2d)+......+\{a+(n-1)d\}}{n}$$

$$= \frac{\frac{n}{2}[a+a+(n-1)d]}{n} = a+\frac{(n-1)d}{2}$$

5. $\because \quad \frac{x_1+x_2+....+x_n}{n} = \bar{x}$ ...(i)

$\therefore$ Req. sum = $(x_1 - \bar{x}) + (x_2 - \bar{x}) + ... + (x_n - \bar{x})$

$= (x_1 + x_2 + ... + x_n) - n\bar{x}$

$= n\bar{x} - n\bar{x} = 0$ (from eq. (i))

6. Let new term is x

$\because \quad \frac{9 \times 15 + x}{10} = 16$

$\Rightarrow \quad x = 25$

7. $\because \quad \frac{1+2+3+......+n}{n} = \frac{n+7}{3}$

$\Rightarrow \quad \frac{n+1}{2} = \frac{n+7}{3}$

$\Rightarrow \quad n = 11$

8. Req. mean$= \frac{3 \times 14 + 2 \times 18}{5} = 15.6 \quad \left(\because \sum_{i=1}^{n} x_i = n\bar{x}\right)$

9. $\because \quad \frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5} = 11$

$\Rightarrow \quad \frac{5x+20}{5} = 11$

$\Rightarrow \quad x = 7$

$\therefore$ Req. mean = $\frac{11+13+15}{3} = 13$

11. $\because$Sum of the 50 observations = 36 50 = 1800

Two observations 30 and 42 are deleted

Sum of the remaining 48 observations

= 1800 – [30 + 42] = 1728

Req. mean = $\frac{1728}{48}$ = 36

13. Sum of n observations = $n\bar{x}$

sum of (n – 4) observations = k

sum of remaining 4 observations = $n\bar{x} - k$

req. mean = $\frac{n\bar{x} - k}{4}$

16. $\because \quad \Sigma f_i x_i = 5 + 8 + 3f + 8 + 15 = 3f + 36$

$\Sigma f_i = 5 + 4 + f + 2 + 3 = f + 14$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 2.6 = \frac{3f+36}{f+14}$

$\Rightarrow \quad f = 1$

20. Let the no. of boys and girls are n_1 and n_2 resp., then

$$\frac{65n_1 + 55n_2}{n_1 + n_2} = 61$$

$\Rightarrow \quad 4n_1 = 6n_2$

$\Rightarrow \quad n_1 : n_2 = 3 : 2$

24. Let $x_1, x_2, ... x_{n_1}$ and $y_1, y_2, ... y_{n_2}$ are two series of size n_1 and n_2 resp.

$G_1 = (x_1 \times x_2 \times .... \times x_{n_1})^{1/n_1}$ ...(1)

$G_2 = (y_1 \times y_2 \times .... \times y_{n_2})^{1/n_2}$ ...(2)

and $G = [(x_1 \times x_2 \times ... \times x_{n_1}) \times (y_1 \times y_2 \times ... y_{n_2})]^{\frac{1}{n_1+n_2}}$

$G = (G_1^{n_1} \times G_2^{n_2})^{1/n_1+n_2}$ [from (1) & (2)]

$\therefore \quad \log G = \frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$

28. Let the distance between home and school = S km

time consumed to reach school from home = $\frac{S}{x}$ hr.

time consumed to reach home from school = S/y hr.

Average speed = $\frac{\text{total covered distance}}{\text{time consume to covered total distance}}$

$$= \frac{S+S}{\frac{S}{x}+\frac{S}{y}} = \frac{2}{\frac{1}{x}+\frac{1}{y}} = \frac{2xy}{x+y} \text{ km/hr.}$$

31. We arranged the value of variate x_i in ascending order with their frequencies

x_i	3	6	7	10	12	15
c.f.	3	7	20	22	30	40

Here N = 40 (even)

$$\therefore \text{ Median } = \frac{\left(\frac{N}{2}\right)\text{th term} + \left(\frac{N}{2}+1\right)\text{th term}}{2}$$

$$= \frac{(20)\text{th term} + (21)\text{th term}}{2}$$

$$= \frac{7+10}{2} = 8.5$$

35. First we changed the classes into continuous form, we have

Class	0.5–10.5	10.5–20.5	20.5–30.5	30.5–40.5	40.5–50.5
f_i	5	7	8	6	4

Here model class is 20.5 – 30.5

$$\therefore \quad \text{Mode} = \ell + \frac{(f_0 - f_1)}{(2f_0 - f_1 - f_2)} \times h$$

$$= 20.5 + \frac{(8-7)}{(16-7-6)} \quad 10 = 23.83$$

40. $\because \quad \bar{x} = \frac{-1+0+4}{3} = 1$

$\Sigma|x_i - \bar{x}| = 2 + 1 + 3 = 6$

Mean deviation $= \frac{1}{n}\Sigma|x_i - \bar{x}| = \frac{1}{3} \quad 6 = 2$

41. First we arranged the observations in ascending order

34, 38, 42, 44, 46, 48, 54, 55, 63, 70

Here n = 10 (even)

$$\therefore \text{ Median (M)} = \frac{\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}}{2}$$

$$= \frac{46+48}{2} = 47$$

$\therefore \Sigma|x_i - M| = 13+9+5+3+1+1+7+8+16 + 23 = 86$

$\therefore$ Mean deviation from Median

$$= \frac{\Sigma|x_i - M|}{n} = 8.6$$

44. Here $u = \frac{x}{h} - \frac{a}{h}$

$\because$ S.D. is not depend on change of origin but it is depend on change of scale.

$$\therefore \quad \sigma_u = \frac{\sigma_x}{h}$$

45. $\because$ S.D. $= \sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2}$

So, S.D. of first n natural numbers

$$= \sqrt{\frac{1}{n}\Sigma n^2 - \left(\frac{1}{n}\Sigma n\right)^2}$$

$$= \sqrt{\frac{1}{n}.\frac{n(n+1)(2n+1)}{6} - \left\{\frac{1}{n}.\frac{n(n+1)}{2}\right\}^2}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}} = \sqrt{\frac{(n+1)(n-1)}{12}}$$

$$= \sqrt{\frac{n^2-1}{12}}$$

46. Given observations 112, 116, 120, 125, 132

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{605}{5} = 121$$

$\Sigma(x_i - \bar{x})^2 = 81 + 25 + 1 + 16 + 121 = 244$

$$\therefore \quad \sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{244}{5} = 48.8$$

49. Given series a, a + d, a + 2d, ... , a + 2nd

$$\bar{x} = \frac{\frac{(2n+1)}{2}[a + a + 2nd]}{(2n+1)} = a + nd$$

$\Sigma(x_i - \bar{x})^2 = n^2d^2 + (1 - n)^2d^2 + ... + d^2 + 0 + d^2 + ... + n^2d^2$

$= 2[n^2d^2 + (n - 1)^2d^2 + ... + d^2]$

$= 2d^2[n^2 + (n - 1)^2 + ... + 1^2]$

$$= 2d^2 \times \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(2n+1)}{3}d^2$$

$$\therefore \quad \text{Variance} = \frac{\Sigma(x_i - \bar{x})^2}{2n+1} = \frac{n(n+1)}{3}d^2$$

55. $\because \quad \sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

Let $y_i = \frac{ax_i + b}{c} = \left(\frac{a}{c}\right)x_i + \frac{b}{c}$

$\Rightarrow \quad \bar{y} = \left(\frac{a}{c}\right)\bar{x} + \frac{b}{c} \quad \Rightarrow \quad y_1 - \bar{y} = \frac{a}{c}(x_i - \bar{x})$

$\Rightarrow$ Req. S.D. $= \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n}}$

$$= \sqrt{\frac{\Sigma\left[\frac{a^2}{c^2}(x_i - \bar{x})^2\right]}{n}} = \left|\frac{a}{c}\right|\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \left|\frac{a}{c}\right|\sigma$$

EXERCISE - 02 BRAIN TEASERS

2. $\because \quad \dfrac{1^2+2^2+3^2+---+n^2}{n} = \dfrac{46n}{11}$

$\Rightarrow \quad \dfrac{n(n+1)(2n+1)}{6n} = \dfrac{46n}{11}$

$\Rightarrow \quad 11(n+1)(2n+1) = 276n$

$\Rightarrow \quad 22n^2 - 243n + 11 = 0$

$\Rightarrow \quad (n-11)(22n-1) = 0$

$\therefore \quad n = 11 \qquad \because \quad n \neq \dfrac{1}{22}$

3. Let the weights of each natural number is W, then

weighted Mean $= \dfrac{1\times W + 2\times W + \ldots\ldots + n\times W}{W + W + \ldots\ldots + W}$

$= \dfrac{n(n+1)\dfrac{W}{2}}{nW} = \dfrac{n+1}{2}$

7. Let $x_1, x_2, ---- x_n$ are n positive numbers such that $x_1 \; x_2 \; ---- \; x_n = 1$ ...(1)

$\because$ A.M. $\geq$ G.M.

So $\dfrac{x_1+x_2+---+x_n}{n} \geq (x_1 \; x_2 \; ---- \; x_n)^{1/n}$

$\Rightarrow \quad x_1 + x_2 + \ldots\ldots + x_n \geq n$ by (1)

8. $\because \quad T_n = (2n-1)(2n+1)(2n+3)$

Req. A.M. $= \dfrac{S_n}{n} = \dfrac{\Sigma T_n}{n} = \dfrac{1}{n}[\Sigma(8n^3+12n^2-2n-3)]$

$= \dfrac{1}{n}[8\Sigma n^3 + 12\Sigma n^2 - 2\Sigma n - \Sigma 3]$

$= 2n^3 + 8n^2 + 7n - 2$

11. $\because \quad \sigma = \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2} = \sqrt{\dfrac{\Sigma x_i^2}{n} - \overline{x}^2}$

$\Rightarrow \quad \Sigma x_i^2 = n(\sigma^2 + \overline{x}^2)$

13. $\because \quad \dfrac{x_1 + x_2 + \ldots.. + x_{10}}{10} = 20$ ...(1)

Req. mean $= \dfrac{(x_1+4)+(x_2+8)+\ldots+(x_{10}+40)}{10}$

$= \dfrac{(x_1+x_2+\ldots.+x_{10})}{10} + \dfrac{4(1+2+\ldots.+10)}{10}$

$= 20 + 22 = 42$ [by eq. (1)]

14. $\text{W.M.} = \dfrac{0\times 1 + 1\times {}^nC_1 + 2\times {}^nC_2 + \ldots\ldots + n\times {}^nC_n}{1 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n}$

$= \dfrac{1\times {}^nC_1 + 2\times {}^nC_2 + \ldots\ldots + n\times {}^nC_n}{{}^rC_0 + {}^nC_1 + \ldots\ldots + {}^nC_n} = \dfrac{\sum\limits_{r=1}^{n} r.{}^nC_r}{2^n}$

$= \dfrac{\sum\limits_{r=1}^{n} r.\dfrac{n}{r}.{}^{n-1}C_{r-1}}{2^n} = \dfrac{n\sum\limits_{r=1}^{n} .{}^{n-1}C_{r-1}}{2^n} = \dfrac{n.2^{n-1}}{2^n} = \dfrac{n}{2}$

16. On arranging the values in the ascending order

$\alpha - \dfrac{7}{2}, \alpha - 3, \alpha - \dfrac{5}{2}, \alpha - 2, \alpha - \dfrac{1}{2}, \alpha + \dfrac{1}{2},$

$\alpha + 4, \alpha + 5$ ($\because \alpha > 0$)

Here number of observations n = 8 (even)

Median $= \dfrac{1}{2}\left[\left(\dfrac{n}{2}\right)\text{th obser.} + \left(\dfrac{n}{2}+1\right)\text{th obser.}\right]$

$= \dfrac{1}{2}\left[(\alpha-2) + \left(\alpha - \dfrac{1}{2}\right)\right] = \alpha - \dfrac{5}{4}$

17. For S.D. of first n odd natural numbers

$1, 3, 5, \ldots., (2n-1)$

$\Sigma x_i = 1 + 3 + 5 + \ldots.. + (2n-1) = n^2$

$\Sigma x_i^2 = 1^2 + 3^2 + 5^2 + \ldots. + (2n-1)^2$

$= [1^2 + 2^2 + 3^2 + \ldots. + (2n)^2] - [2^2 + 4^2 + \ldots. + (2n)^2]$

$= [1 + 2^2 + 3^2 + \ldots + (2n)^2] - 2^2[1^2 + 2^2 + \ldots + n^2]$

$= \dfrac{2n(2n+1)(4n+1)}{6} - 4 \,.\, \dfrac{n(n+1)(2n+1)}{6}$

$= \dfrac{n(2n+1)}{3}[(4n+1) - 2(n+1)]$

$= \dfrac{n(2n+1)(2n-1)}{3} = \dfrac{n(4n^2-1)}{3}$

$\therefore$ Req. S.D. $= \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2}$

$= \sqrt{\dfrac{4n^2-1}{3} - n^2} = \sqrt{\dfrac{n^2-1}{3}}$

19. Let $\overline{x} = \dfrac{x_1 + x_2 + \ldots.. + x_n}{n}$

New Mean $= \dfrac{(x_1+1)+(x_2+2)+\ldots..+(x_n+n)}{n}$

$= \dfrac{(x_1+x_2+\ldots..+x_n)}{n} + \dfrac{(1+2+\ldots..+n)}{n} = \overline{x} + \dfrac{n+1}{2}$

20. $\because \quad \dfrac{x_1 + x_2 + x_3 + --- + x_n}{n} = \overline{x}$ ...(i)

When x_2 is replaced by λ then

New Mean $= \dfrac{x_1 + \lambda + x_3 + --- + x_n}{n}$

$= \dfrac{(x_1+x_2+x_3+---+x_n)+\lambda-x_2}{n} = \dfrac{n\overline{x} + \lambda - x_2}{n}$ by (i)

21. $\because \quad G_1 = (x_1 \; x_2 \; \ldots. \; x_n)^{1/n}$

and $\quad G_2 = (y_1 \; y_2 \; \ldots. \; y_n)^{1/n}$

$\therefore \quad G = \left(\dfrac{x_1}{y_1} \times \dfrac{x_2}{y_2} \times \ldots\ldots \times \dfrac{x_n}{y_n}\right)^{1/n}$

$= \dfrac{(x_1 \times x_2 \times \ldots\ldots \times x_n)^{1/n}}{(y_1 \times y_2 \times \ldots\ldots \times y_n)^{1/n}} = \dfrac{G_1}{G_2}$

24. Let man spends x Rs. on purchasing each kind of pens at the rate 5 Rs/pen, 10 Rs/pen, 20 Rs/pen

$$\text{Average cost} = \frac{\text{Total cost}}{\text{no. of total pens}}$$

$$= \frac{x + x + x}{\frac{x}{5} + \frac{x}{10} + \frac{x}{20}} = \frac{3}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} = \frac{60}{7} \text{ Rs/pen}$$

27. Here N = 10, $u_i = \frac{x_i - 25}{10}$

$\therefore \Sigma f_i u_i = 1 \quad (-2) + 3 \quad (-1) + 4 \quad 0 + 2 \quad 1 = -3$

and, $\Sigma f_i u_i^2 = 1 \quad 4 + 3 \quad 1 + 4 \quad 0 + 2 \quad 1 = 9$

$$\sigma = h\sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} = 10\sqrt{\frac{9}{10} - \left(\frac{-3}{10}\right)^2} = 9$$

29. Let the set of n observations $x_1, x_2, \ldots\ldots x_n$

$$\therefore \quad \bar{x} = \frac{\Sigma x_i}{n}$$

$$\text{Mean of new set} = \frac{\Sigma\left(\frac{x_i}{\alpha} + 10\right)}{n} = \frac{1}{\alpha}\frac{\Sigma x_i}{n} + \frac{10n}{n} = \frac{\bar{x} + 10\alpha}{\alpha}$$

30. Let the age of each student is x years then the age of teacher is (x + 20) years

$$\frac{(x + 20) + 3x}{4} = 20 \quad \Rightarrow \quad x = 15$$

Hence age of the teacher = 35 years

32. Here number of terms = n + 1 (odd)

$$\therefore \text{ Median} = \left(\frac{n+2}{2}\right)\text{th term} = \left(\frac{n}{2} + 1\right)\text{th term} = {}^{2n}C_{n/2}$$

33. $\because$ M.D. from mean of the series

a, a + d, a + 2d, ... , a + 2nd is $\text{M.D.} = \frac{n(n+1)}{2n+1}|d|$

for given series 5, 10, 15, ... , 85

a = 5, d = 5, a+ 2nd = 85 i.e. n = 8

$$\therefore \text{ M.D.} = \frac{8 \times 9}{17} \quad 5 = 21.17$$

35. $\because$ S.D. of the A.P. a, a + d, a + 2d, ... a + 2nd

$$\text{S.D.} = \sqrt{\frac{n(n+1)}{3}}\ |d|$$

Given numbers 31, 32, 33, ... 47 are in A.P.

Here a = 31, d = 1 and a + 2nd = 47 i.e. n=8

$$\therefore \text{ S.D.} = \sqrt{\frac{8 \times 9}{3}} \times 1 = 2\sqrt{6}$$

36. Given $\Sigma(x_i - \bar{x})^2 = 250$, n = 10, $\bar{x} = 50$

$$\because \quad \sigma = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{1}{10} \times 250} = 5$$

Hence coefficient of variation

$$= \frac{\sigma}{\bar{x}} \quad 100 = \frac{5}{50} \quad 100 = 10\%$$

38. Let the number of terms of two series are n_1 and n_2 whose means are $\bar{x}_1$ and $\bar{x}_2$ resp.

$$\because \quad \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\text{Now} \quad \bar{x} - \bar{x}_1 = \frac{n_2(\bar{x}_2 - \bar{x}_1)}{(n_1 + n_2)} > 0 \qquad (\because \ \bar{x}_2 > \bar{x}_1)$$

$\Rightarrow \quad \bar{x} > \bar{x}_1 \quad$ ...(i)

$$\text{Again} \quad \bar{x} - \bar{x}_2 = \frac{n_1(\bar{x}_1 - \bar{x}_2)}{(n_1 + n_2)} < 0 \qquad (\because \ \bar{x}_1 < \bar{x}_2)$$

$\Rightarrow \quad \bar{x} < \bar{x}_2 \quad$ ...(ii)

by (i) & (ii) $\quad \bar{x}_1 < \bar{x} < \bar{x}_2$

40. Arrange the given observations in ascending order

40, 54, 62, 68, 76, 90

no. of terms (n) = 6 (even)

$\therefore$ Median (M)

$$= \frac{\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2} + 1\right)\text{th term}}{2} = \frac{62 + 68}{2} = 65$$

$\Sigma|x_i - M| = 25 + 11 + 3 + 3 + 11 + 25 = 78$

$$\text{M.D. from median} = \frac{\Sigma|x_i - M|}{n} = \frac{78}{6} = 13$$

$$\therefore \text{ Coefficient of M.D.} = \frac{\text{M.D.}}{\text{median}} = \frac{13}{65} = 0.2$$

41. Here $n_1 = 10$, $\bar{x}_1 = 5$, $\sigma_1 = 2\sqrt{6}$ (for I group)

$n_2 = 20$, $\bar{x}_2 = 5$, $\sigma_2 = 3\sqrt{2}$ (for II group)

S.D. of combined group

$$\sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\bar{x}_1 - \bar{x}_2)^2}$$

$$= \sqrt{\frac{10 \times 24 + 20 \times 18}{30} + 0} \qquad (\because \bar{x}_1 = \bar{x}_2)$$

$$= \sqrt{20} = 2\sqrt{5}$$

42. Since mean deviation is minimum when it is taken by median, so here K is median of given observations.

$$K = \text{median} = \left(\frac{n+1}{2}\right)\text{th observation}$$

$= 51^{th}$ observation

$\therefore \quad K = x_{51}$

43. $\text{S.D.} = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$ and $\text{M.D.} = \frac{\Sigma|x_i - \bar{x}|}{n}$

let $\quad |x_i - \bar{x}| = y_i$

i.e. $(x_i - \bar{x})^2 = |x_i - \bar{x}|^2 = y_i^2$

$$\text{Now} \quad (\text{S.D.})^2 - (\text{M.D.})^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} - \left(\frac{\Sigma|x_i - \bar{x}|}{n}\right)^2$$

$$= \frac{\Sigma y_i^2}{n} - \left(\frac{\Sigma y_i}{n}\right)^2 = \sigma_y^2 > 0$$

($\because$ σ^2 is non negative and all values are not same)

$\Rightarrow \quad (\text{S.D.})^2 - (\text{M.D.})^2 > 0$

$\Rightarrow \quad$ S.D. > M.D.

EXERCISE - 03 PREVIOUS YEAR QUESTION

2. Given n = 15, $\Sigma x = 170$, $\Sigma x^2 = 2830$

Since one observation 20 was found be wrong and it replaced by its correct value 30

correct sum of observation = 170 – 20 + 30 = 180

correct sum of squares of obsorvations

$= 2830 - 20^2 + 30^2 = 3330$

The correct variance $= \frac{3330}{15} - \left(\frac{180}{15}\right)^2 = 78$

6. Number of observations = 2n

and observations are

a, a, ----- n times, –a, –a, ---- n times

Here $\bar{x} = \frac{na + (-na)}{2n} = 0$

$\because$ S.D.$= \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x_i^2}{2n}} \quad (\because \bar{x} = 0)$

$2 = \sqrt{\frac{2na^2}{2n}} = |a| \qquad (\because \text{S.D.} = 2)$

10. Population A has 100 obser 101, 102, ... 200 and variance V_A.

Population B has 100 obser. 151, 152, ... 250 i.e. (101 + 50), (102 + 50), ... (200 + 50) and variance V_B

$\because$ Variance is independent of change of origin.

i.e. $V_B = V_A \Rightarrow \frac{V_A}{V_B} = 1$

11. Let number of boys and girls are n_1 and n_2 resp.

$\because \quad \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 50 = \frac{52n_1 + 42n_2}{n_1 + n_2}$

$\Rightarrow \quad n_1 = 4n_2 \qquad ...(1)$

$\therefore$ Percentage of boys $= \frac{n_1}{n_1 + n_2}$ 100 = 80 [using (1)]

12. $\because \quad \frac{\Sigma x_i}{n} = \bar{x}$

$\Rightarrow \quad \frac{a + b + 8 + 5 + 10}{5} = 6 \Rightarrow a + b = 7 \quad ...(1)$

and $\quad \frac{\Sigma x_i^2}{n} - \bar{x}^2 = \sigma^2$

$\Rightarrow \quad \frac{a^2 + b^2 + 64 + 25 + 100}{5} - 36 = 6.8$

$\Rightarrow \quad a^2 + b^2 = 25 \qquad ...(2)$

On solving equation (1) & (2) a = 4, b = 3 or a = 3, b = 4

13. Sum of first n even natural numbers

$\Sigma x_i = 2 + 4 + 6 + ... + 2n$

$= 2[1 + 2 + 3 + ... + n] = n(n + 1)$

Sum of square of first n even natural numbers

$\Sigma x_i^2 = 2^2 + 4^2 + 6^2 + ... + (2n)^2$

$= 2^2[1^2 + 2^2 + 3^2 + ... + n^2]$

$= \frac{2n(n+1)(2n+1)}{3}$

Variance of first n even natural numbers

$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$= \frac{2(n+1)(2n+1)}{3} - (n + 1)^2 = \frac{n^2 - 1}{3}$

So, statement-1 is false and statement-2 is true.

14. $\because$ M.D. of A.P. a, a + d, a + 2d, ... a + 2nd about mean is

M.D. $= \frac{n(n+1)}{2n+1}|d|$

given numbers 1, 1 + d, 1 + 2d, ... 1 + 100d are in A.P. and its M.D. is 255.

Here a = 1, 2nd = 100 d i.e. n = 50

$\therefore \quad \frac{50 \times 51}{101}|d| = 255 \quad \Rightarrow \quad d = 10.1$

15. Here $n_1 = n_2 = 5$, $\sigma_1^2 = 4$, $\sigma_2^2 = 5$, $\bar{x}_1 = 2$, $\bar{x}_2 = 4$

Variance of combined set

$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\bar{x}_1 - \bar{x}_2)^2$

$= \frac{5 \times 4 + 5 \times 5}{(5 + 5)} + \frac{5 \times 5}{(5 + 5)^2}(2 - 4)^2 = \frac{9}{2} + 1 = \frac{11}{2}$

16. Mediam = 25.5 a

x_i : a 2a 3a .... 50 a

$|x_i - M|$: 24.5a 23.5a 24.5a

$\sum |x_i - M| = [24.5a + 23.5\,a + ... + 0.5\,a + 0.5\,a + ... + 24.5a]$

$= 2a\,[0.5 + 1.5 + ... + 24.5]$

$\sum |x_i - M| = 25 \quad 25\,a$

M. D. $= \frac{25 \times 25a}{50}$

$50 = \frac{25 \times 25a}{50} \Rightarrow a = 4$

17. $\bar{x} = \text{mean} = \frac{\Sigma x_i}{n}$

mean of x_i is given as 30 gm

If each data is increased by some number (i.e. 2) the mean is also increased by 2.

i.e. corrected mean = 30 + 2 = 32 gm

σ = standard deviation $= \sqrt{\frac{\Sigma|x_i - \bar{x}|^2}{n}}$

standard deviation does not depend on change of origin so if every data is increased by same number (i.e. 2) then standard deviation remains same.

So the corrected standard deviation = 2gm.